What Econ720 is About

Prof. Lutz Hendricks

Econ720

August 14, 2024

What Econ720 is about

Macro is built around a small number of workhorse models:

1. Overlapping generations
2. "Standard growth" in continuous and discrete time
3. Stochastic growth
4. Endogenous growth
5. Search and matching models

We study basic versions of the **models** and the **tools** needed to analyze them.

What is not covered

Computational issues

- see Econ821 (sometimes)

Empirical issues

How to take models to data?

We will talk about empirical applications later in the semester (and in Econ721).

2. Models: Why and How?

Setting the Stage: Goals

We will develop a lot of models ... why?

- What kinds of things can we do with models?
- How should models be designed?

One example application to fix your mind: **fiscal policy**

Fiscal Policy Questions

We want to answer questions such as:

1. Progressive income taxes:
   How do they affect output / wealth inequality?
2. Is it "better" to tax consumption or income?
3. Government debt:
   How does a debt-financed tax cut affect future generations?

... and many more

Fiscal Policy Findings

The pioneering study of fiscal policy: Auerbach and Kotlikoff (1987)

Some of their findings:

> Deficits arising from income tax cuts of short duration "crowd in" saving and investment in the short run even though saving and capital formation are crowded out in the long run by such policies.
> Consumption taxation stimulates considerably greater savings than income or wage taxation.
> Most of the long-run welfare gains that would result from a move to consumption taxation are due to intergenerational transfers of the tax burden rather than gains in economic efficiency.
> Officially reported government deficits can be highly misleading indicators of the "tightness" or "looseness" of fiscal policy.

How do we get results like these?

Methods

How could we answer the fiscal policy questions?

Run an **experiment** (randomized controlled trial)

- Great, but not feasible

Regressions:

- Run a regression of saving rates on tax rates and controls
- Cross-country data?
- What goes wrong?

Case studies:

- Compare outcomes before after a big tax reform.
- What goes wrong?

Methods

What else could we do?

Models

The only method we have left to answer fiscal policy questions:

- **Build a model**.
- Somehow set its parameter values so they "match" data.
- Run counterfactual experiments in the model
  E.g.: switch from income tax to consumption tax.

The **model as a laboratory** where we can run experiments.

The big downside:

- The answer is only as good as the model.
- And it's hard to know if the model is any good.

What is a good model?

Simple, but not too simple

- Only model what is truly needed to answer the question.
- Especially first generation models should be simple, understandable "benchmark" models.
- Add complexity later to check what matters.

Complex models are black boxes.

Simple models are good

Keep that in mind when you think "this is a toy model"

What is a good model?

Standard ingredients

- Ideally, we would use a single model to study all topics.
- That model would be too complex.

But we can re-use model "blocks" (households, firms, ...)

What is a good model?

General equilibrium

- Ensure that all markets clear.
- Ensure that all agents satisfy their constraints.
- Keep track of the fact that "everything affects everything".

What is a good model?

Individuals respond to **incentives**

- Rule of thumb behavior will not do

The defining feature of economics:

- Agents solve **optimization problems**.
- Discussion below

Agents have **reasonable expectations**

- They understand how the economy works.
- In economics: rational expectations.

Digression

Are people really this rational?

Related: What is economics?

The Auerbach-Kotlikoff Model

The AK model is the simplest model in which dynamic fiscal policy can be studied.

Key ingredients:

- Households make consumption/saving and labor supply decisions.
- Firms hire workers and capital.
- The government imposes various kinds of taxes.
- All markets are competitive.
- There is no uncertainty.

This is a good (first) model precisely because it is simple.

We get non-trivial results, but we can really understand them.

Households

Households

- live for many periods (why?)
- make decisions about
  - consumption / saving
  - labor supply / leisure
  - why those decisions?
- perfect foresight (why?)
- make all of their life choices simultaneously

Household Problem

$$\max_{\{c_t, l_t, a_t\}} \sum_{t=1}^{55} \beta^t \mathscr{U}(c_t, l_t) \tag{1}$$

subject to the budget constraint

$$a_{t+1} + (1+\tau_t) c_t = y_t - T(y_t) + R_t a_t \tag{2}$$

where labor income is given by

$$y_t = w_t e_t (1 - l_t) \tag{3}$$

Government vary income taxes $T(y)$ and consumption taxes τ.

Households respond, fully understanding changes in future taxes and prices.

Firms

Firms are trivial.

There is one competitive firm.

It rents capital and labor from households.

It pays wage w_t and gross interest R_t.

Government

The government issues debt D and collects taxes to pay for useless government consumption G.

Budget constraint:

$$D_{t+1} + T_t = G_t + R_t D_t \tag{4}$$

Social security collects taxes and pays transfers to the old...

Competitive Equilibrium

Key point: An equilibrium covers all time periods.

- Households need to know the future to make their decisions.

Objects:

- Sequences of consumption, leisure, firm capital, government debt, tax rates, prices.
- For all time periods.

These satisfy:

- Given prices and tax rates, the households choose $\{c_t, l_t\}\ \forall t$
- Given prices, firms are happy to hire capital and labor $\forall t$.
- The government budget constraint clears $\forall t$
- Markets clear $\forall t$ (goods, labor, bonds)

Sample Result: Choice of Tax Base

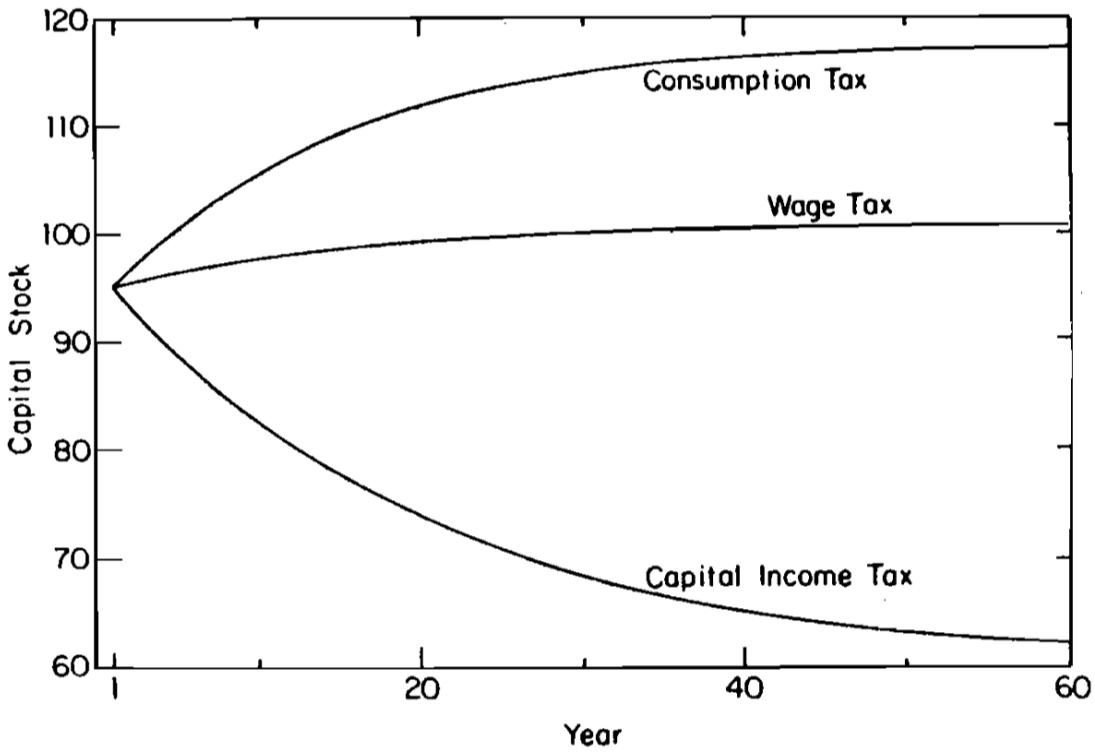

Figure 5.3. The impact on capital formation of tax reform.

We can now run counterfactual experiments in the model.

Trace out what happens over time to capital, output, ...

Welfare Effects

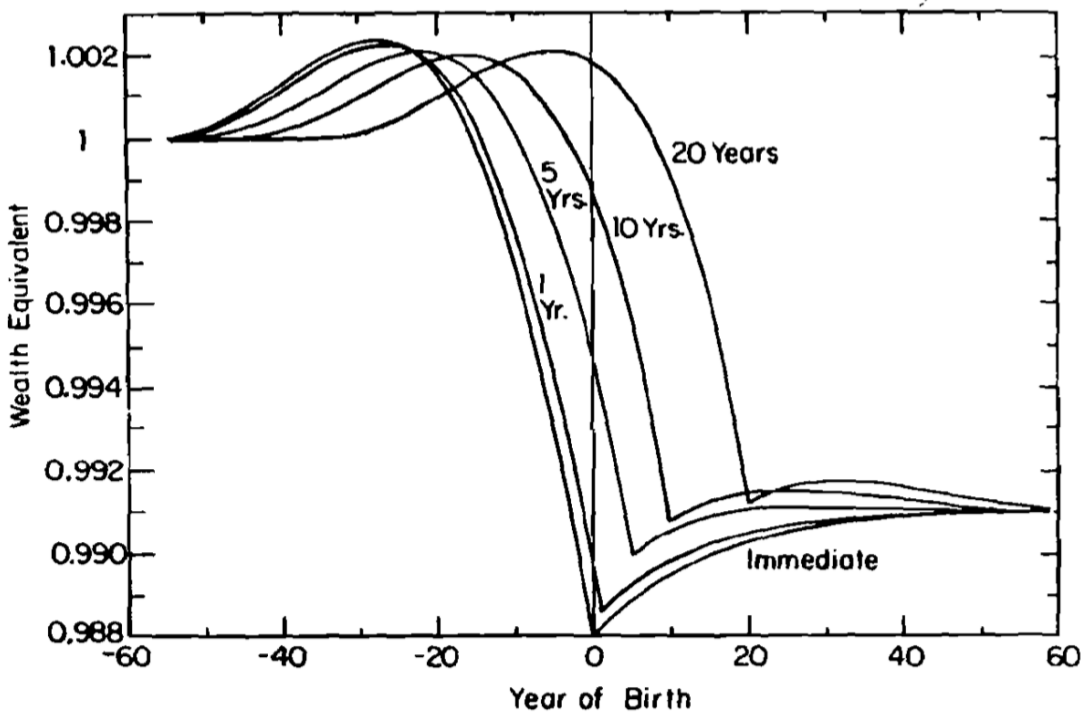

Figure 5.8. The welfare effects of preannounced switches to wage taxation.

We can compute how utility of each generation is affected.

We can dig into their consumption and leisure paths to gain intuition...

How to think about these results?

Models are simple; reality is not.

- Many policy effects are omitted.
- Models are misspecified ("wrong")

Quantitative results (the actual numbers) don't mean much.

What we gain:

- Insights into the **main forces** at work.
  E.g.: progressive taxes reduce hours worked.
- An idea of **magnitudes**
  Which forces are likely large or small.
- Insights into what is **robust / fragile**
  By comparing results from several models.

Key Points

1. A main goal of macro: policy analysis
2. We use **quantitative models**
   because we don't have any other tools
3. Good models start **simple**
4. Quantitative results are indications of orders of magnitude.

3. Competitive Equilibrium

How to set up a model

In the past: macro models looked very different from micro models.

- IS/LM, AS/AD

Now: all models are micro models.

An economy is

- a list of agents
- with preferences and constraints
- who solve optimization problems
- and interact in markets

Model Primitives

An economy is described by

- the list of **agents**,
- their **demographics**,
- their **preferences**,
- their **endowments**,
- the **technologies** they have access to
- the **markets** in which they can trade.

Every model description should start with these elements.
You are not allowed to analyze anything until you have described these model elements.

Competitive Equilibrium

Once we put all the pieces together and let agents interact in markets, we get a Competitive Equilibrium.

A key skill we will learn:

How to translate the description of an economy into a set of equations that characterize the **competitive equilibrium**.

Definition

A competitive equilibrium is an **allocation** (a list of quantities) and a **price system** (a list of prices) such that
- the quantities **solve all agents' problems**, given the prices;
- all **markets clear**.

How to Set Up a Competitive Equilibrium

Steps:

1. Describe the economy
2. Solve each agent's problem
3. State the market clearing conditions
4. Define an equilibrium

All of this is really mechanical.

The hard part is to say something about what the equilibrium looks like.

(Ask your computer.)

Step 1: Describe the Economy

1. List the agents (households, firms).
2. For each agent define:
   - **Demographics**:
     AK: Each period, N households are born; live for 55 periods
   - **Preferences**:
     AK: $\sum_{t=1}^{55} \beta^t \mathscr{U}(c_t, l_t)$
   - **Endowments**:
     AK: each household has one unit of time each period.
   - **Technologies**:
     AK: output is produced using $F(K, L)$.
3. Define the **markets** in which agents interact.
   - AK: households work for firms; households purchase goods from firms.

Step 2: Solve Each Agent's Problem

Write down the maximization problem each agent solves.

Derive a set of equations that determine the agent's choice variables.

- E.g.: A consumption function, saving function.
- We call these **policy functions** or **decision rules**.

Example

A household chooses c and s to maximize utility, subject to a budget constraint.
Policy rules: $c = f(y,p)$ and $s = g(y,p)$

Step 3: Market Clearing

For each market, calculate supply and demand by each agent.

- Aggregate supply = $\sum$ individual supplies.
- Aggregate demand = $\sum$ individual demands.

Market clearing is simply:

- Aggregate supply = aggregate demand.

Tip

The market clearing condition for apples contains only quantities of apples.

- If there are prices or bananas, it's wrong.

Step 4: Define the Equilibrium

From steps 2-3:

Collect all endogenous objects

- e.g., consumption, output, wage rate, ...

Collect all equations

- first order conditions or policy functions
- market clearing conditions

You should have N equations that could (in principle) be solved for N endogenous objects

- prices
- quantities (the allocation)

Notes

Competitive equilibrium covers all periods

- not a period-by-period concept

Expectations are consistent with equilibrium

- agents solve for equilibrium to forecast prices

Review

1. What are the steps required for defining a competitive equilibrium?

Step	Outcome

2. What goes into market clearing conditions (and what doesn't)?

4. Static example

Static Example

- We study a very simple one period economy.
- There are many identical households.
- They receive **endowments** which they eat in each period.
- Nothing interesting happens in this economy - it merely illustrates the method.

Step 1: Describe the Economy

Demographics:

- There are N identical households.
- They live for one period.
- For now, there are no other agents (firms, government, . . .).

Preferences:

- Households value consumption of two goods according to a utility function $u(c_1, c_2)$

Step 1: Describe the Economy

Endowments:

- Each agent receives endowments of the two goods (e_1, e_2).

Technology:

- There is no production. Endowments cannot be stored.
- Resource constraint: $Ne_1 = Nc_1$ and $Ne_2 = Nc_2$.

Tip

A technology is always described by a **resource constraint** that equates how much stuff is made with how much stuff is used. Resource constraints usually become market clearing conditions in equilibrium.

Step 1: Describe the Economy

Markets:

- There are competitive markets for the two goods
- The prices of the two goods are p_1 and p_2.

Econ101 question

What are prices denoted in?

Step 2: Solve each Agent's Problem

There is only one agent: the household.

Households maximize $u(c_1, c_2)$ subject to a budget constraint.

State variables the household takes as given:

- market prices for the two goods, p_1 and p_2.
- endowments e_1 and e_2.

The **choice variables** are c_1 and c_2.

- We can normalize the price of one good to one (numeraire): $p_1 = 1$.
- Call the relative price $p = p_2/p_1$.

Household problem

Budget constraint: Value of endowments = value of consumption.

The household solves the **problem**:

$$\begin{aligned} & \max u(c_1, c_2) \\ s.t.\ c_1 + p\ c_2 &= e_1 + p\ e_2 \end{aligned}$$

A solution to the household problem is a pair (c_1, c_2)

More precisely: a pair of **decision rules**

- $c_1 = \mathscr{C}_1(e_1, e_2, p)$ and $c_2 = \mathscr{C}_2(e_1, e_2, p)$

Solving the household problem

To find the optimal choices set up a **Lagrangean**:

$$\Gamma = u(c_1, c_2) + \lambda \ [e_1 + p \ e_2 - c_1 - p \ c_2]$$

It would actually be easier to substitute the constraint into the objective function and solve the unconstrained problem

$$\max \ u(e_1 + p \ e_2 - p \ c_2, c_2)$$

but the Lagrangean is instructive.

Household first-order conditions

$$\Gamma = u(c_1, c_2) + \lambda \ [e_1 + p \ e_2 - c_1 - p \ c_2]$$

The **first order conditions** are

$$\partial \Gamma / \partial c_i = u_i(c_1, c_2) - \lambda p_i = 0 \tag{5}$$

where $p_1 \equiv 1$ and $p_2 \equiv p$.

The multiplier λ has a useful interpretation:

- It is the marginal utility of relaxing the constraint a bit,
- i.e. the marginal utility of wealth.

Household first-order conditions

$$\frac{u_i(c_1, c_2)}{p_i} = \lambda \ \forall i \tag{6}$$

Important: Always state the first-order conditions in words.

- Key part of understanding how the model "works"
- If you cannot explain FOCs, they are probably wrong.

Household solution

The solution to the household problem is a pair (c_1, c_2, λ) that solves

- 2 FOCs
- the budget constraint.

Some tips

- Always explicitly state what **variables** constitute a solution and which **equations** do they have to satisfy.
  - This makes it easy to keep track of the pieces that go into the competitive equilibrium.
- You should have a FOC for each choice variable and all the constraints.
- Make sure you have the same number of variables and equations.
- When you write down an equation, **pause and think**.
  - Make sure you understand what the equation says in words.
  - If you cannot make sense of it, it's probably wrong!

Simplify the optimality conditions

It is useful to substitute out the Lagrange multiplier λ.

The ratio of the FOCs implies

$$u_2/u_1 = p \tag{7}$$

This is the familiar tangency condition: marginal rate of substitution equals relative price. [Graph]

Now the solution is a pair (c_1, c_2) that satisfies (7) and the budget constraint.

Note: I can keep the Lagrange multiplier or drop it. If I keep it, I also need to keep another equation (e.g., the FOC for c_1).

Log utility example

Assume log utility:

$$u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$$

Then the problem can be solved in closed form:

$$\frac{u_2}{u_1} = \beta \frac{c_1}{c_2} = p$$

Substitute this back into the budget constraint:

$$\begin{aligned} c_1 + \beta\ c_1 &= W = e_1 + p\ e_2 \\ c_1 &= \frac{W}{1+\beta} \\ c_2 &= \frac{\beta W}{1+\beta} \end{aligned}$$

Log utility example

Tip: A useful feature of log utility: the expenditure shares are independent of p.

- The reason is exactly the same as that of constant income shares resulting from a Cobb-Douglas production function: unit elasticity of substitution.

Tip: Recall that taking a monotone transformation of u doesn't change the optimal policy functions.

- In particular, we can replace u by

$$u(c_1, c_2) = c_1 \; c_2^{\beta}$$

- Convince yourself that this yields exactly the same consumption functions.

Step 3: Market Clearing

There are two markets (for goods 1 and 2).

- Why isn't there just 1 market where good 1 is traded for good 2?

Each agent

- supplies the endowments e_i and
- demands consumption c_i in those markets.

Goods are traded for **units of account**.

I don't use the word **money** because there is no such thing in this economy.

Market Clearing

The market clearing condition is

"aggregate supply = aggregate demand."

Aggregate supply is simply the sum of individual supplies:

$$S_i = \sum_{h=1}^{N} e_i = N\ e_i \tag{8}$$

Aggregate demand:

$$D_i(p, e_1, e_2) = \sum_{h=1}^{N} c_i = N\ c_i(p, e_1, e_2) \tag{9}$$

Market clearing:

$$c_i = e_i \tag{10}$$

Everybody eats their own endowments.

Definition of Equilibrium

A **competitive equilibrium** is an allocation (c_1, c_2) and a price p that satisfy:

- 2 household optimality conditions (FOC and budget constraint).
- 2 goods markets clearing conditions.

Now we count equations and variables.

- We have $2N+1$ objects: $2N$ consumption levels and one price.
- We have $2N$ household optimality conditions and 2 market clearing conditions.

Econ201 question

Why do we have one equation too many?

What goes into the Equilibrium?

Equations:

- for each agent: equations that define solution here: FOCs and budget constraint
- market clearing conditions

Objects:

- go through all equations
- collect all endogenous objects

Recap of key points

1. A macro model consists of exactly these parts:
   - Demographics
   - Preferences
   - Endowments
   - Technologies
   - Markets
2. A competitive equilibrium is an allocation (think quantities) and a price system such that
   - all agents solve their optimization problems, given prices;
   - markets clear.
3. Market clearing conditions only contain quantities of one good (no prices!).
4. Prices are in units of account that can be chosen arbitrarily at each trading date.
5. Walras' law allows you to drop one market clearing condition **or** one budget constraint.

Where Is the Money?

We just wrote down a model without money.

The vast majority of macro models have no money.

But money is important ... are macroeconomists crazy?[1]

[1] They may well be, but not because they write down models without money.

Reading

There is a remarkable shortage of discussions on how to properly set up a model in textbooks.

But see

- Krusell (2014), ch. 2 describes the ingredients of modern macro models.
- Ch. 5 talks about Arrow-Debreu versus sequential trading.

Hopefully up to date links to unpublished texts are in the syllabus. Let me know if you can't find something.

References

Auerbach, A. J. and L. J. Kotlikoff (1987): *Dynamic fiscal policy*, Cambridge University Press.

Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.