Midterm Exam. Econ602. Spring 2008

Professor Lutz Hendricks

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:45 hours.

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1 Education Costs

[40 points] Consider the following version of a standard growth model with human capital. The planner solves

$$\max \sum_{t=1}^{\infty} \beta^t u(c_t) \quad (1)$$

s.t.

$$k_{t+1} = (1-\delta) k_t + x_{kt} \quad (2)$$

$$h_{t+1} = (1-\delta) h_t + x_{ht} \quad (3)$$

$$c_t + x_{kt} + \eta x_{ht} = f(k_t, h_t) \quad (4)$$

with k_1 and h_1 given. Here c is consumption, k is physical capital, h is human capital, and η is a constant representing education costs. Assume that the production function is Cobb-Douglas:

$$f(k,h) = z k^{\alpha} h^{\varepsilon} \quad (5)$$

where z is a constant technology parameter and $\alpha + \varepsilon < 1$.

(a) Derive the first-order condition for the planner's problem using Dynamic Programming. Define a solution in sequence language and in functional language.

(b) Solve for the steady state levels of k/h and k.

(c) Characterize the impact of cross-country differences in education costs (η) on output per worker in steady state. In particular, calculate the ratio of outputs per worker for two countries that only differ in their η's.

2 Relative Wealth Preferences

[60 points] Consider the following version of the growth model in continuous time.

Demographics: There is one representative household.

Preferences:

$$\int_0^{\infty} e^{-\rho t} \left[U(c_t) + V\left(k_t/\bar{k}_t\right) \right] dt \quad (6)$$

Endowments: The household starts with k_0.

Technology:

$$\dot{k}_t = f(k_t) - c_t \tag{7}$$

Government budget constraint: The government taxes consumption at rate τ_c and lump-sum rebates the revenues R_t to the household.

$$R_t = \tau_c c_t \tag{8}$$

Household budget constraint:

$$\dot{k}_t = f(k_t) - (1+\tau_c) c_t + R_t \tag{9}$$

Notation: c: consumption, k: capital, $\bar{k}$: average capital in the economy.
Assumptions: U, V, f are strictly increasing and strictly concave. $f'(0) = \infty$. $f'(\infty) = 0$.

1. State the household's current value Hamiltonian and derive the first-order conditions. Do not yet substitute out the co-state. Define a solution to the household problem.

2. Define a competitive equilibrium.

3. Derive an equation that implicitly solves for the steady state capital stock.

4. Draw the phase diagram. Start with $\dot{k} = 0$ and discuss its shape.

5. Derive $\dot{c} = 0$ and discuss its slope / intercept. For which values of k does $\dot{c} = 0$ have a solution? Hint: It is easier to write down $\dot{\lambda} = 0$, where λ is the co-state. Then use the fact that $\dot{\lambda} > 0$ implies $\dot{c} < 0$.

6. Assume that $\dot{c} = 0$ is concave,
$$\partial^2 c / \partial k^2 |_{\dot{c}=0} < 0 \tag{10}$$
and that it intersects $\dot{k} = 0$ twice. Discuss the stability properties of the two steady states.