Exam 1. Econ520. Spring 2021

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UNC

Instructions:

- Answer all questions.
- Write legibly.
- If you need more space, attach additional pages. Number your answers. Do not write on the back of the pages.
- *Explain* your answers – do not just state them.
- *Show* your derivations – do not just state the final result.
- Do not refer to any notes or books. You may use a calculator.
- The total time is 75 minutes.
- The total number of points is 100.

Name:	PID:

1 Short Questions

1. [10 points] Suppose we regress output per worker on capital per worker: $\log(Y/L) = \alpha + \beta \log(K/L) + \varepsilon$. Suppose that the true specification would also include human capital h: $\log(Y/L) = \alpha + \beta \log(K/L) + \gamma h + \varepsilon$. Capital and h are positively related in the data.

   (a) Would the OLS regression coefficient $\hat{\beta}$ be higher or lower than the true β? Explain the intuition.

   (b) Explain, using this example, why regressions cannot be used to answer cause-effect questions.

   Solution: (a) The OLS coefficient would be higher than the true β. It also picks up the effect of the omitted h.

   (b) Omitted variables are one problem with the causal interpretation of OLS coefficients. It may be that $h \to k$ and $h \to y$, but not $k \to y$.

2. [10 points] Assume that the aggregate production function is given by $Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}$. The interest rate is $r = q - \delta$ where q is the marginal product of capital. Suppose that $\alpha = 1/3$ and $\delta = 0.06$. If the U.S. interest rate is 4% and India has 1/20 of the U.S. capital stock per worker, what would the interest rate be in India?

   In the data, the interest rate in India is about the same as in the U.S. How could that be explained in the model?

   Solution: The interest rate is $r = q - \delta$ where $q = \partial Y / \partial K = \alpha (K/(AL))^{\alpha - 1}$

   Cutting K by 90% increases q by factor $20^{1-\alpha} = 7.3$.

   If the interest rate in the U.S. is 4% and $\delta = 0.06$, then $q_{US} = 0.1$ and $q_{IND} = 0.73$ and $r_{IND} = 0.68$.

   If India has the same interest rate as the U.S., one interpretation would be that productivity is lower by factor 20.

3. [10 points] The Solow model tells us that growth cannot be sustained through capital accumulation. The Romer models tells us that growth can be sustained through knowledge accumulation. Explain where the difference comes from.

Solution: The difference is rivalry. Both types of capital are accumulated subject to diminishing returns. Without some other force to counteract diminishing returns, the marginal product of capital goes to zero as capital is accumulated. Eventually, the MPK is too small to support growth.

Nonrival knowledge capital introduces increasing returns. Recall the example of a drug manufacturer. The increasing returns offset the diminishing returns that set in as knowledge is accumulated.

Another way of saying the same thing: With physical capital, growth requires that K/L grows. That runs into diminishing returns. With knowledge, it is enough that A grows (not A/L). Even if diminishing returns imply that A/L falls over time, we can sustain growth.

4. [10 points] If production requires non-renewable resources, such as oil, does this mean that growth cannot be sustained in the long-run (because oil extraction will eventually have to shrink)? Explain the intuition.

Solution: The short answer is "no" - growth can be sustained.

There are various ways of explaining why. A fixed factor puts a drag on growth because of diminishing returns. This is really what happens in the Solow model as we try to grow through capital accumulation.

However, if there is another factor that grows, it counteracts the diminishing returns. This is basically why the Solow model with technical change can have sustained growth in per capita output. K/L can grow, even though K runs into diminishing returns.

Put differently: any fixed input acts like a TFP term. In effect, the TFP growth rate in the model with an exhaustible factor is Ax. TFP A grows. The exhaustible input x falls over time. As long as Ax grows, the economy can grow.

2 Solow Model

Consider the standard Solow model where output is produced according to

$$Y_t = K_t^{\alpha} (AL_t)^{1-\alpha} \tag{1}$$

and capital is accumulated according to $\dot{K}_t = sY_t - \delta K_t$. Population growth is n.

Questions:

1. [10 points] Graph investment sy and depreciation $(n + \delta) k$ against capital and illustrate how a higher saving rate increases steady state k^*.

   Solution: We did this in class.

2. [20 points] In the same graph, show that k^* increases more (for a given increase in s) when the capital share is higher. What is the intuition for this result?

   Solution: For a precise answer, the key is to realize that, for given k, the sy curve shifts up by the same amount, regardless of α.

   Higher α means less curvature of the sy function. Hence, a given upward shift of sy implies a larger increase in steady state capital.

   Intuition: Higher s means higher k. The economy accumulates more capital. The limit for this stems from diminishing returns to capital. Higher α means that diminishing returns set in more slowly. The MPK falls more gradually as k rises. Hence the point where the additional savings just pay for the addititional depreciation (the steady state) arrives later.

3 Romer Model

Consider a Romer model where ideas are produced according to

$$\dot{A}_t = B\left(s_A L_t\right)^{\lambda} A_t^{\phi} \tag{2}$$

where $\lambda > 0$, $\phi < 1$, $B > 0$, and $0 < s_A < 1$ are parameters.

Questions:

1. [10 points] Derive the balanced growth rate

$$g\left(A\right) = \frac{\lambda}{1-\phi} n \tag{3}$$

Solution: We did this in class.

2. [20 points] Using a graph of $g(A)$ versus A, explain why population growth is required for sustained growth of A. Also explain why faster population growth leads to faster output growth. Explain the economic intuition for this result.

Solution: We did this in class.

End of exam.