Exam 1. Econ520. Spring 2015

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UNC

Instructions:

- Answer all questions.
- Clearly number your answers. Write legibly.
- Do *not* write your answers on the question sheets.
- *Explain* your answers – do not just state them.
- *Show* your derivations – do not just state the final result.
- Do not refer to any notes or books. You may use a calculator.
- The total time is 75 minutes.
- The total number of points is 100.

1 Short Questions

1. Ideas:

   (a) [7 points] Provide an example of how the non-rivalry of "ideas" leads to increasing returns to scale.

   (b) [7 points] Using international trade as an example, explain how the non-rivalry of ideas leads to scale effects.

2. [7 points] Briefly describe the main costs and benefits of granting long patents to innovators.

3. [9 points] Suppose that $y_t = k_t^{\alpha}$ and the growth rate of k is $g(k) = 0.05$. Compute the growth rate of y for $\alpha = 1/3$ and for $\alpha = 2/3$. What is the intuition for the difference?

2 Cross-country Income Gaps

Consider the production function $Y = K^{\alpha}(AL)^{1-\alpha}$.

We would like to assess whether variation in capital can plausibly account for a 5 fold gap in output per worker ($y = Y/L$) across two countries (Rich and Poor). Both countries share the same parameters (α and A).

Calculate each answer below for $\alpha = 1/3$ and $\alpha = 2/3$.

1. [10 points] By how much would $k = K/L$ have to differ across countries to generate a 5 fold gap in y?

2. [10 points] Suppose the rich country's interest rate is 5%. What would the poor country's interest rate be? Assume a depreciation rate of $\delta = 0.05$.

3. [10 points] Explain how your answers to the previous sub-questions differ between the cases of high and low α.

3 Solow Model

Recall the key equation of the Solow model: $\dot{k}_t = sA^{1-\alpha}k_t^{\alpha} - (n+\delta)\,k_t$.

Consider two Solow economies, labeled I and II, that differ in their capital share parameters α. Specifically, $\alpha_I < \alpha_{II}$.

The economies also differ in their productivity parameters A. To make things simple, assume that A_I and A_{II} imply the same steady state capital stocks. This means their steady state outputs must also be the same (recall $sy_{ss} = (n+\delta)\,k_{ss}$).

Questions:

1. [15 points] Plot one Solow diagram containing both economies. By Solow diagram I mean: k on the horizontal axis and sy and $(n+\delta)\,k$ on the vertical axis.

   Explain how the plots differ for the two countries.

2. [15 points] Now suppose that s doubles in both countries. In a second Solow diagram, illustrate how the higher s affects the Solow diagram for both economies.

   Explain your answer.

3. [10 points] Which economy experiences the larger change in steady state output when s increases? What is the intuition?

   (You do not need to calculate steady state output. A graphical answer suffices.)

End of exam.

4 Answers

4.1 Answers: Short questions

1. Ideas:

   (a) Suppose you have constant returns to scale to a bundle of rival inputs X: $Y = AX$. It takes F units of X to design the product. Then the first unit of Y costs $F + 1/A$. More generally, average costs are $F/Y + 1/A$, which is obviously decreasing in Y.

   (b) Scale effect: larger markets produce more innovation. Trade allows countries to pool their innovations (potentially). Therefore, mechanically, innovation inputs increase. Since each country can use the other country's innovations, total innovation increases. Then there is the effect of larger markets on the profits generated by innovations.

2. Long patents: strong incentives to innovate. But innovators have monopolies that distort prices.

3. $g(y) = \alpha g(k)$. With higher α, growing k does not run into as severe diminishing returns compared with lower α. Hence y grows faster.

4.2 Answer: Cross-country Income Gaps

1. $y = k^{\alpha} A^{1-\alpha}$. Therefore, $y_R/y_P = (k_R/k_P)^{\alpha}$. Capital would have to differ by factor $5^{1/3}$. That would be 125 for $\alpha = 1/3$ and about 11 for $\alpha = 2/3$.

2. The interest rate is $r = \alpha k^{\alpha-1} A^{1-\alpha} - \delta$. In the rich country, the marginal product must be 0.1. In the poor country, the marginal product is $\alpha (k_P/k_R)^{\alpha-1} k_R^{\alpha-1} A^{1-\alpha}$. It is higher by factor $(k_R/k_P)^{1-\alpha}$. For $\alpha = 1/3$ this would be $1,000^{2/3} = 100$, so the interest rate would be about 1,000%. For $\alpha = 2/3$ this would be $31^{1/3} = 3.1$, so the interest rate would be about 30%.

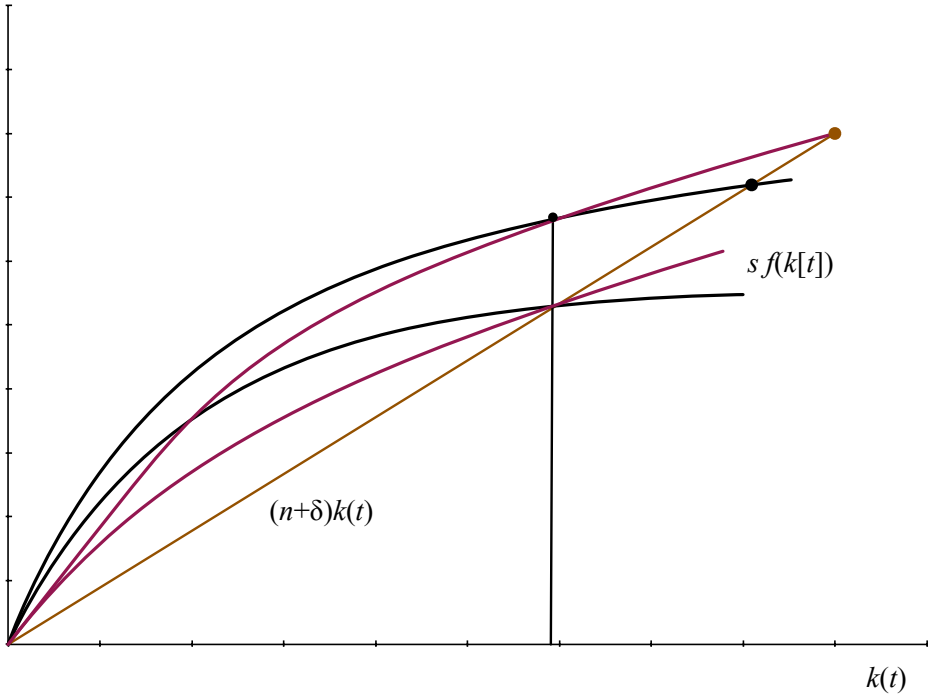
3. The difference stems from how quickly MPK rises when k falls. With low α, a country can produce a lot of output with little k (b/c MPK is very high). For the same reason, the interest rate is then very high.

4.3 Answer: Solow Model

1. See the Solow diagram in Figure 1. High α implies that the sy curve has little curvature. By construction, both cases have the same steady state k at the start.

2. In Figure 1, both sy curves shift up by the same amount (by y_{ss} times the change in s).

Figure 1: Solow Diagram



3. The economy with higher α attains the higher steady state. Graphically, this is because the sy curve has less curvature than with low α. Economically, this is because it runs into diminishing returns less quickly.

End of answers.