Applying the Solow Model

Prof. Lutz Hendricks

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Topics

We apply the Solow model to study:

1. Cross-country variation in growth rates
2. "New economy" innovations

Long-run Growth

Long-run Growth

What does the Solow model imply for long-run growth?

Main result

The principle of transition dynamics
Countries grow faster when they are far below their steady state.

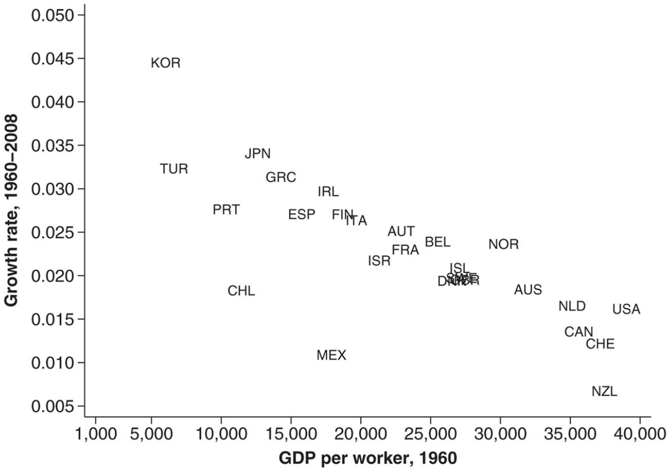
Main reference: Jones (2013), ch. 2, 3

What is the evidence supporting the principle?

- One exercise: if countries have similar steady states, their income levels should converge over time
- initially poor countries should grow faster

Convergence: Evidence

FIGURE 3.5 **CONVERGENCE IN THE OECD, 1960–2008**



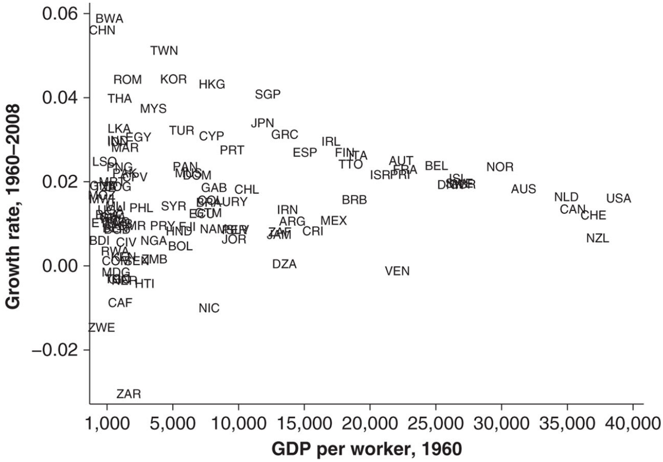
Among OECD countries: those that were initially poor grew faster.

Empirical Evidence

- Should we conclude that transitional growth explains cross-country differences in output growth?
- No!
- Figure 5.8 only shows OECD countries - mostly rich Western European countries + North America.

Empirical Evidence

FIGURE 3.6 THE LACK OF CONVERGENCE FOR THE WORLD, 1960–2008



No luck for a broad set of countries.

Empirical Evidence

- But figure 5.9 is the wrong experiment!
- The Solow model does not say: "poor countries grow faster"
- It says: "countries that are poor **relative to their steady states** grow faster."
- That is true in the data.

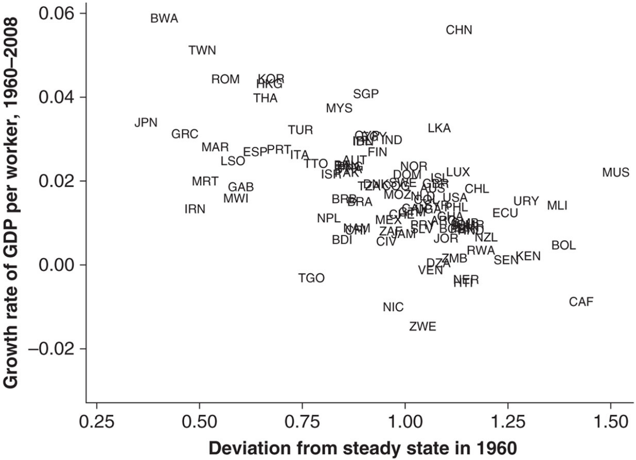
Empirical Evidence

Exercise

For a set of countries gather data on s, n.
Compute steady state output: y^*
Compute output in 1960 relative to steady state: y/y^*
Compute average growth 1960-2000
Plot average growth against y/y^*
What do you expect to find?

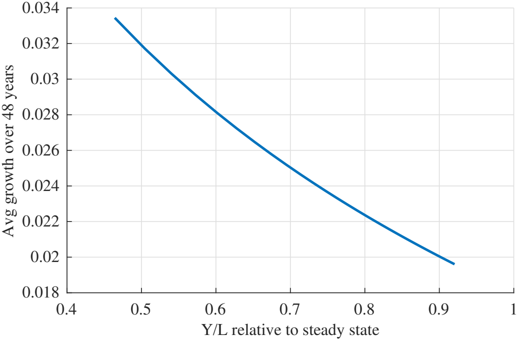
Conditional Convergence

FIGURE 3.8 "CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–2008



Source: Jones (2013)

Convergence: Solow Model

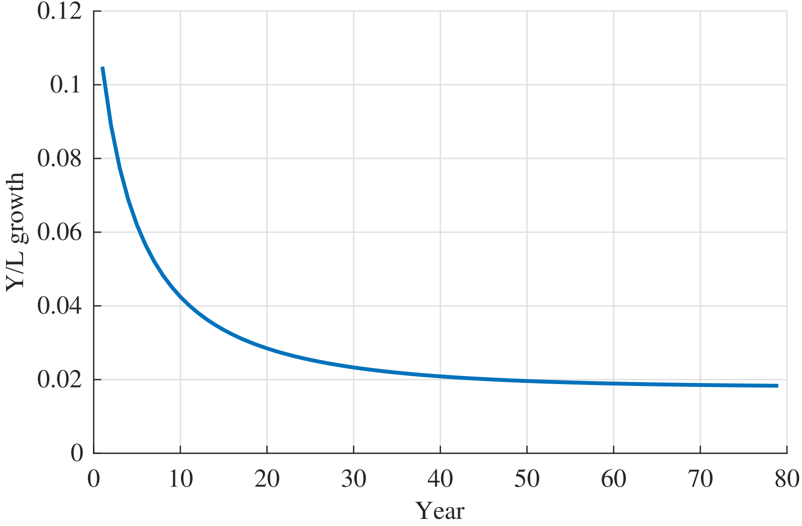


Prediction from a Solow model with capital share 1/3

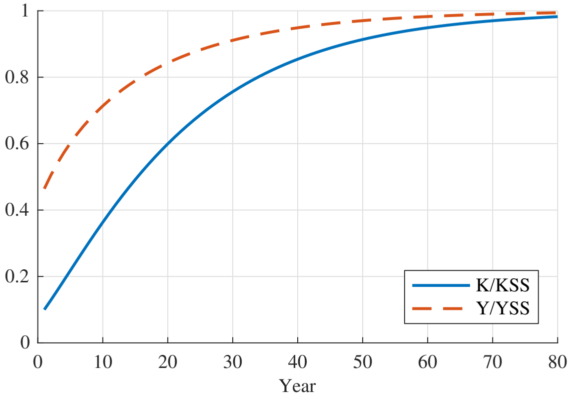
The fit is not bad, though the growth rate varies less than in the data.

Simulating the Solow Model

High growth rates do not last as long as in the data.



Simulating the Solow Model



Convergence is too fast.

In the data, the "**half-life**" is about 30 years – 10 years in the model.

Convergence is even faster when the saving rate is endogenous.

Convergence implications

The Solow model makes a quantitative prediction about growth rates.

Countries **converge fairly quickly** to their steady states (perhaps within 20 years).

Then they all should grow at almost the same rates.

Fact

The Solow model cannot explain why countries grow at different rates for long periods of time.

"Growth accounting" shows that much of variation in long-run growth is due to A, not k.

Did we just invalidate the Principle of Transition Dynamics?

- No, we did not.
- Countries grows faster when their capital stocks are low.
- But this does not account for the observed differences in long-run (40 year) growth rates across countries.
- It does account for growth rates at shorter horizons.

The Tigers

There are a few countries that sustained growth by capital accumulation for a long period of time.

How?

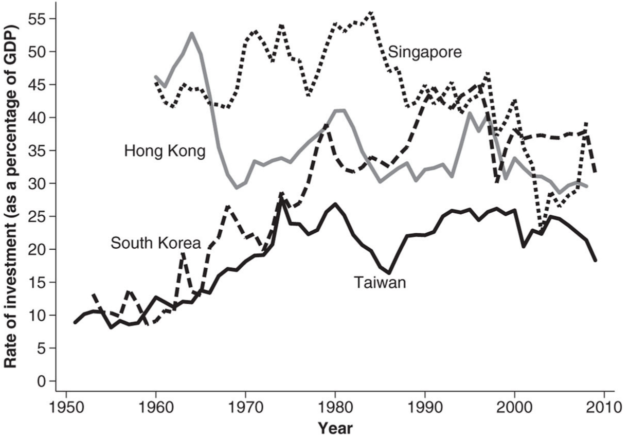
It cannot work with a constant saving rate s - the Solow model shows this.

Such countries must have saving rates that **rise over time**.

Examples are: South Korea, Singapore, Hong-Kong.

The Tigers

FIGURE 2.14 **INVESTMENT RATES IN SOME NEWLY INDUSTRIALIZING ECONOMIES**



Source: Jones (2013)

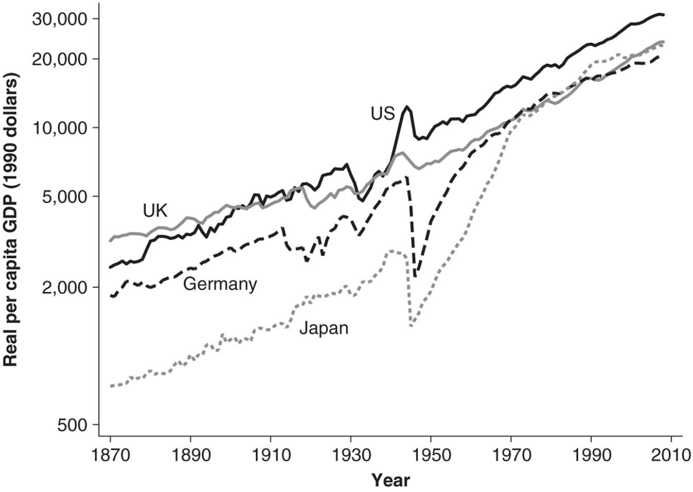
Exercise: Rising saving rate

- Simulate the Solow model with a saving rate that rises from 10% to 40% (Singapore).
- Start the model in steady state: $K_0 = K^*$.
- Show that the growth rate of y stays positive for a long time.
- You could now compare that growth path with data for Singapore and convince yourself that a large share of Singapore's spectacular growth since 1960 is indeed due to capital accumulation (as shown by Alwyn Young).

Convergence and Post-war Growth

One episode where convergence was very fast: growth after WW2

FIGURE 3.3 **PER CAPITA GDP, 1870–2008**



Convergence and Post-war Growth

Convergence to pre-war trends was very fast after WW2

Many countries were back on their trend paths after 5-7 years

Much of the convergence after the initial years was growth in TFP, not capital accumulation.

Exercise

- Take a spreadsheet.
- Fix parameters at plausible empirical values: $\alpha = 1/3$, $d = 0.08$, $s = 0.2$.
- Compute the steady state.
- Fix K_0 at some multiple of K^*.
- Compute the transition path for K_t by iterating over $K_{t+1} = sY_t + (1-d)K_t$.
- Plot the growth rate of Y_t against time.
  - You should see that growth is very high initially, if K_0 is small. But growth slows dramatically very quickly.
- Now plot the growth rate of Y_t against over 40 years against initial Y_t - this is the model analogue of figure 5.8.
  - You should see that the model relationship is much flatter than the data relationship.
  - The model fails to explain large variation in 40 year average growth rates.

Summary

The Solow model's main prediction is the Principle of Transition Dynamics.

In the data:

- no unconditional convergence
- but conditional convergence (consistent with the model).

But the convergence in the data is mostly not due to capital accumulation.

- the model implies very fast convergence
- we see this in the data after capital destruction

Empirical long-run growth rate differences are mostly due to A, not K.

The End of Economic Growth?

The Issues

We discuss the claims made in Frey (2015): "How to Prevent the End of Economic Growth"

What does the article claim?

Proposed policy solutions

1. Support investment in labor intensive industries
2. Redistribute income to raise aggregate demand
3. Encourage more entrepreneurial risk taking

Thoughts?

A Solow Interpretation

Innovations raise productivity (presumably, which is why they are worth a lot).

- A rises.

But the additional income accrues to neither capital nor labor.

- it goes to innovators
- their saving rate is high

Defer concerns about aggregate demand (this is a long-run model)

A modified Solow model

There is a new input X that represents innovation

$$Y = AX^{\alpha}K^{\beta}L^{\gamma} \tag{1}$$

Constant returns to scale:

$$\alpha + \beta + \gamma = 1 \tag{2}$$

Per capita output:

$$y = Y/L = Ax^{\alpha}k^{\beta} \tag{3}$$

Early innovation: A rises; $\alpha = 0$.

"New economy:" A rises; α rises.

- x gets a larger income share.
- capital and labor get smaller shares.

Law of motion

Capital accumulation is unchanged $\dot{k} = sy - (n+\delta)k$

- This fixes steady state $k/y = s/(n+\delta)$.

Production function:

$$y/k = Ax^{\alpha}k^{\beta-1} \tag{4}$$

Steady state capital stock:

$$k^{1-\beta} = Ax^{\alpha}\frac{s}{n+\delta} \tag{5}$$

Factors are paid marginal products

As always with Cobb-Douglas: factor income shares are constant

- capital gets β
- labor gets γ
- x gets the rest: $\alpha = 1 - \beta - \gamma$

Details: factor shares

- Labor:

$$w = \gamma A x^{\alpha} K^{\beta} L^{\gamma-1} \tag{6}$$
$$= \gamma y \tag{7}$$

- Capital gets share β:

$$q = \beta A x^{\alpha} K^{\beta-1} L^{\gamma} \tag{8}$$
$$= \beta y/k \tag{9}$$

- x gets share α:

$$p = \alpha A x^{\alpha-1} K^{\beta} L^{\gamma} \tag{10}$$
$$= \alpha y/x \tag{11}$$

"Old fashioned" innovation

A rises by factor $\lambda > 1$

α unchanged.

Implications:

- $k/y = s/(n+\delta)$ unchanged
- k rises by $\lambda^{\alpha/(1-\beta)}$ (from the steady state k solution)
- w and p and y do the same
  - from the factor price equations
- q unchanged

"New economy:" Higher α

To focus on redistributional effect: adjust A so that y unchanged

$k/y = s/(n+\delta)$ unchanged

Then k unchanged

w, q fall;

p rises

Redistribution of income from factors to x

Combined Effect

"New economy:" A rises while income is redistributed from factors to x (α rises).

- or: A is constant, but X rises due to innovation (at the same time α rises)
- we probably don't have the right production function for that!

x owners (innovators) get richer.

Wages: may stagnate, even though output rises

- labor share declines (true in the data!)

Investment

- marginal product of capital q falls
- I/Y may fall (but then c/y would have to rise!)

Policy implications

What has changed relative to old-fashioned A growth?

Should we subsidize labor intensive industries?

Policy implications

A key idea of economic policy

Separate redistribution from efficiency
If you want to redistribute income, use transfers, not subsidies.

One additional concern:

What if the marginal product of some workers falls so much to make them unemployable?

Automation

Automation has replaced "routine" jobs.

Figure 6. Employment Growth Has Polarized Between High- and Low-Paid Occupations

CHANGES IN OCCUPATIONAL EMPLOYMENT SHARES AMONG WORKING-AGE ADULTS, 1980–2015

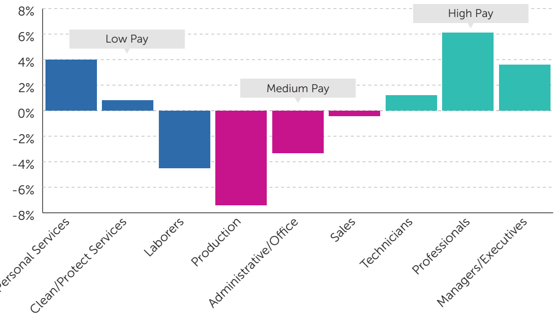
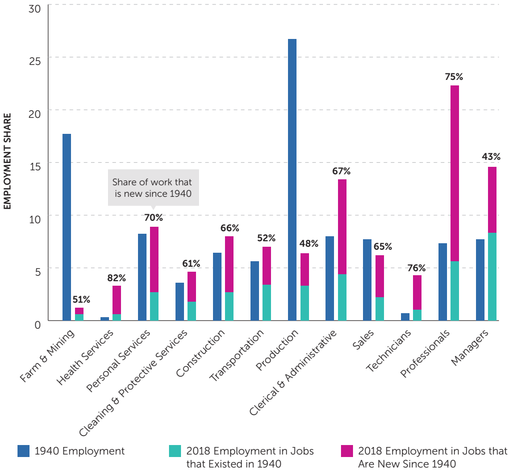


Figure is constructed using U.S. Census of Population data for 1980, 1990, and 2000, and pooled American Community Survey (ACS) data for years 2014 through 2016, sourced from IPUMS (Ruggles et al., 2018). Sample includes working-age adults ages 16 – 64 excluding those in the military. Occupational classifications are harmonized across decades using the classification scheme developed by Dorn (2009).

Source: Autor (2020)

Automation also creates new jobs

Figure 2. More Than 60% of Jobs Done in 2018 Had Not Yet Been "Invented" in 1940



Source: Autor (2020)

What does the future hold?

We don't know.

> *"No economic law dictates that the creation of new work must equal or exceed the elimination of old work. Still, history shows that they tend to evolve together." – Autor (2020), p. 12*

References I

Autor, D. (2020): "The Work of the Future," Tech. rep., MIT Work of the Future Task Force.

Frey, C. B. (2015): "How to Prevent the End of Economic Growth," *Scientific American*.

Jones, Charles; Vollrath, D. (2013): *Introduction To Economic Growth*, W W Norton, 3rd ed.