# Review Problems: Innovation and Growth

Econ520. Spring 2021. Prof. Lutz Hendricks. November 11, 2020

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Jones, Macroeconomics, problems 6.1-6.8.

# 1 Basics

1. What is meant by the non-rivalry of ideas?
   (a) Give examples of rival and non-rival goods.
   (b) If Roche holds a patent on a drug, does that make it a rival good?
2. Explain how non-rivalry lead to increasing returns to scale and scale effects.
3. Explain why increasing returns to scale are incompatible with perfect competition. Why does this destroy the presumption that market equilibria are efficient?
4. What is meant by scale effects? Explain why they arise.
5. Define "balanced growth path."
6. Consider a product like a smartphone. It contains many parts, each of which is covered by a separate patent. Now consider a medical drug which is basically a patented chemical compound with no other ingredients. Which case do you think calls for longer patent protection?

## 1.1 Answers: Basics

1. See slides
2. See slides
3. Increasing returns means falling average costs, so that marginal cost $<$ average cost. Perfect competition requires pricing goods at marginal cost. With increasing returns, firms would incur losses. If goods are not priced at marginal cost, this is inefficient. Example: the marginal cost of a drug may be only $1. It may the worth $100 to the patient, but the firm may price it at $200.
4. See slides.
5. An equilibrium path along which all variables grow at constant rates.

6. The drug deserves longer patent protection. Reasons:

   (a) Each smartphone patent creates a hold-up problem. The patent holder can block production of all smartphones (or extract large rents). The problem is that a small innovation that accounts for a small part of the entire product can be used to extract rents that are in no relationship to the value of the original innovation.

   (b) The patent on one phone part can hinder innovation for all other parts. Because after the fact much of the rents generated by any innovation can be extracted by that one patent holder.

# 2 Romer Model

1. Why is there sustained growth in the Romer model, but not in the Solow model?

2. Derive the balanced growth rate of ideas in the Romer model.

3. Suppose there is a one-time increase in the productivity of research. Describe the effect on the level and the growth rate of technology ($A$).

4. The government uses patent protection and R&D subsidies to foster growth. Could such policies overshoot their targets and actually reduce output and consumption, even in the long-run?

## 2.1 Answer: Romer model

**1.** Romer: constant returns to $A$ in the production of $A$. Solow: diminishing returns to $K$ in the production of $K$.

Of course, the Romer model with diminishing returns grows, if there is population growth. This is due to the non-rivalry of ideas.

So there are four cases: rival / non-rival $\times$ constant returns / diminishing returns. All of them have sustained growth (with population growth), except for the Solow case (diminishing returns / rival).

**2.** We did this in class.

**3.** Increase in $\bar{z}$: faster growth $\left(g = \bar{z}\ell\bar{N}\right)$. No change in $y$ at impact.

**4.** The short answer is: of course. Suppose we set the fraction of labor working in R&D to 1. Then output is zero.

# 3 Modified Romer Model

Consider the following modified Romer model:

- Production functions:

$$
\begin{aligned}
Y_t &= A_t^{\alpha} L_{yt} && (1)\\
\dot{A}_t &= B A_t L_{at} - d A_t && (2)
\end{aligned}
$$

- Resource constraint:

$$L = L_{yt} + L_{at} \tag{3}$$

- Allocation of labor:

$$
\begin{aligned}
L_{at} &= \ell \bar{N} && (4)\\
L_{yt} &= (1-\ell)\,\bar{N} && (5)
\end{aligned}
$$

There are two changes relative to the original Romer model: the exponent $\alpha$ on labor in the production function for goods and the depreciation term $dA_t$ in the production function for ideas (which is the same as the depreciation term in the Solow model). Assume that $0 < \alpha < 1$ and $0 < d < 1$.

**Questions:**

1. Derive the growth rates of $Y_t$ and of $A_t$ as functions of exogenous parameters.
2. Plot the time paths of $log(Y_t)$ and $log(A_t)$ for an economy that experiences a permanent increase in depreciation ($d$ rises) at date $t_0$. Explain what you plot.
3. Explain why the non-rivalry of ideas leads to increasing returns to scale. What does non-rivalry mean?

## 3.1 Answers: Modified Romer Model

1. $g(Y) = \alpha g(A)$ and $g(A) = B\ell\bar{N} - d$.
2. Straight lines with kinks at $t_0$. The $\log(Y)$ line is flatter than $\log(A)$ line. Explanation: higher $d$ reduces $g(A)$. But no jump in $A$ at $t_0$; we are not changing the stock of ideas, just the growth rate.
3. We think constant returns to all rival factors - the replication argument. But non-rival ideas do not need to be replicated. Example: build 2 factories to double output. No need to double the number of blueprints. Nonrivalry means: an idea can be used at the same time by multiple users.

# 4 Romer Model with Diminishing Returns

Consider the Romer model with $\dot{A}_t = BA_t^{\phi} s_R L_t$, $L_t = e^{nt}$ and $\rho < 1$.

1. Derive the balanced growth rate.
2. Intuitively, why does the balanced growth rate rise with $n$?
3. [Harder] What is the effect of a permanent increase in $s_R$ on the time path of $A_t$? Set $n = 0$. Hint: use the law of motion for $A_t$ to plot $g(A_t)$ against $A_t$.

## 4.1 Answers: Romer Model with Diminishing Returns

1. The same as in the slides, but with $\lambda = 1$: $g(A) = n/(1-\phi)$.
2. We did this in class: Without population growth, diminishing returns imply that growth peters out. Each time the population increases, there is an upward push to innovation (scale effect). We had a graph in the slides.
3. Use $g(A) = Bs_R LA^{\phi-1}$. This declines in $A$ and has a steady state where $g(A) = 0$. Start in that steady state. Raising $s_R$ pushes the $g(A)$ curve up. The growth rate is now positive, but declines over time (moving along the $g(A)$ curve towards the new steady state with higher $A$).