

# Exam 1. Econ499. Spring 2011

Professor Lutz Hendricks

UNC

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## **Instructions:**

- The exam consists of 3 questions.
- Answer all questions.
- *Explain* your answers – do not just state them.
- *Show* your derivations – do not just state the final result.
- Do not refer to any notes or books. You may use a calculator.
- The total time is 75 minutes.
- The total number of points is 100.

# 1 Production Model

Consider the production function  $Y = AK^\alpha(hL)^{1-\alpha}$ .  $h$  is a measure of human capital per person. Assume  $\alpha = 0.5$ . You have data for two countries:

Country	Y/L	K/L	h
1	\$10,000	\$20,000	1
2	\$3,000	\$6,000	0.5

- [4 points] Write the production function in per capita terms ( $y = Y/L$  as a function of  $k = K/L$ ).
- [6 points] Compute each country's productivity level,  $A$ .
- [12 points] Decompose the output gap between the two countries,  $y_1/y_2$ , into the contributions of capital, human capital, and productivity.
- [12 points] Plot the production functions of the two countries (not to scale). Show the contributions of  $K/L$  and ( $A$  and  $h$ ) (combined) to the  $Y/L$  gap between the two countries. (It's hard to separately plot the contributions of  $A$  and  $h$ .)

# 2 Solow Model

Consider a modified Solow model that gives rise to the following law of motion for per capita capital:

$$\dot{k}_t = sAk_t^\alpha - [\bar{\delta} + \delta k_t^\alpha] \quad (1)$$

Relative to the usual Solow model, the depreciation part has changed. Some amount  $\bar{\delta}$  depreciates no matter what. In addition, depreciation is no longer linear in capital ( $\delta k_t^\alpha$  instead of  $\delta k_t$ ). Assume that  $sA - \delta > 0$ .

- [8 points] What happens to the change in capital ( $\dot{k}_t$ ) as capital goes to 0? What happens as capital gets large?
- [8 points] Derive the steady state capital stock.
- [12 points] Draw the Solow diagram for this model. Hint: it is easiest to plot  $(sA - \delta)k_t^\alpha$  as one line and  $\bar{\delta}$  as the second.
- [7 points] Is the steady state stable? Explain.
- [8 points] What happens to the *growth rate* of capital as time goes by. Assume that the economy starts with  $k_0$  above the steady state. Explain.

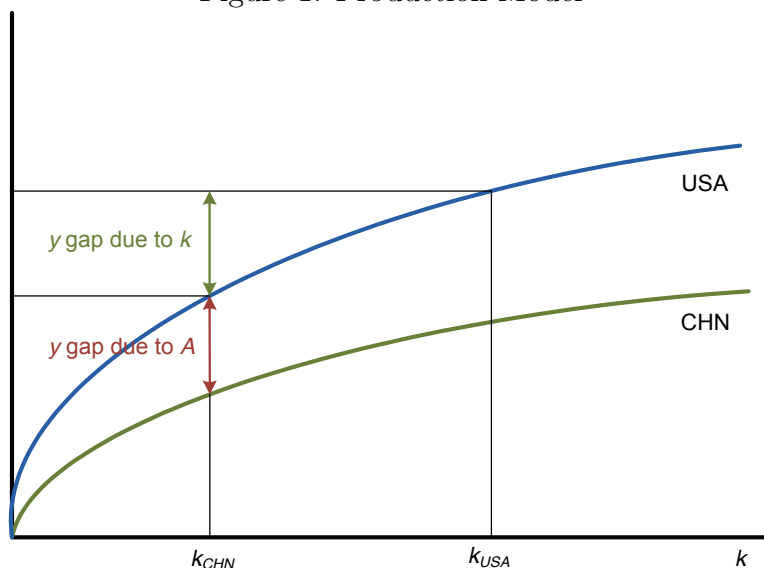
### 3 Shorter Questions

1. [9 points] Assume that a variable  $x$  starts at value 1, then grows at 5% per year for the first 20 years and at 3% per year for the next 30 years. Plot  $\log(x)$  over time and explain your graph. Calculate the value of  $x$  at the end of the 50 year period.
2. [14 points] Briefly summarize how the evidence on former colonies supports the hypothesis that institutions are important for cross-country income gaps.

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End of exam.

Figure 1: Production Model



## 4 Answers

### 4.1 Production Model

1.  $y = Ak^\alpha h^{1-\alpha}$ .
2.  $A = y / (k^\alpha h^{1-\alpha})$ .  $A_1 = 70.7$  and  $A_2 = 54.8$ .
3. Just take the ratio of the two production functions. Productivity:  $A_1/A_2 = 1.29$ . Capital:  $(k_1/k_2)^\alpha = 1.83$ . Human capital:  $(h_1/h_2)^{(1-\alpha)} = 1.41$ .
4. See figure 1.

### 4.2 Solow Model

1. Write  $\dot{k}_t = (sA - \delta)k_t^\alpha - \bar{\delta}$ . As  $k \rightarrow 0$  we find  $\dot{k} < 0$ . As  $k \rightarrow \infty$  we find  $\dot{k} \rightarrow \infty$ .
2. Steady state:

$$k^* = \left( \frac{\bar{\delta}}{sA - \delta} \right)^{1/\alpha} \quad (2)$$

3. This looks like a Solow model, except that the depreciation line is horizontal at  $\bar{\delta}$ .
4. The steady state is not stable. To its right,  $k$  keeps growing forever.
5. The growth rate is  $\dot{k}/k = (sA - \delta)k^{\alpha-1} - \bar{\delta}/k$ . It goes to zero as time goes by. Capital keeps growing, but at ever smaller rates. This is due to diminishing marginal products.

### 4.3 Short Questions

1.  $x(50) = x(0)(1.05)^{20}(1.03)^{30} = 6.44$ . The plot is a straight line, starting at  $\log(x) = 0$  with a slope of 0.05 for 20 years and a slope of 0.03 for 30 years.
2. See the class slides. Key points:
  - (a) The evidence: reversal of fortune among colonies, but not among non-colonies. Current institutions are correlated with measures of GDP and settler mortality in the 16th century.
  - (b) Interpretation: Some colonies could be settled (low population density; few diseases). In those colonies, “European style” institutions were put in place. Other colonies could not be settled (large indigenous population; diseases). In those colonies, repressive institutions were put in place. Institutions are highly persistent over time and still affect economic outcomes today.

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