Asset Pricing

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Objectives

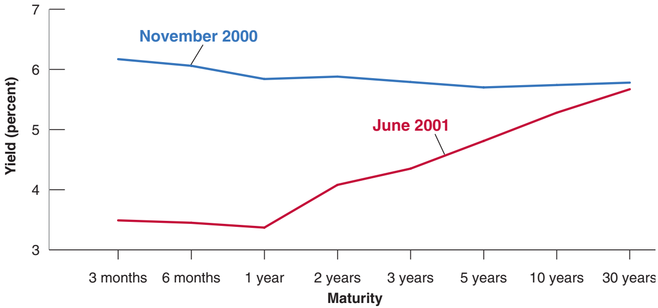
In this section you will learn:

1. how asset prices are determined as present discounted values of "dividends."
2. how asset prices are affected by policy.
3. how the yield curve can be used to predict future interest rates.

Bond Yields

The Yield Curve

The Yield Curve



The yield curve plots interest rates (yields) against bond maturities.

Here: treasury bonds

Bond Prices

A **discount bond** pays \$100 in T years.

The price of a bond is the discounted expected present value of its payments

- complication: risk

1 year discount bond: $P_{1t} = \$100/(1+i_{1t})$

- this actually defines i_{1t}, the 1 year yield from t to $t+1$

$$1+i_{1t} = \frac{\$100}{p_{1t}} \quad (1)$$

2 year bond

2 year discount bond:

- price P_{2t}
- payoff tomorrow: $P_{1,t+1}$ (it's now a 1 year bond)
- without risk: it's 1 year return must be i_{1t}

$$P_{2t} = \frac{P_{1,t+1}}{1+i_{1,t}} = \frac{\$100}{(1+i_{1t})(1+i_{1,t+1})} \tag{2}$$

Of course, $i_{1,t+1}$ is not known at t – we must use its expected value

Bond Prices

Yield on 2 year bond:
the constant interest rate that produces the bond price as present value:

$$P_{2t} = \frac{\$100}{(1+i_{2t})^2} \tag{3}$$

Therefore:

$$(1+i_{2t})^2 = (1+i_{1,t})(1+i_{1,t+1}) \tag{4}$$

Or approximately

$$i_{2,t} = \frac{1}{2}(i_{1,t} + i_{1,t+1}) \tag{5}$$

The Yield Curve

The yield curve plots yields against maturities

If the yield curve is upward sloping: $i_{1,t+1} > i_{1,t}$

- agents expect rising short term interest rates

We can interpret the yield on a T year bond as the average short term interest rate agents expect over the next T years

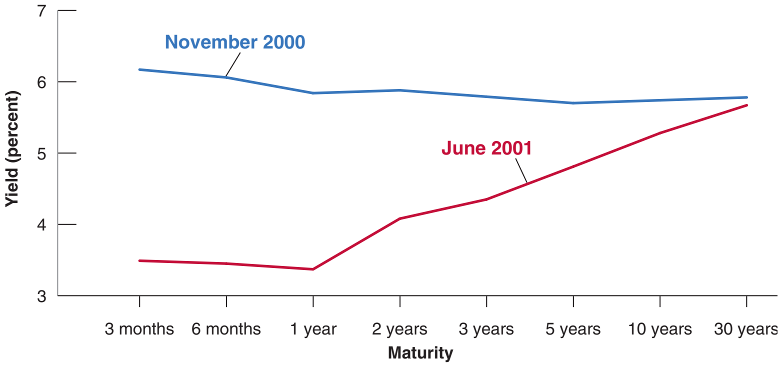
The Yield Curve

Long bond yields reveal investor expectations about future interest rates.

Qualifications:

- risk
- liquidity

Example: The 2001 Recession



Nov-2000: the tail end of the tech bubble expansion

2001 Recession

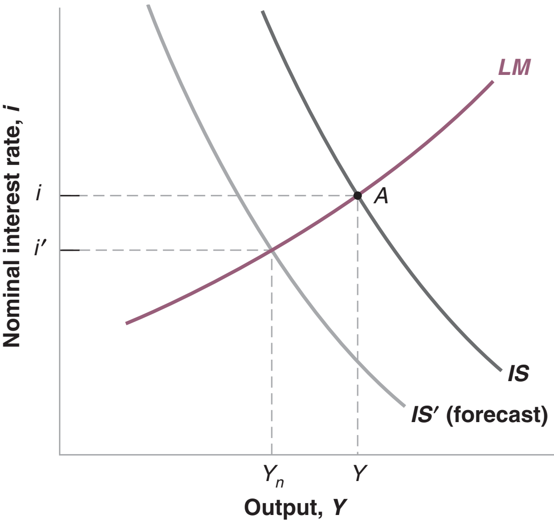
Nov-2000

- the Fed tries to slow growth with unemployment below the natural rate
- high short term interest rates
- expected slowdown of activity $\rightarrow$ lower long term interest rates

Jun-2001

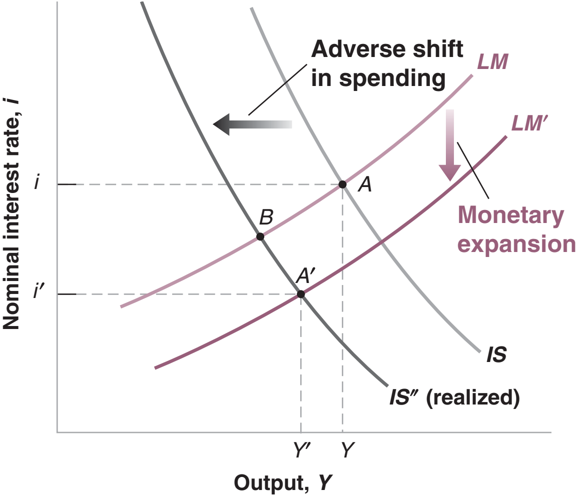
- the tech bubble burst $\rightarrow$ recession
- the Fed lowers short term interest rates
- investors expect a recovery, but not soon
- the yield curve slopes upwards

The 2001 Recession in the Model



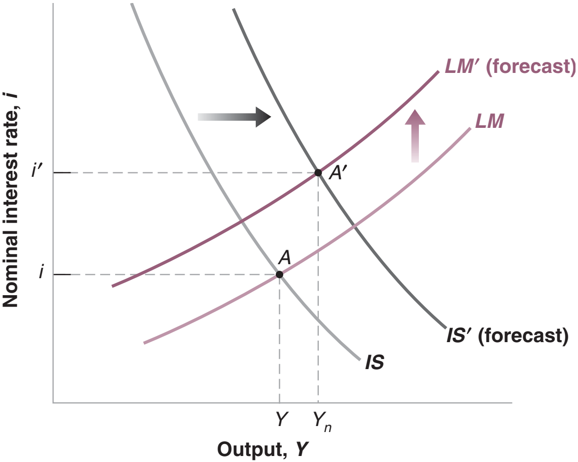
Nov-2000:
$Y > Y_n$
IS expected to shift gradually left
Yield curve is downward sloping

The 2001 Recession



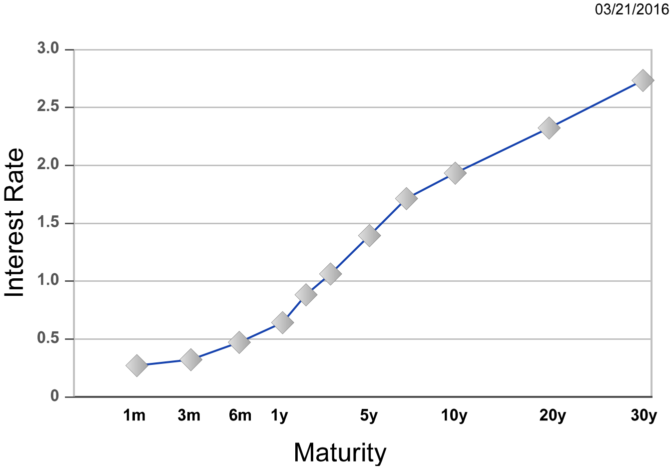
Jun-2001:
A larger than expected reduction in demand.
The Fed responds by lowering i

The 2001 Recession



Jun-2001:
Expections of gradual demand recovery
plus Fed tightening

The Current Yield Curve



Source: treasury.gov

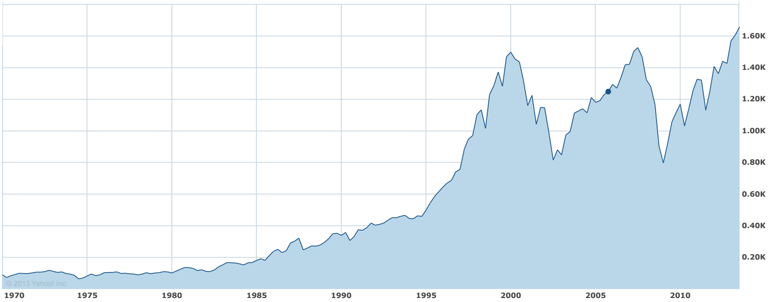
The Current Yield Curve

Key features:

1. Liquidity trap
   short rates are essentially 0
2. Rates are below 2% up to maturities of 10 years
   investors expect low interest rates for a long time
3. Long yields are quite low (2.7% nominal)
   investors expect low inflation (or low real returns)

The Stock Market

The S&P 500



Source: yahoo.com

A Simple Model of Stock Prices

- Abstract from risk premia for now and assume
  - the return on stocks = the return on bonds
- Return on bonds: $i_{1,t}$
- Return on stocks:
  - t: invest Q_t
  - $t+1$: earn dividend D_{t+1} and sell stock for Q_{t+1}
  - rate of return: $(D_{t+1}+Q_{t+1})/Q_t$

A Simple Model of Stock Prices

Equal rates of return

$$1 + i_{1,t} = \frac{D_{t+1} + Q_{t+1}}{Q_t} \qquad (6)$$

Solve

$$Q_t = \frac{D_{t+1} + Q_{t+1}}{1 + i_{1,t}} \qquad (7)$$

Now apply the same equation for Q_{t+1}

$$Q_t = \frac{D_{t+1}}{1 + i_{1,t}} + \frac{D_{t+2} + Q_{t+2}}{(1 + i_{1,t})(1 + i_{1,t+1})} \qquad (8)$$

Now apply the same for Q_{t+2}, then for Q_{t+3}, etc...

A Simple Model of Stock Prices

We end up with

$$Q_t = \frac{D_{t+1}}{Z_{t,1}} + \frac{D_{t+2}}{Z_{t,2}} + \frac{D_{t+3}}{Z_{t,3}} + ... + \frac{D_{t+n}}{Z_{t,n}} + \frac{Q_{t+n}}{Z_{t,n}} \tag{9}$$

where

$$Z_{t,n} = (1+i_{1,t})(1+i_{1,t+1})...(1+i_{1,t+n}) \tag{10}$$

discounts payoffs in $t+n$ to t.

How does this change with inflation?

A Simple Model of Stock Prices

Result

Stock prices equal the present discounted value of future dividends. This is called the fundamental value.

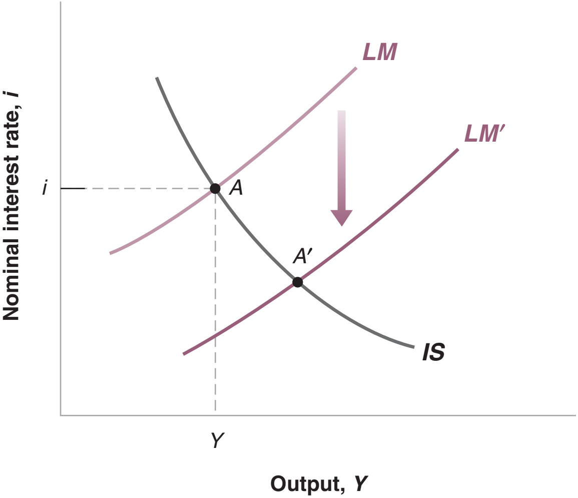
Implications:

1. High interest rates (now or in the future) depress stock prices.
2. Low dividends (now or in the future) depress stock prices.
3. Stock returns are generally unpredictable
   3.1 really: the difference between stock and bond returns is unpredictable
   3.2 this is called the **equity premium**

Caveats

1. Stocks are risky, so they generally earn higher returns than bonds
    1.1 We should discount by $r_{1,t}$ plus a **risk premium**.
2. Stock prices often deviate from fundamental values for reasons that are not well understood
    2.1 The deviations are called **bubbles**

Shocking the Stock Market

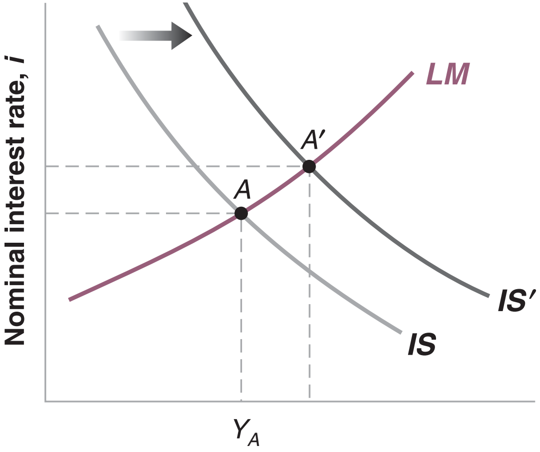


Monetary expansion:

1. Low interest rates
2. High future output and dividends

Both raise stock prices

Rise in Consumption

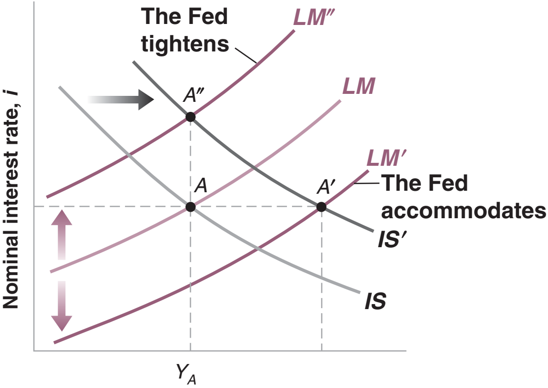


Two offsetting effects

1. higher Y
2. higher i

Change in stock prices is ambiguous

Rise in Consumption



Possible Fed reactions:

1. Accomodate
2. Stabilize

One implication: No stable relationship between output and stock prices

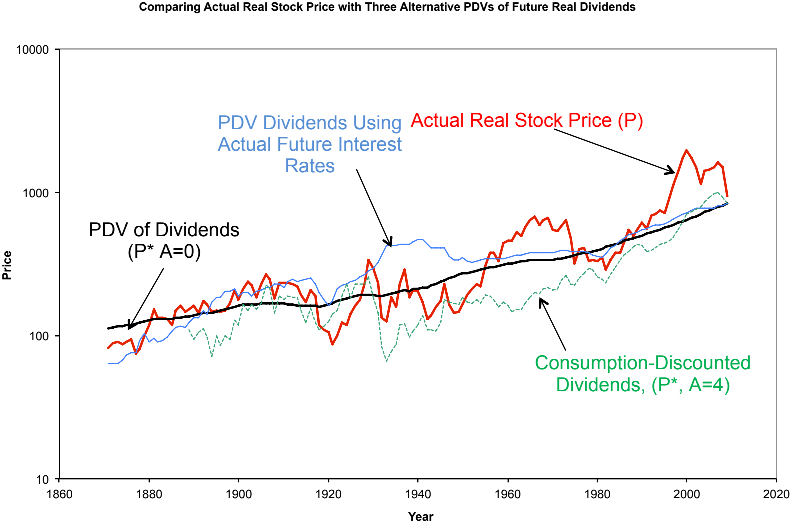
Stocks Are Too Volatile

How volatile should stocks be?

A rough approximation:

- go to a period t (before today)
- collect data on future interest rates and dividends
- compute the present value of dividends
- compare with actual stock prices

Stocks Are Too Volatile



Source: Robert Shiller
(http://www.econ.yale.edu/~shiller/data.htm)

Stocks Are Too Volatile

Notes on the Shiller graph:

1. Actual real price is the raw stock price, deflated with a price index
2. P*, A=0: present value of dividends, assuming a constant interest rate
3. P*, actual future interest rates: present value of dividends, discounted by realized future long-term interest rates
4. P*, A=4: uses the Consumption Asset Pricing Model with risk aversion of 4

Each line is scaled such that

1. the terminal value equals that last observed stock price
2. the geometric mean of the growth rates is the same

The values are actually computed backwards as $P_t = \frac{P_{t+1}+D_{t+1}}{1+r_{t+1}}$.

Why Might Stocks Be so Volatile?

1. Tail risk:
   stock prices include the risk of rare (bad) events
   e.g. major depressions; financial crises
2. Long-term risk:
   the long-run growth rate of dividends (or productivity) could fluctuate
3. Bubbles:
   fluctuations are random; not related to fundamentals

Asset Pricing With Risk

What Is Risk?

Possible answers (with counter-examples):

A Simple Model of Risk

A household lives for 2 periods.

In period 1, he receives income y and eats c_1.

In period 2, there are 2 possible states:

1. good state: income y_H with probability π_H
2. bad state: income y_L with probability π_L

Prefences:

$$u(c_1) + \underbrace{\pi_L u(c_L) + \pi_H u(c_H)}_{\text{expected utility in pd. 2}} \tag{11}$$

A Simple Model of Risk

There are 2 assets:

1. asset L pays 1 unit of consumption in state L
2. asset H pays 1 unit of consumption in state H

Budget constraints:

$$c_1 = y - p_L s_L - p_H s_H \tag{12}$$

$$c_L = y_L + s_L \tag{13}$$

$$c_H = y_H + s_H \tag{14}$$

Household Problem

$$\max_{s_L, s_H} u(y - p_L s_L - p_H s_H) + \underbrace{\sum_{j=L,H} \pi_j u(y_j + s_j)}_{\text{expected utility in pd 2}} \tag{15}$$

First-order conditions:

$$u'(c_1) p_j = \pi_j u'(y_j + s_j); j = L, H \tag{16}$$

This solves for the household's willingness to pay for the assets:

$$p_j = \pi_j \frac{u'(c_j)}{u'(c_1)}, j = L, H \tag{17}$$

Asset Prices

$$\frac{p_L}{p_H} = \frac{\pi_L}{\pi_H} \frac{u'(c_L)}{u'(c_H)} > 1 \tag{18}$$

Simplify by assuming that $\pi_j = 1/2$

Result

An asset is valuable, if it pays in a state with low consumption (high marginal utility).

What Is Risk in the Model?

Safe asset: pays $1/2$ in each state of the world.

$$p_{safe} = 0.5\,(p_L + p_H) \tag{19}$$

It follows that the "risky" asset p_L is more valuable (pays a lower expected return) than the safe asset:

$$p_H < p_{safe} < p_L \tag{20}$$

What then is risk?

CAPM

Consumption Asset Pricing Model.

Generalizes the logic of our simple model to many assets, many periods, and many states of the world.

Main result

An asset pays a high return, if its dividends are positively correlated with consumption (it pays out in good states of the world).

Simplified CAPM

2 periods, 1 asset

2 states of the world in $t+1$

$$\max_s u(y-s) + \sum_{j=L,H} \pi_j u(y_j + R_j s) \tag{21}$$

First-order condition for k:

$$u'(c) = \sum_j \pi_j u'(c_j) R_j = \mathbb{E}\left\{u'(c_j) R_j\right\} \tag{22}$$

This is a very general asset pricing equation.

It holds for many periods, many assets, many states of the world

Rates of return

Which assets pay high rates of return?

Assume $u(c) = \ln(c)$ (just to simplify notation)

$$\begin{aligned} \frac{1}{c} &= \sum_j \pi_j \frac{R_j}{c_j} && (23) \\ &= \sum_j \pi_j \frac{R_j - \bar{R}}{c_j} + \sum_j \pi_j \frac{1}{c_j} \bar{R} && (24) \end{aligned}$$

where $\bar{R} = \sum_j \pi_j R_j$ is the expected return

Expected return $\bar{R}$ is high for assets with low $\sum_j \pi_j \frac{R_j - \bar{R}}{c_j}$

Example

2 states of the world with equal probabilities

returns are $\bar{R} + \Delta R$ and $\bar{R} - \Delta R$

Case 1: asset pays high returns in **good** states

$$\sum_j \pi_j \frac{R_j - \bar{R}}{c_j} = \frac{1}{2}\left[\frac{\Delta R}{c_H} + \frac{-\Delta R}{c_L}\right] < 0 \tag{25}$$

- asset has **high** expected return

Case 2: asset pays high returns in **bad** states

$$\sum_j \pi_j \frac{R_j - \bar{R}}{c_j} = \frac{1}{2}\left[\frac{\Delta R}{c_L} + \frac{-\Delta R}{c_H}\right] > 0 \tag{26}$$

- asset has **low** expected return (lower than the safe asset)

CAPM Asset Pricing Equation

The price of an asset is the present value of dividends, discounted at the marginal rate of substitution:

$$p_t = \mathbb{E}\sum_{j=1}^{\infty} d_{t+j}\frac{u'(c_{t+j})}{u'(c_t)} \tag{27}$$

This is the basis for computing the β risk measure commonly used in finance.

Our 2 period model is a special case:

$$p_L = \pi_L \times 1 \times \frac{u'(c_L)}{u'(c_1)} + (1-\pi_L) \times 0 \times \frac{u'(c_H)}{u'(c_1)} \tag{28}$$

Excess Volatility Puzzle

In the data, consumption is very smooth.

Therefore, $u'(c)$ is very smooth.

The price of a stock should then equal the present value of smooth dividends, discounted at a roughly constant rate.

Stock prices should be smooth.

They are not.

Equity Premium Puzzle

The same asset pricing equation should hold for a riskless bond.

If consumption is very smooth, a riskless bond with a divdend of 100 should cost about the same as a a stock with a dividend of 100.

The expected return on stocks and bonds should be very similar.

They are not similar in the data.

How severe is the puzzle?

Investors forego very large returns.

Table 3

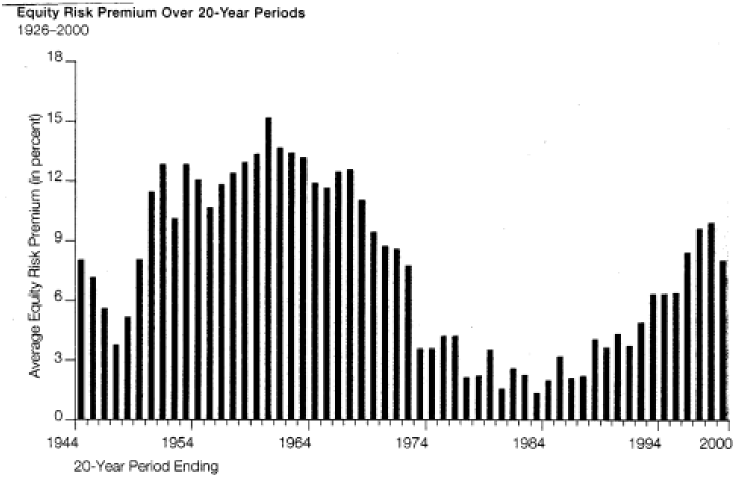
Terminal value of $1 invested in Stocks and Bonds

Investment Period	Stocks		T-bills	
	Real	Nominal	Real	Nominal
1802-1997	$558,945	$7,470,000	$276	$3,679
1926-2000	$266.47	$2,586.52	$1.71	$16.56

Source: Mehra and Prescott (2003)

Long holding periods

Over 20 year holding periods: stocks dominate bonds.



Source: Mehra and Prescott (2003)

Possible Explanations

What could explain the equity premium puzzle?

Reading

- Blanchard and Johnson (2013), ch 15

Advanced Reading:

- Mehra and Prescott (1985): the paper that discovered the equity premium puzzle
- Mehra and Prescott (2003): later perspective on the equity premium puzzle

References I

Blanchard, O. and D. Johnson (2013): *Macroeconomics*, Boston: Pearson, 6th ed.

Mehra, R. and E. C. Prescott (1985): "The equity premium: A puzzle," *Journal of monetary Economics*, 15, 145–161.

——— (2003): "The equity premium in retrospect," *Handbook of the Economics of Finance*, 1, 889–938.