Exam 1. Econ520. Fall 2012

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UNC

Instructions:

- Answer all questions.
- Clearly number your answers. Write legibly.
- Do *not* write your answers on the question sheets.
- *Explain* your answers – do not just state them.
- *Show* your derivations – do not just state the final result.
- Do not refer to any notes or books. You may use a calculator.
- The total time is 75 minutes.
- The total number of points is 100.

1 Solow Model

Consider the Solow model with the following modification: The saving rate depends on the capital stock:

$$s = \bar{s}(1 - [\bar{k}/k_t]^\beta) \tag{1}$$

where $\bar{s} > 0, \bar{k} > 0$ and β are parameters. The law of motion for capital is still given by

$$\begin{aligned} \dot{k}_t &= sAk_t^\alpha - \delta k_t \quad (2) \\ &= A\bar{s}(1 - [\bar{k}/k_t]^\beta)k_t^\alpha - \delta k_t \quad (3) \end{aligned}$$

Questions:

1. [20 points] For the case $\alpha = \beta$ draw the Solow diagram (saving and depreciation against k_t).
   (a) How does the graph change when $\bar{k}$ increases?
   (b) How many steady states are there for a small $\bar{k}$? Indicate them in your graph. Which ones are stable?
2. [20 points] For the case $\alpha - \beta = 1$ draw the alternative Solow diagram (growth of k against k).
   (a) How does your graph change when $\bar{k}$ increases?
   (b) How many steady states are there for a small $\bar{k}$? Indicate them in your graph. Which ones are stable?

2 Romer Model

Recall the key equations:

$$\begin{aligned} \dot{K}_t &= sY_t - dK_t \quad (4) \\ Y_t &= A_t K_t^\alpha (s_Y L_t)^{1-\alpha} \quad (5) \\ \dot{A}_t &= \delta (s_A L_t)^\lambda A_t^\phi \quad (6) \\ n &= \dot{L}_t / L_t \quad (7) \end{aligned}$$

1. [20 points] We have learned that the difference between capital and ideas is the non-rivalry of ideas.
   (a) Explain what non-rivalry means.

(b) Where does non-rivalry show up in the equations of the Romer model? How would the equations change if ideas were rival? *Hint:* What happens to output as we double rival inputs?

(c) Here is a puzzle: the law of motion for A in the Romer model has exactly the same properties as the law of motion for K in the Solow model (without depreciation, but that is not important). Yet the Romer model generates persistent growth in per capita income, whereas the Solow model does not. How is this possible?

2. [40 points] Set $\lambda = 1 - \phi$.

   (a) Derive the balanced growth rate of A. Show your steps.

   (b) Plot $g(A) = \dot{A}/A$ against A/L. Explain the properties of your plot.

   (c) Show the effect of one-time increase in A, which might stem from opening up the economy to foreign ideas, on the time paths of $g(A)$ and A.

   If you have trouble with this question, you can set $\lambda = 1$ and $\phi = 0$, but this will not earn full credit.

End of exam.

3 Answers

3.1 Solow Model

Note that

$$\dot{k} = A\bar{s}\left[1 - \bar{k}^{\beta}k^{-\beta}\right]k^{\alpha} - \delta k \tag{8}$$

$$= A\bar{s}\left[k^{\alpha} - \bar{k}^{\beta}k^{\alpha-\beta}\right] - \delta k \tag{9}$$

1. For $\alpha = \beta$ we have $k^{\alpha-\beta} = 1$ and $\dot{k} = A\bar{s}k^{\alpha} - A\bar{s}\bar{k}^{\beta} - \delta k$.
   (a) The usual curve shifts down in parallel as $\bar{k} \uparrow$.
   (b) There are 2 steady states. The low one is unstable. The high one is stable.
2. With $\alpha - \beta = 1$: $g(k) = \dot{k}/k = A\bar{s}\left[k^{\alpha-1} - \bar{k}^{\beta}\right] - \delta$
   (a) The usual curve shifts up as $\bar{k} \uparrow$ (because $\beta < 0$).
   (b) There is 1 steady state, which is stable as usual.
3. For $\alpha = \beta$ we have $g(k) = A\bar{s}k^{\alpha-1} - \delta - A\bar{s}\bar{k}^{\beta}/k$.

   For $k \to \infty$ both $k^{\alpha-1}$ and $1/k$ go to 0 and $g(k) \to -\delta$. This is as usual. But for $k \to 0$ we have $g(k) \to -\infty$ (because the saving rate goes to $-\infty$). Alternatively, without making much of a difference, you could impose in both parts of the questions that the saving rate cannot be negative. Then $g(k) \to -\delta$ as $k \to 0$. In either case, the graph looks like Figure 1.
   (a) Since higher $\bar{k}$ lowers s, the graph must shift down everywhere. It does not matter exactly how.
   (b) Of course, the answer must be the same as in #1.

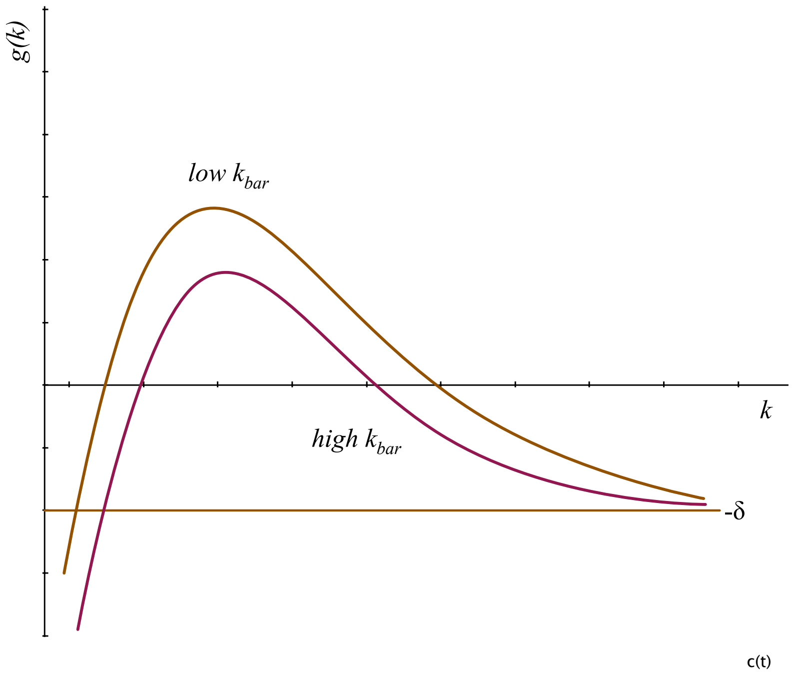

Figure 1: Solow diagram

3.2 Romer Model

1. Non-rivalry

   (a) See slides.

   (b) It shows up in the goods production function. With rival ideas, we would have constant returns to all inputs as in $Y = A^{\beta}K^{\alpha}L^{1-\alpha-\beta}$. Nothing would change in the R&D production function or in the behavior of A_t. However, increasing per capita output would now require that A/L and K/L grow over time, which is not sustainable.

   It is tempting to think the difference stems from A entering the production function linearly. But you can easily convince yourself that a model where $Y = A^{\beta}K^{\alpha}L^{1-\alpha}$ has persistent growth for any $\beta > 0$. The key is increasing returns to all inputs, not the lack of diminishing returns to A.

   (c) A and K grow in basically the same way in both models. The difference: growth in K is not enough for growth in per capita output; you need growth in K/L (rival input). But growth in A is enough, even if A/L does not grow (cf the Romer balanced growth path).

2. We have $g(A) = \delta(s_A L/A)^{\lambda}$.

   (a) On the balanced growth path we need L/A to be constant, so that $g(A) = n$.

   (b) The plot is downward sloping, as usual.

(c) No shift in the curve. Higher A moves the economy to the right. The growth rate of A is now below n and A/L moves down over time. A/L experiences a one-time jump up. But then the growth rate is negative until it falls back to its original level. There are no permanent effects of this "gift of knowledge."

End of answers.