Growth through Innovation: The Romer Model

Prof. Lutz Hendricks

Econ520

March 31, 2023

Questions

- 1. How does economic growth come about?
- 2. What types of policies could manipulate long-run growth?

The dominant view today:

Innovation (the production of new "ideas") is what drives TFP growth.

Ideas

Ideas are discoveries that improve productivity. They include:

- Designs for **new products**: the microchip, the steam engine,...
- New ways of organizing production: Walmart, the assembly line.
- Key assumption: Ideas are produced like other goods.
 - By profit maximizing firms.
 - ► The profit of innovation is the rent of owning a patent.
- The stock of knowledge is a form of capital.

Ideas resemble physical capital

- they are produced by investing goods
- they are accumulated over time

If ideas are like physical capital, how is growth sustainable? The key difference: ideas are **non-rival**

Non-rivalry

Most goods are rival

- only a limited number of people can use a good at the same time
- examples: cars, computers, ...

Ideas can be used by many at the same time.

- software, music
- product designs (blueprints)
- production methods (just-in-time production, assembly line).

Why Does Non-rivalry Matter?

The Solow logic implies:

- physical capital accumulation cannot sustain growth.
- why not?

The Romer model shows:

- Accumulation of non-rival "knowledge capital" generates increasing returns to scale.
- Growth can be sustained.

Romer Model

Issues

- We study models where intentional innovation drives productivity growth.
- Romer model:
 - ▶ The standard model of R&D goes back to **Romer** (1990).
 - Innovations are produced like any other good using R&D labor as input.

Policy effects

- Policies, such as R&D subsidies, can change the rate at which innovations are produced.
- Surprisingly, it turns out that policies have no effect on long-run growth.

In this section you will learn:

- $1. \ \mbox{how to analyze the Romer model}$
- 2. why R&D policies do not change the long-run growth rate of the economy

The Romer model

Solow block

Production of goods works exactly like in the Solow Model

Aggregate production function:

$$Y_t = K_t^{\alpha} \left(A_t L_{Yt} \right)^{1-\alpha} \tag{1}$$

Capital accumulation as in the Solow model

$$\dot{K}_t = s_K Y_t - \delta K_t \tag{2}$$

Labor input grows at a constant rate

$$g(L) = n \tag{3}$$

Final goods production function has:

- constant returns to rival inputs: K and L_Y .
- has increasing returns to all inputs (including A)

Labor is divided into production (L_Y) and R&D (L_A) .

R&D Block

- Ideas are produced just like other goods.
- The input is labor (L_{At})

not much changes if capital is an input, too.

- The output is a number of new ideas.
 - A_t is the number of ideas that have been invented up to t.
 A_t is the number of ideas discovered today (or the rate at which they are discovered).

R&D Block

► The ideas production function:

$$\dot{A}_t = \bar{B} L_{At}^{\lambda} \tag{4}$$

- \triangleright λ determines returns to scale.
- $\blacktriangleright \overline{B}$ is a productivity parameter.

Ideas are inputs to innovation

How easy it is to produce a new idea depends on how much has already been discovered.

$$\bar{B} = B A^{\phi} \tag{5}$$

If ideas help produce new ideas: $\phi > 0$: $A \uparrow \Longrightarrow \overline{B} \uparrow$. If there is "fishing out:" $\phi < 0$. Assume $\phi \le 1$. (If $\phi > 1$ odd things happen...). The ideas production function is then

$$\dot{A} = B L_A^{\lambda} A^{\phi}$$

$$g(A) \equiv \dot{A}/A = B L_A^{\lambda} A^{\phi-1}$$
(6)
(7)

Ideas production function



Even though ideas foster innovation ($\phi > 0$), more ideas imply slower g(A).

Ideas production function

Note how similar this is to the law of motion for capital in the Solow model

Model			Productivity	"Capital"	Labor	Depreciation
Solow	\dot{K}_t	=	$sA^{1-\alpha}$	K_t^{α}	L_t^{1-lpha}	$-\delta K_t$
Romer	\dot{A}_t	=	В	A_t^{ϕ}	L_{At}^{λ}	-0

It follows that there cannot be long-run growth in A/L when $\lambda+\phi<1$ (details follow).

But we still can get long-run growth in Y/L.

Why is growth sustainable?

We now accumulate a second capital good:

$$Y_{t} = \underbrace{K_{t}^{\alpha}}_{\text{Solow Romer}} \underbrace{A_{t}^{1-\alpha}}_{\text{Romer}} L_{Y_{t}}^{1-\alpha}$$
(8)

In per capita terms

$$Y_t/L_t = \underbrace{\left(\frac{K_t}{L_t}\right)^{\alpha}}_{\text{Solow}} \underbrace{A_t^{1-\alpha}}_{\text{Romer}}$$
(9)

Note that A_t enters in the aggregate (not as A_t/L_t).

The Romer model

Behavior

So far we have described technologies.

To describe behavior, we make a Solow assumption:

A constant saving rate

$$S/Y = I/Y = s_K$$



$$L_A = s_A L \tag{10}$$

$$L_Y = (1 - s_A) L \tag{11}$$

Model summary

The Solow block:

$$Y = K^{\alpha} (A L_Y)^{1-\alpha}$$
(12)

$$\dot{K} = s_K Y - \delta K$$
(13)

$$L_t = L_0 e^{nt}$$
(14)

Production of ideas:

$$\dot{A} = B \ L_A^\lambda \ A^\phi \tag{15}$$

Constant behavior:

$$L_Y = s_Y L; \quad L_A = s_A L \tag{16}$$

The growth rate of ideas:

$$g(A) = B (s_A L)^{\lambda} A^{\phi-1}$$
(17)

Model summary

- This looks complicated, but isn't.
- We have tricked the model such that Y and K don't matter for how A evolves.

$$\dot{A} = B \ L_A^\lambda \ A^\phi \tag{18}$$

This would change, if we let A depend on K
 but that would not affect the results
 only the algebra would be more complicated (see Romer 2011)

Does the Model Make Sense?

- The production functions are arbitrary.
 - But what matters are certain qualitative features, not the exact functional form.
 - We will get back to this.
- There is only one input. Only one good.
 - All of this can be relaxed without changing anything too important.
- ▶ Where are the households, consumption, population growth ...
 - We can add those it does not make any difference.
- ► The labor allocation is fixed.
 - This is important.
 - The literature does not make this assumption. It can talk about patents, policy, ...
- Ideas are produced like goods.

Balanced growth path

Definition

A BGP is a path along which all variables grow at constant rates.

Why might this be interesting?

At what rates do the endogenous objects grow on the BGP? Result 1: g(k) = g(y)

as in the Solow model (same technology)

Proof:

• Law of motion: $g(k) = s y/k - \delta$.

• Constant g(k) requires constant k/y.

Balanced growth path

Result 2: g(y) = g(A)

as in the Solow model (same technology)

Proof:

- Production function: $y = k^{\alpha} A^{1-\alpha}$.
- Take growth rates: $g(y) = \alpha g(y) + (1 \alpha) g(A)$

Result

All long-run growth is due to R&D.

Growth rate of ideas

On the balanced growth path

$$g(A) = \frac{\lambda n}{1 - \phi} \tag{19}$$

This is the key result of the model.

Important:

This only holds on the balanced growth path.

Derivation

Ideas production:

$$g(A) = B \frac{L_A^{\lambda}}{A^{1-\phi}} \tag{20}$$

BGP: g(A) is constant $\implies L_A^{\lambda}/A^{1-\phi}$ is constant. Or:

$$g\left(L_{A}^{\lambda}\right) = g\left(A^{1-\phi}\right) \tag{21}$$

$$\lambda g(L_A) = (1 - \phi) g(A) \tag{22}$$

With constant time allocation, s_A : $g(L_A) = n$. Therefore

$$\lambda n = (1 - \phi) g(A) \tag{23}$$

Rearrange. Done.

Balanced growth in the Romer model is characterized by:

$$g(y) = g(k) = g(A)$$
(24)
$$g(A) = \frac{\lambda n}{1 - \phi}$$
(25)

All growth is due to innovation. Why is this true?

Why is all growth due to innovation?

Solow model:

- K is rival
- What matters for per capita output is K/L
- K does growth in the Solow model (due to population growth), but not K/L (diminishing returns)

Romer model:

A is non-rival

- ▶ What matters for per capita output is A, NOT A/L
- A grows (due to population growth), even if A/L falls over time

Balanced growth: Intuition

$$g(A) = \frac{\lambda n}{1 - \phi}$$
(26)

Growth is simply a multiple of population growth Behavior does not matter: s_K and s_A do not appear in (26).

Intuition

- Consider the case $\phi = 0$.
- Ideas production is then

$$\dot{A} = B L_A^{\lambda} \tag{27}$$

- If the population is **constant**, L_A is constant.
- In each period, the economy produces a constant number of ideas.
- The growth rate of ideas, $g(A) = B L_A/A$, falls to zero over time.
- A fixed number of people cannot produce a growing stream of ideas.

Population growth is necessary for sustained innovation (at a constant rate).



A

$$g(A) = BA^{\phi-1}L_A^{\lambda}$$

Special Case: Phi = 1

With $\phi = 1$, idea production becomes

$$g(A) = B L_A^{\lambda} \tag{28}$$

This is the case studied by Romer (1990).

The model has exploding growth, unless the population is constant. This is clearly contradicted by post-war data: L_A rose dramatically, while g(y) was at best constant.

Reality check

- 1. The model says: constant population no growth.
 - But we are still producing new ideas all the time.
 - How can we reconcile this?
- 2. What if the population shrinks over time?
 - Is the long-run growth rate negative?

Summary

- Innovations are produced just like regular goods, but they are non-rival.
- Therefore, we have scale effects: larger markets support more rapid innovation.
- The growth rate of Y/L is proportional to the population growth rate.
- ▶ A one-time increase in R&D effort (higher *L*_A) raises the rate of innovation permanently.
 - But this is not enough to sustain higher long-run growth.

What is the effect of a permanent increase in

- 1. research productivity (easy)
- 2. population (holding k fixed or not)
- 3. population growth (Europe)

Review Questions

- 1. Why is growth of Y/L not sustainable in the Solow model?
- 2. Why is population growth needed to sustain growth in the Romer model?

Reading

▶ Jones (2013b), ch. 5.

Optional:

- Romer (2011), ch. 3.1-3.4
- ▶ Jones (2013a), ch. 6

Advanced Reading

- Jones (2005) talks in some detail about the economics of ideas.
 Lucas (2000) and McCrattan and Proceett (2000) on openness
- Lucas (2009) and McGrattan and Prescott (2009) on openness and growth

References I

Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.

------ (2013a): *Macroeconomics*, W W Norton, 3rd ed.

- Jones, Charles; Vollrath, D. (2013b): Introduction To Economic Growth, W W Norton, 3rd ed.
- Lucas, R. E. (2009): "Trade and the Diffusion of the Industrial Revolution," *American Economic Journal: Macroeconomics*, 1–25.
- McGrattan, E. R. and E. C. Prescott (2009): "Openness, technology capital, and development," *Journal of Economic Theory*, 144, 2454–2476.

Romer, D. (2011): Advanced macroeconomics, McGraw-Hill/Irwin.