The Solow Diagram

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Analyzing the Solow Model

What are the implications of the Solow model?

- Why do economies grow over time?
- Does the economy settle down in the long-run?
- What are the long-run and short-run effects of changes in saving?

To answer that:

1. Study the balanced growth path

   where all growth rates are constant over time
2. Plot the law of motion for k.

2. Balanced Growth

Where does the economy settle in the long run?

We will show: the economy converges to a **balanced growth path** (or **BGP**)

Definition

On a balanced growth path, all growth rates are constant over time.

Characterizing the BGP

Start from the law of motion

$$g(k) = s\bar{k}^{\alpha-1} - (n+\delta) \tag{1}$$

Constant $g(k)$ requires constant $\bar{k}$ or

$$g(\bar{k}) = g\left(\frac{K}{AL}\right) = g(K/L) - \gamma = 0 \tag{2}$$

or

$$\bar{g}(K/L) = \gamma \equiv g(A) \tag{3}$$

BGP output growth

Output per worker:

$$Y/L = \frac{K^{\alpha}(AL)^{1-\alpha}}{AL} \times A = \left(\frac{K}{AL}\right)^{\alpha} \times A \tag{4}$$

By the growth rate rule $(g(x^{\alpha}) = \alpha g(x))$:

$$g(Y/L) = \alpha g\left(\bar{k}\right) + \gamma \tag{5}$$

On the BGP $\bar{k}$ is constant:

$$\bar{g}(Y/L) = \gamma \tag{6}$$

Note: $\bar{g}(x)$ is the balanced growth rate of x.

What drives long-run growth?

Result:

Growth cannot be sustained without productivity growth:

$$\gamma = 0 \implies \bar{g}(K/L) = \bar{g}(Y/L) = 0 \tag{7}$$

The balanced growth rate of output is completely determined by productivity growth.

It is not possible to achieve long-run growth by saving alone.

Brief intuition (more below):

- If we try to growth K/L faster than A, the marginal product of capital falls

BGP Levels

What is the balanced growth level of $\bar{k}$?

From

$$\bar{g}\left(\bar{k}\right) = s\bar{k}^{\alpha-1} - (n+\gamma+\delta) = 0 \tag{8}$$

we need

$$\bar{k} = \left(\frac{s}{n+\gamma+\delta}\right)^{1-\alpha} \tag{9}$$

Higher saving rates increase output **levels**, but not growth rates.

Intuition:

$$sY = I = (n+\delta)K \tag{10}$$

BGP Levels

$$\bar{k} = \left(\frac{s}{n+\gamma+\delta}\right)^{1-\alpha} \tag{11}$$

1. Unique
2. Higher saving increase $\bar{k}$ and output
3. Higher depreciation or population growth reduce $\bar{k}$ and output

How big these effects are is governed by α.

- curvature of the production function
- more curvature $\implies$ smaller changes in $\bar{k}$
- intuition...

3. Dynamics of the Capital Stock

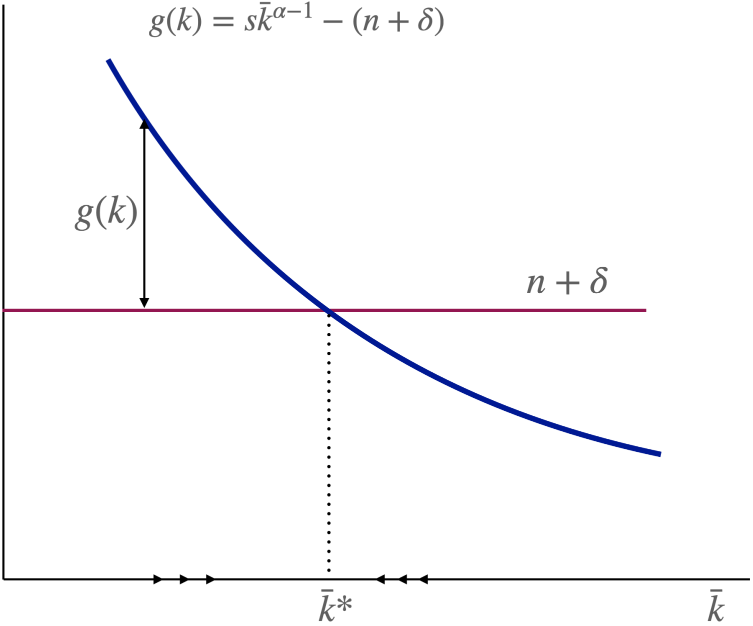
How do economies grow when they are not on the BGP?

Start from the law of motion:

$$g(k) = s\bar{k}^{\alpha-1} - (n+\delta) \tag{12}$$

$\bar{k}^{\alpha-1}$ is downward sloping in $\bar{k}$.

Dynamics



Key ideas

1. Growth is driven by investment $>$ depreciation.
2. Low $k \implies$ high $MPK = f'(k) \implies$ saving generates a lot of output $\implies$ output grows
3. High $k \implies$ high depreciation $\implies$ output shrinks
4. Therefore, the economy always converges to a steady state where investment $=$ depreciation

Interactive Solow Diagram, Demonstration of Solow dynamics

The Principle of Transition Dynamics

Fact

In the Solow model, the farther away the economy is from its steady state, the faster it grows (or shrinks)

What is the intuition?

Why does investment not sustain growth?

The problem is the diminishing MP_K.

Giving up one unit of C today yields $MP_{K'} - \delta$ in additional output tomorrow.

As k grows, MP_K eventually falls below δ:

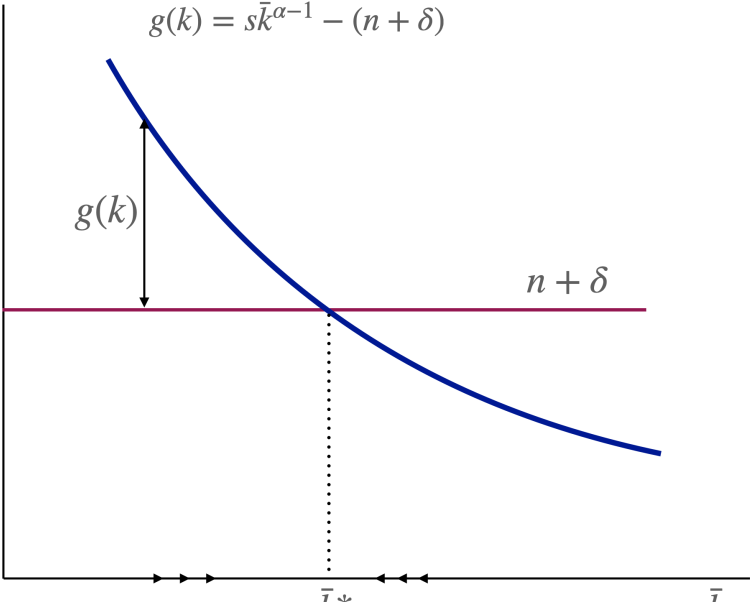
- Additional investment no longer even pays for its own depreciation.

Key result

Sustained growth through capital accumulation requires that MP_K stays above δ, even as k grows without bounds.

Comparative statics (or dynamics)

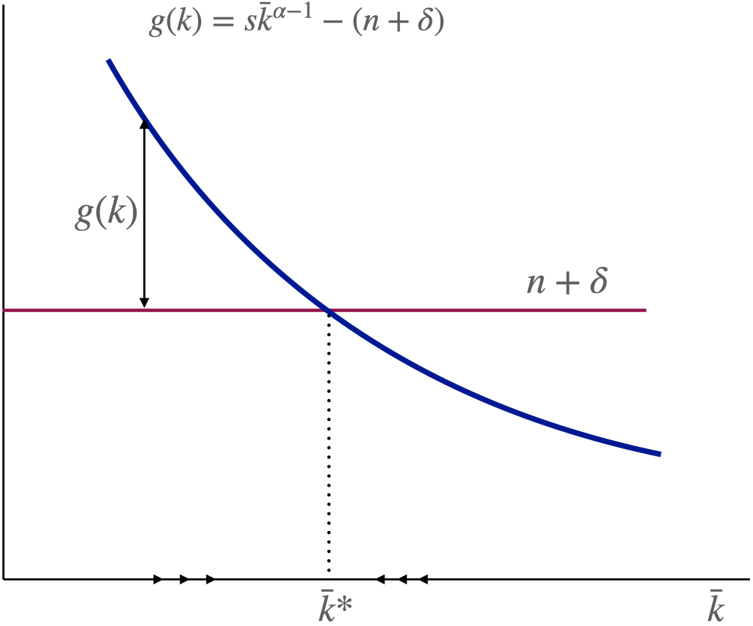
What happens if households save more?



Reality Check

- The model says: more investment (or **lower consumption**) generates a period of **faster** growth.
- Isn't everybody saying that we can get more growth by stimulating consumption?
- How does the contradiction get resolved?
- Where is the effect of lower consumption demand in the Solow model?
- Where is the demand side anyway?

Lower Population Growth



Important Points

- The Solow model reveals how choices (saving, fertility) affect capital and output (levels and growth).
- Capital cannot sustain long-run growth.
  - the reason: diminishing returns
- Therefore policies have level effects.
- In the short run: countries grow fast when they are far below their steady states.
- In the long run: growth is determined by productivity improvements.

Reading

- Jones / Vollrath, Introduction to Economic Growth, 3rd or 4th ed., ch. 2
- Blanchard and Johnson (2013), ch. 11

Further Reading:

- Romer (2011), ch. 1
- Barro and Sala-i Martin (1995), ch. 1.2

References I

Barro, R. and X. Sala-i Martin (1995): "Economic growth," *Boston, MA*.

Blanchard, O. and D. Johnson (2013): *Macroeconomics*, Boston: Pearson, 6th ed.

Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.