Review Problems: Innovation and Growth

Econ520. Fall 2022. Prof. Lutz Hendricks. November 2, 2022

Jones, Macroeconomics, problems 6.1-6.8.

1 Basics

1. What is meant by the non-rivalry of ideas?
   
   (a) Give examples of rival and non-rival goods.
   
   (b) If Roche holds a patent on a drug, does that make it a rival good?

2. Explain how non-rivalry lead to increasing returns to scale and scale effects.

3. What is meant by scale effects? Explain why they arise.

4. Define "balanced growth path."

1.1 Answers: Basics

1. See slides

2. See slides

3. See slides.

4. An equilibrium path along which all variables grow at constant rates.

2 Romer Model

1. Why is there sustained growth in the Romer model, but not in the Solow model?

2. Derive the balanced growth rate of ideas in the Romer model.

3. Suppose there is a one-time increase in the productivity of research. Describe the effect on the level and the growth rate of technology ($A$).

4. The government uses patent protection and R&D subsidies to foster growth. Could such policies overshoot their targets and actually reduce output and consumption, even in the long-run?
2.1 Answer: Romer model


Of course, the Romer model with diminishing returns grows, if there is population growth. This is due to the non-rivalry of ideas.

So there are four cases: rival / non-rival $\times$ constant returns / diminishing returns. All of them have sustained growth (with population growth), except for the Solow case (diminishing returns / rival).

2. We did this in class.

3. Increase in $\bar{z}$: faster growth ($g = \bar{z}\ell\bar{N}$). No change in $y$ at impact.

4. The short answer is: of course. Suppose we set the fraction of labor working in R&D to 1. Then output is zero.

3 Modified Romer Model

Consider the following modified Romer model:

- Production functions:
  \[ Y_t = A_t^\alpha L_{yt} \]  
  \[ \dot{A}_t = BA_t L_{at} - dA_t \]  

- Resource constraint:
  \[ L = L_{yt} + L_{at} \]  

- Allocation of labor:
  \[ L_{at} = \ell\bar{N} \]  
  \[ L_{yt} = (1 - \ell)\bar{N} \]

There are two changes relative to the original Romer model: the exponent $\alpha$ on labor in the production function for goods and the depreciation term $dA_t$ in the production function for ideas (which is the same as the depreciation term in the Solow model). Assume that $0 < \alpha < 1$ and $0 < d < 1$. 
Questions:

1. Derive the growth rates of $Y_t$ and of $A_t$ as functions of exogenous parameters.

2. Plot the time paths of $\log(Y_t)$ and $\log(A_t)$ for an economy that experiences a permanent increase in depreciation ($d$ rises) at date $t_0$. Explain what you plot.

3. Explain why the non-rivalry of ideas leads to increasing returns to scale. What does non-rivalry mean?

3.1 Answers: Modified Romer Model

1. $g(Y) = \alpha g(A)$ and $g(A) = B \ell \bar{N} - d$.

2. Straight lines with kinks at $t_0$. The $\log(Y)$ line is flatter than $\log(A)$ line. Explanation: higher $d$ reduces $g(A)$. But no jump in $A$ at $t_0$; we are not changing the stock of ideas, just the growth rate.

3. We think constant returns to all rival factors - the replication argument. But non-rival ideas do not need to be replicated. Example: build 2 factories to double output. No need to double the number of blueprints. Nonrivalry means: an idea can be used at the same time by multiple users.

4 Romer Model with Diminishing Returns

Consider the Romer model with $\dot{A}_t = BA_t^\phi s_R L_t$, $L_t = e^{nt}$ and $\rho < 1$.

1. Derive the balanced growth rate.

2. Intuitively, why does the balanced growth rate rise with $n$?

3. [Harder] What is the effect of a permanent increase in $s_R$ on the time path of $A_t$? Set $n = 0$. Hint: use the law of motion for $A_t$ to plot $g(A_t)$ against $A_t$.

4.1 Answers: Romer Model with Diminishing Returns

1. The same as in the slides, but with $\lambda = 1$: $g(A) = n/(1 - \phi)$.

2. We did this in class: Without population growth, diminishing returns imply that growth peters out. Each time the population increases, there is an upward push to innovation (scale effect). We had a graph in the slides.
3. Use \( g(A) = B s_{R} L A^{\phi - 1} \). This declines in \( A \) and has a steady state where \( g(A) = 0 \). Start in that steady state. Raising \( s_{R} \) pushes the \( g(A) \) curve up. The growth rate is now positive, but declines over time (moving along the \( g(A) \) curve towards the new steady state with higher \( A \)).