Midterm Exam. Take 3.

Professor Lutz Hendricks – Econ720. Spring 2009

- This is a takehome exam. To be handed in on Monday, Nov 9, by noon.
- Each student is to work on this alone.
- You can earn up to 20 additional points for your midterm grade.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 A growing economy

Consider the standard growth model with leisure. The population size is $N_t = (1+n)^t$. The representative household solves

$$\max \sum_{t=0}^{\infty} \beta^t \; u(c_t, l_t) \tag{1}$$

s.t.

$$k_{t+1}(1+n) = R_t k_t + w_t (1-l_t) - c_t \tag{2}$$

where c is per capita consumption, l is per capita leisure, and k is per capita capital. The firm is standard with constant returns to scale production function $F(K_t, A_t L_t)$ where $A_t = (1+g)^t$. Capital depreciates at rate δ. Assume the utility function $u(c,l) = \frac{c^{1-\sigma} \; l^{\rho(1-\sigma)}}{1-\sigma}$ in what follows. Assume $\sigma > 1$.

1. Find the balanced growth rates of k, c, l, w, R, assuming a balanced growth path exists.
2. Define a *stationary* competitive equilibrium (emphasis on stationary).
3. Define a steady state of the stationary version of the economy.
4. Which steady state condition restricts the set of preferences required for a steady state to exist? Show that the preferences assumed satisfy this condition, while $u(c,l) = c^{1-\sigma} + \phi l^{1-\sigma}$ does not.
5. Derive the consumption growth rate as a function of current and future prices.
6. How does a change in the interest rate affect consumption growth? Compare variable leisure ($\rho > 0$) with fixed leisure ($\rho = 0$). In which case does consumption growth respond more to a given change in the interest rate? Provide intuition. Assume that wages are constant.
7. How does a change in wage growth affect consumption growth? Again, compare the cases of $\rho > 0$ and $\rho = 0$. In which case is the response of consumption growth larger? Explain the intuition.

2 Answer: A growing economy

[Due to Rajesh Singh]

1. Balanced growth rates: $g(k) = g(c) = g(w) = g$. $g(l) = g(R) = 0$.

2. Define detrended variables: $\hat{c}_t = c_t / g^t$ etc. Then the household solves

$$\max \sum_t \beta^t u(\hat{c}_t A_t, l_t) = \sum_t \hat{\beta}^t u(\hat{c}_t, l_t) \tag{3}$$

where $\hat{\beta} = \beta (1+g)^{1-\sigma}$ subject to

$$\hat{k}_{t+1}(1+n)(1+g) = R_t \hat{k}_t + \hat{w}_t (1 - l_t) - \hat{c}_t \tag{4}$$

The firm solves

$$\max F\left(\hat{k}_t, \hat{n}_t\right) - \hat{w}_t n_t - q_t \hat{k}_t \tag{5}$$

where $\hat{n} = L/AN$.

Market clearing requires

$$\begin{aligned} F\left(\hat{k}_t, \hat{n}_t\right) + (1-\delta)\hat{k}_t &= \hat{k}_{t+1}(1+g)(1+n) + \hat{c}_t \qquad (6) \\ \hat{n} &= 1 - l \qquad (7) \end{aligned}$$

Stationary CE: Allocation and prices that satisfy household FOC, firm FOC, market clearing, $R = q + 1 - \delta$.

Household FOCs:

$$\begin{aligned} u_c(\hat{c}_t, l_t) &= \hat{\beta} R_{t+1} u_c(\hat{c}_{t+1}, l_{t+1}) \qquad (8) \\ \frac{u_l(\hat{c}_t, l_t)}{u_c(\hat{c}_t, l_t)} &= \hat{w}_t \qquad (9) \end{aligned}$$

Firm FOCs: factor prices equal marginal products.

3. Steady state: $\left(\hat{k}, \hat{c}, l, \hat{w}, q, R\right)$ that satisfy: Household: $\hat{\beta} R = 1$ and static condition. Firm: 2 FOCs. Markets:

$$F\left(\hat{k}, 1 - l\right) = \delta \hat{k} + \hat{c}$$

and $R = q + 1 - \delta$.

4. The condition is $u_c / u_l = 1/\hat{w}$. With the assumed preferences:

$$\frac{u_l}{u_c} = \rho \frac{\hat{c}}{l}$$

which is stationary. With the alternative preferences (not stationary)

$$\frac{u_l}{u_c} \phi (c/l)^\sigma$$

which does not grow at rate g.

5. Derive consumption growth by combining Euler and static conditions. $u_c = \hat{c}^{-\sigma} l^{\rho(1-\sigma)}$. $u_c/u_l = \frac{l}{\rho \hat{c}} = \frac{1}{\hat{w}}$. Thus, $u_c = \hat{c}^{-\gamma} \ (\rho/\hat{w})^{\rho(1-\sigma)}$ where $\gamma = \sigma + (\sigma - 1)\rho > \sigma$. Consumption growth

$$\frac{u_c\left(\hat{c}, \hat{l}\right)}{u_c\left(\hat{c}', \hat{l}'\right)} = \left(\frac{\hat{c}'}{\hat{c}}\right)^{\gamma} \ \left(\frac{\hat{w}'}{\hat{w}}\right)^{\rho(1-\sigma)} = \hat{\beta} R'$$

6. Higher ρ implies larger γ and smaller change in consumption growth. Intuition: leisure and consumption are complements in the sense that lower leisure raises u_c. When R rises, the household postpones both c and l. He postpones c by less because the lower l makes this more costly.

7. Wage growth reduces consumption growth when $\rho > 0$. Intuition: Without leisure, wages do not affect consumption growth (pure income effect). With leisure, wage growth induces higher leisure today. Leisure complements consumption, which therefore rises.