Arrow-Debreu and Sequential Trading

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Introduction

Macro models are dynamic (have many periods).

Then we have a choice of how to represent equilibrium:

▶ Arrow-Debreu: all trading takes place at date 0
▶ Sequential trading: markets open in each period

This is where the details matter (units of account, Walras’ law, ...)
Two Period Example

Demographics:

- $N$ identical households live for 2 periods, $t = 1, 2$.

Commodities:

- there is one good in each period

Preferences: $u(c_1, c_2)$

Endowments: $e_t$

“Technology”: $c_t = e_t$
Markets

Now we have a choice between 2 equivalent arrangements

- Arrow-Debreu: all trades take place at $t = 1$
- Sequential trading: markets open in each period
Arrow-Debreu Trading

The arrangement:

- All trades take place at $t = 1$
- Agents can buy and sell goods for delivery at any date $t$
- Prices are $p_t$

Can we normalize prices to 1?

Surprise:
If we write out this model, it looks exactly like the static 2 good model (see above).
Arrow-Debreu Equilibrium

Household budget constraint:

\[ \sum_{t} p_t e_t = \sum_{t} p_t c_t \] (1)

Interpretation:
The household sells \( e_t \) to and buys \( c_t \) from the Walrasian auctioneer at a single trading date.

Market clearing:

\[ e_t = c_t \] (2)

▶ Again the same as resource constraints.
Equilibrium

Objects: $c_t, p_t, \ t = 1, 2$

Equations:

- Household policy rules: $c_t(p_1, p_2)$ implicitly defined by first-order condition and budget constraint
- Market clearing: $e_t = c_t$

Notes:

- only $p_2/p_1$ is determined in equilibrium (choice of unit of account)
- only one equation is redundant by Walras’ law (why?)
Equivalence of Dates and Goods

**Fact**

*A model with T goods is equivalent to a model with T periods.*

This is only true under “complete markets”

- roughly: there are markets that allow agents to trade goods across all periods and states of the world
- we will talk about details later
Sequential Trading

An alternative trading arrangement.
Markets open at each date.
Only the date \( t \) good can be purchased in the period \( t \) market.

Now we have **one numeraire for each trading period**: \( p_t = 1 \).
We need assets to transfer resources between periods.
Markets

At each date we have

1. a market for goods \((p_t = 1)\);
2. a market for 1 period discount bonds (price \(q_t\))

A discount bond pays 1 unit of \(t + 1\) consumption.
Digression: Modeling bonds

**Definition**

A one period bond promises to pay one unit of consumption in $t + 1$.

Call its price $q_t$.

Then the real interest rate is: $R_{t+1} = 1/q_t$.

What is a real interest rate?

Alternative normalization:

- set $q_t = 1$ and let each bond pay $R_{t+1}$ units of consumption
- why can I do this?
Household problem

Now we have one budget constraint per period:

\[ e_t + b_{t-1} = c_t + b_t q_t \]  

(3)

With \( b_0 = 0 \).

Household solves:

\[ \max_{b_1} u(e_1 - b_1 q_1, e_2 + b_1) \]  

(4)
Household solution

FOC:

\[ u_1 q_1 = u_2 \]  \hspace{1cm} (5)

\( q_1 \) is the relative price of period 2 consumption.

Give up 1 unit of \( c_1 \) and get \( 1/q_1 \) units of \( c_2 \).

Solution: \( c_1, c_2, b_1 \) that solve FOC and 2 budget constraints.
Market Clearing

- Goods: \( e_t = c_t \)
- Bonds: \( b_t = 0 \)

Why does bond market clearing look so odd?
Equivalence

Note that the relative price is the same under both trading arrangements:

\[ p = q = \frac{u_2}{u_1} \]  

Fact

*When markets are complete, Arrow-Debreu and sequential trading equilibria are identical.*
Macro is micro

or

IS-LM is dead. Long-live general equilibrium

- The method outlined here is central to all of (macro) economics.
- Being able to translate a description of an economy into the definition of a competitive equilibrium is an important skill.
Final example

Demographics: There are $N$ households. Each lives for $T > 1$ periods.

Preferences: $\sum_{t=1}^{T} u(c_{1,t}, \ldots, c_{J,t})$ where $J$ is the number of goods available in each period.

Endowments: Household $i$ receives $e_{i,j,t}$.

Technologies: Endowments can only be eaten in the period they are received.

- Resource constraint:

Markets:

- Sequential trading: there are competitive markets for the $J$ goods; there are one period discount bonds in each period.
- Arrow-Debreu: the $J \times T$ goods are traded in $t = 1$. 
Final example: Equilibrium
Krusell (2014), ch. 5 talks about Arrow-Debreu versus sequential trading.
References