

# **Technical Appendix:**

## **How Do Taxes Affect Human Capital? The Role of Intergenerational Mobility**

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When using material from this technical appendix, please reference the underlying paper “How Do Taxes Affect Human Capital? The Role of Intergenerational Mobility”, *Review of Economic Dynamics*, 2002.

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# 1. Labor-Leisure Choice

This section extends the baseline model by allowing the household to choose the amount of time consumed as leisure. For simplicity, I abstract from altruistic bequests. In order to determine how abstracting from leisure choice affects the outcomes of the policy experiments, I also study a special case of the model in which leisure is fixed.

## 1.1 Household Problem

The structure of the household problem is as follows. The household first solves the problem of lifetime utility maximization conditional on choosing each schooling level. The household then picks the level of  $s$  with the highest utility net of education costs.

### 1.1.1 Household Problem for Given $(q, s)$

Once the household has chosen a schooling level  $s$ , his optimization problem is as follows.

$$\max \sum_{a=a_C}^{a_L} \beta^a u(c_{b,a}, l_{b,a})$$

subject to the law of motion for human capital, which is unchanged,

$$(1) \quad h_{b,a+1} = (1 - \delta_h) h_{b,a} + G(h_{b,a}, x_{b,a}, v_{b,a}, q)$$

and subject to a sequence of budget constraints (displayed in the main text). Alternatively, a present value budget constraint can be imposed:

$$(2) \quad \sum_{a=a_0}^{a_L} \frac{y_{b,a} + z_{b,a} - p c_{b,a} - d_{b,a}^s}{D_{b,a}} = 0$$

Here,  $l$  denotes leisure,  $d$  is a tuition payment during school time, and  $y$  represents the earnings flow net of wage and job-training taxes. The Lagrangean for this problem is

$$\begin{aligned} & \max \sum_{a=a_C}^{a_L} \beta^a u(c_{b,a}, l_{b,a}) \\ & + \lambda_b \left\{ \sum_{a=a_0}^{a_L} D_{b,a}^{-1} \left[ \bar{y}_{b,a} - T_w(\bar{y}_{b,a}) - d_{b,a}^s - p c_{b,a} - \tau_{x,b+a-1} x_{b,a} + z_{b,a} \right] \right\} \\ & + \sum_{a=a_S+1}^{a_L} \frac{\mu_{b,a}}{D_{b,a}} \left\{ G(h_{b,a}, x_{b,a}, v_{b,a}, q) + (1 - \delta_h) h_{b,a} - h_{b,a+1} \right\} \end{aligned}$$

$\lambda$  and  $\mu$  denote Lagrange multipliers. Assuming an interior solution, the first-order conditions are:

$$(3) \quad \beta^a u_c(b, a) = \lambda_b p / D_{b,a}; \quad a = a_C, \dots, a_L$$

$$(4) \quad \beta^a u_l(b, a) = \lambda_b w_{b+a-1} h_{b,a} / D_{b,a}$$

where  $w_{b+a-1} = \omega_{b,a}^S [1 - T'_w(b, a)]$  is the marginal wage rate after taxes.

$$(5) \quad \lambda_b w_{b+a-1} h_{b,a} = \mu_{b,a} G_v(b, a)$$

$$(6) \quad \lambda_b \pi_{b+a-1} = \mu_{b,a} G_x(b, a)$$

where  $\pi_{b+a-1} = 1 - T'_w(b, a) + \tau_{x,b+a-1}$  is the marginal cost of  $x$  to the household.

$$(7) \quad \mu_{b,a-1} R_{b,a} = \lambda_b w_{b+a-1} (1 - l_{b,a} - v_{b,a}) + \mu_{b,a} [G_h(b, a) + 1 - \delta_h]$$

Here, it is understood that the previous four equations only hold for  $a = a_S + 1, \dots, a_R - 2$ . At age  $a_R - 1$ , the household does not invest in human capital and  $v_{b,a} = x_{b,a} = \mu_{b,a} = 0$  and  $l_{b,a} = l^R$  which is the exogenous level of leisure during retirement. During schooling ( $a = a_S$ ) the household enjoys the exogenous leisure level  $l^S$  and job-training investment is, of course, zero. The first-order conditions can be simplified as follows. Leisure is determined by

$$(8) \quad \frac{u_c(b, a)}{u_l(b, a)} = \frac{p}{w_{b+a-1} h_{b,a}}$$

if  $l$  is interior. The first-order conditions for  $x$  and  $v$  imply

$$(9) \quad \frac{G_v(b, a)}{G_x(b, a)} = \frac{w_{b+a-1} h_{b,a}}{\pi_{b+a-1}}$$

The law of motion for the shadow price of human capital is

$$(10) \quad \mu_{b,a-1} R_{b,a} = \lambda_b w_{b+a-1} \left\{ 1 - l_{b,a} - v_{b,a} + \frac{G_h(b, a) + 1 - \delta_h}{G_v(b, a) / h_{b,a}} \right\}$$

A solution of the household problem then consists of age profiles for  $c$ ,  $l$ ,  $v$ ,  $x$ ,  $h$ ,  $\mu$  and a scalar  $\lambda$  that solve (1), (3), (6), (8), (9), (10), and the present value budget constraint (2). Asset holdings follow residually from the flow budget constraint.

If leisure is fixed, the problem is modified as follows. The first-order condition for leisure (4) is replaced by an exogenous age-leisure profile. The other conditions determining household behavior and equilibrium remain unchanged.

Note that the problem with fixed leisure is still not identical to the one reported in the main text. One minor difference is that the agents' endowment available for market time differs from the one in the main text. A more important difference is due to a peculiar feature of the household problem adopted from Heckman et al. (1998, 1999). In their education choice the households maximize the present value of earnings despite the fact that utility consists not only of the discounted stream of the  $u(c_{b,a})$ , but also subtracts the nonpecuniary education cost  $p_s$ . In the main text I nonetheless adopt Heckman et al.'s formulation because it provides a well-known and empirically successful benchmark model of education choice. Moreover, none of the results reported in the paper are modified, if agents are assumed to choose the education level that yields maximum lifetime utility instead. In other words, the model with fixed leisure presented in this appendix yields the same qualitative insights as the model presented in the main text.

### 1.1.2 Solution Algorithm

The household problem is solved by iterating over guesses for  $\lambda_b$ ,  $h_{b,a}$ , and  $v_{b,a}$ . Given  $\lambda$ , the first-order condition for consumption (3) determines the marginal utility of consumption in all periods. The first-order condition (8) determines  $c/l$ . Together, these can be solved for the levels of  $c$  and  $l$  at all dates. The shadow price  $\mu$  is then updated using (7), starting from the last date using the guesses for  $v$ ,  $x$ ,  $h$  and the updated  $l$ . Next, the guess for  $v$  is updated by solving (5) for  $v$ :

$$(vh)^{\phi+\psi-1} = \frac{G_v}{h\phi B} \left( \frac{x}{vh} \right)^{-\psi},$$

where  $x/vh = (w/\pi)\phi/\psi$  from (9) and  $G_v/h = w\lambda/\mu$ . The values of  $x$  are obtained from the identity  $x = (x/vh)vh$ . Finally, the guesses for  $h$  are updated by iterating over its law of motion.

### 1.1.3 Functional Forms

The utility function is  $u(c,l) = (cl^\xi)^{1-\sigma}/(1-\sigma)$ . Hence,  $u_c = (1-\sigma)u(c,l)/c = c^{-\sigma}l^{\xi(1-\sigma)}$  and  $u_l = \xi(1-\sigma)u(c,l)/l = \xi c^{1-\sigma}l^{\xi(1-\sigma)-1}$ , so that  $u_c/u_l = l/(\xi c)$ . When  $l$  is interior, consumption is then determined by (3) together with  $u_c(b,a) = c^{\xi(1-\sigma)-\sigma} [\xi u_c/u_l]^{\xi(1-\sigma)}$ .

When  $l$  is fixed, as during retirement, consumption is determined by (3) together with  $u_c = c^{-\sigma}l^{\xi(1-\sigma)}$ .

## 1.2 Education Choice

After solving the above problem for each possible choice of  $s$ , the household compares the present discounted values of utility associated with each choice. It chooses the level of  $s$  that offers the largest present value of utility, net of its idiosyncratic draw of the non-pecuniary education cost. The education cost is determined as in the main text.

## 1.3 Equilibrium

The definition of competitive equilibrium is nearly the same as for the baseline model. Leisure profiles  $l_{b,a}^{q,s}$  are added to the list of equilibrium objects. The labor market clearing condition now reads

$$L_t^s = \sum_q \sum_{a=a_S+1}^{a_R} \Pr_{t-a+1}(q, s) (1 - l_{t-a+1,a}^{q,s} - v_{t-a+1,a}^{q,s}) h_{t-a+1,a}^{q,s}.$$

## 1.4 Parameters

The only parameters that need to be chosen in addition to those described in the main text are those related to leisure preferences. Based on time-use studies (Juster and Stafford 1991) the leisure preference parameter  $\xi$  is chosen such that households spend on average 55 percent of their time endowment on leisure during ages 25 to 64. This leisure share is lower than the values typically used in the literature, but the figures are not comparable because the literature uses infinite horizon models. Leisure during retirement is set to  $l^R = 1$  and during school it is set to  $l^S = 0.7$ . For the case of fixed leisure, I assume at constant leisure level of 0.7 during work life.

## 1.5 Results

This section presents numerical simulation results for the same experiments as reported in the main text, except that the household is now allowed to choose labor supply. The calculations show that all of the findings reported in the paper continue to hold. Labor-leisure choice alters the outcomes of most experiment substantially, but the comparisons between models with alternative assumptions about intergenerational persistence are similar to those reported in the paper.

First, consider experiments that do not directly distort job-training decisions. Table 1 shows the changes in aggregates due to a move from progressive to proportional income taxation. The first column shows the outcomes for the model with fixed leisure and no intergenerational

persistence. The second column adds labor-leisure choice. The third column adds intergenerational persistence of ability and education.

Leisure choice implies much larger level effects of the tax change (compare columns 1 and 2). This finding is well-known from the literature (e.g. Trostel 1993). However, the discrepancy between the models with and without intergenerational persistence are even smaller than in the baseline model without labor-leisure choice (compare columns 2 and 3). Similar findings hold for other experiments that do not directly distort the incentives for job-training investment, such as the move to a consumption tax or the introduction of an education subsidy. The results are therefore not reported.

[INSERT TABLE 1 HERE]

Next consider a policy change that directly distorts job-training decisions. As in the main text, the experiment subsidizes job-training inputs by 40% ( $\tau_x = -0.4$ ), financed by adjusting lump-sum transfers. The results are shown in table 2 for the same three models as above. As in the case of the proportional tax system, leisure choice substantially magnifies the effects of the policy change. Consistent with the results reported in the main text, the discrepancy between models with and without intergenerational persistence is somewhat larger than for experiments that do not strongly distort job-training investment. In sum, allowing for labor-leisure choice does not alter any of the steady state conclusions derived from the baseline model.

[INSERT TABLE 2 HERE]

## **2. Alternative Policy Experiments**

This section reports the outcomes for several policy experiments commonly studied in the literature. The main conclusion is that in each case a conventional life-cycle model yields outcomes that are quite similar to the baseline model with realistic intergenerational mobility.

The first experiment is a revenue neutral move to a consumption tax (see Heckman et al. 1999; Davies and Whalley 1991). This experiment sets all income tax rates to zero and adjusts the consumption tax rate to maintain government revenues unchanged. The results are reported in table 3, which mirrors the structure of table 4 in the text. The first column shows the changes in aggregates generated by a standard life-cycle model. The second column adds intergenerational persistence of education. The third column adds persistence of ability and represents the findings from a model with realistic intergenerational mobility. As in the proportional tax experiment, intergenerational persistence has little impact on the tax effects.

[INSERT TABLE 3 HERE]

Table 4 shows the results from eliminating capital income taxation, which is a commonly studied tax reform in the literature (e.g., Lucas 1990, Davies and Whalley 1991). The wage tax rate is adjusted to maintain constant government revenues. Table 5 displays the outcomes of subsidizing tuition by 25 percent, financed by adjusting lump-sum transfers (e.g., Heckman et al. 1999). In both cases, the results confirm the main finding of the paper that a conventional life-cycle model closely approximates the properties of the baseline model.

[INSERT TABLE 4 HERE]

[INSERT TABLE 5 HERE]

The conclusions for transitional dynamics are similar. To illustrate, figure 1 shows the trajectory of aggregate output following the move to consumption taxation (the experiment underlying table 3). The trajectory generated by the model without intergenerational persistence is close to those of the baseline model with realistic persistence.<sup>1</sup>

[INSERT FIGURE 1 HERE]

Table 6 replicates an experiment of Davies and Whalley (1991). It is a move from proportional income to consumption taxation, where job-training investment is not tax-deductible. The initial steady state has proportional labor and capital income taxes ( $\tau_K = 0.5$ ;  $\tau_w = 0.2$ ). The non-deductibility of job-training inputs is captured by setting  $\tau_x = -\tau_w$ . The experiment sets income tax rates and  $\tau_x$  to zero. A consumption tax is introduced that keeps tax revenues unchanged. For this experiment, intergenerational persistence magnifies the tax effects by similar amounts as for the job-training subsidy studied in the main text. In both cases output increases by an additional 1.5% due to intergenerational persistence.

[INSERT TABLE 6 HERE]

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<sup>1</sup> The transition path is shown only for the first 140 years, but is computed for 500 years to ensure complete convergence to the new steady state.

### 3. Tables for Technical Appendix

Table 1. Steady state effects of moving to a proportional income tax.

Percentage changes	Fixed leisure. No intergenerational persistence	Labor-leisure choice. No intergenerational persistence	Labor-leisure choice. Realistic persistence
Output	1.4	9.5	9.5
Capital stock	-0.8	2.1	2.0
Labor input	2.3	12.9	12.8
High school labor input ( $L^1$ )	1.0	11.8	11.8
College labor input ( $L^2$ )	5.0	15.2	15.0
High school wage rate ( $\omega^1$ )	0.0	-2.3	-2.4
College wage rate ( $\omega^2$ )	-2.7	-4.3	-4.3
Quintile ratio	10.2	15.5	15.8
College premium	54.6	86.1	94.7
Galton coefficient for earnings	--	--	12.9
Percentage point changes			
Fraction with college	-3.2	-5.9	-6.3
$q = 1$	-8.3	-8.4	-8.3
$q = 2$	-7.3	-10.5	-11.4
$q = 3$	-5.7	-10.3	-11.9
$q = 4$	8.2	5.7	6.1
Fraction among college educated with $q = 1$	-9.0	-9.0	-8.7
with $q = 4$	18.5	25.0	28.2
$\Pr(q = 4)$	0.0	0.0	0.3
$\Pr(s = 2   s^P)$ with $s^P = 1$	-3.2	-5.9	-3.4
with $s^P = 2$	-3.2	-5.9	-4.3



Table 2. Steady state effects of a 40% job-training subsidy

Percentage changes	Fixed leisure. No intergenerational persistence	Labor-leisure choice. No intergenerational persistence	Labor-leisure choice. Realistic persistence
Output	3.2	9.4	11.0
Capital stock	-7.1	-5.1	-3.9
Labor input	7.9	16.2	18.1
High school labor input ( $L^1$ )	9.5	18.3	20.3
College labor input ( $L^2$ )	4.6	11.9	13.4
High school wage rate ( $\omega^1$ )	-5.4	-7.0	-7.2
College wage rate ( $\omega^2$ )	-2.3	-3.4	-3.3
Quintile ratio	-3.2	-5.1	-5.2
College premium	53.5	82.5	79.3
Galton coefficient for earnings	--	--	13.9
Percentage point changes			
Fraction with college	-5.8	-8.2	-8.1
$q = 1$	-8.4	-8.4	-8.4
$q = 2$	-11.5	-14.3	-13.9
$q = 3$	-8.9	-14.6	-16.5
$q = 4$	5.5	4.4	2.9
Fraction among college educated with $q = 1$	-9.0	-8.7	-8.8
with $q = 4$	24.5	34.5	37.4
Pr( $q = 4$ )	0.0	0.0	2.0
Pr( $s = 2 \mid s^P$ ) with $s^P = 1$	-5.8	-8.2	-4.1
with $s^P = 2$	-5.8	-8.2	-7.7

Table 3. Steady state effects of a consumption tax.

Percentage changes	No intergenerational transmission	Only transmission of schooling	Baseline model
Output	5.5	5.5	6.0
Capital stock	9.8	9.8	10.3
Labor input	3.7	3.7	4.2
High school labor input ( $L^1$ )	1.6	1.7	2.1
College labor input ( $L^2$ )	8.1	7.8	8.6
High school wage rate ( $\omega^1$ )	3.2	3.1	3.1
College wage rate ( $\omega^2$ )	-1.1	-1.0	-1.2
Quintile ratio	10.9	11.5	11.2
College premium	66.8	67.4	68.5
Galton coefficient for earnings	--	--	5.3
Percentage point changes			
Fraction with college	-3.8	-3.8	-3.9
$q = 1$	-8.1	-7.6	-7.7
$q = 2$	-9.4	-9.9	-10.2
$q = 3$	-10.3	-10.9	-11.5
$q = 4$	12.7	13.0	12.8
Fraction among college educated with $q = 1$	-8.7	-8.0	-8.2
with $q = 4$	26.3	27.0	28.6
Pr( $q = 4$ )	0.0	0.0	0.6
Pr( $s = 2 \mid s^P$ ) with $s^P = 1$	-3.8	0.8	-2.1
with $s^P = 2$	-3.8	-15.0	-2.6

Table 4. Steady state effects of eliminating the capital income tax

Percentage changes	No intergenerational transmission	Only transmission of schooling	Baseline model
Output	3.7	3.7	4.0
Capital stock	10.1	10.1	10.4
Labor input	1.0	1.0	1.4
High school labor input ( $L^1$ )	0.8	0.8	1.2
College labor input ( $L^2$ )	1.5	1.4	1.8
High school wage rate ( $\omega^1$ )	2.8	2.7	2.7
College wage rate ( $\omega^2$ )	2.3	2.3	2.3
Quintile ratio	-0.5	-0.5	-0.5
College premium	10.1	10.9	10.4
Galton coefficient for earnings	--	--	0.8
Percentage point changes			
Fraction with college	-0.5	-0.6	-0.5
$q = 1$	-1.5	-1.6	-1.5
$q = 2$	-0.9	-1.0	-1.2
$q = 3$	-0.8	-0.9	-1.0
$q = 4$	1.2	1.3	1.2
Fraction among college educated with $q = 1$	-1.5	-1.6	-1.6
with $q = 4$	2.5	2.7	3.1
Pr( $q = 4$ )	0.0	0.0	0.3
Pr( $s = 2 \mid s^P$ ) with $s^P = 1$	-0.5	0.0	-0.3
with $s^P = 2$	-0.5	-1.4	-0.2

Table 5. Steady state effects of an education subsidy

Percentage changes	No intergenerational transmission	Only transmission of schooling	Baseline model
Output	-0.4	-0.4	-0.1
Capital stock	0.2	0.1	0.4
Labor input	-0.6	-0.7	-0.3
High school labor input ( $L^1$ )	-2.2	-2.5	-2.1
College labor input ( $L^2$ )	2.9	3.4	3.5
High school wage rate ( $\omega^1$ )	1.4	1.6	1.5
College wage rate ( $\omega^2$ )	-2.1	-2.5	-2.4
Quintile ratio	8.7	9.4	8.7
College premium	-71.3	-76.6	-73.4
Galton coefficient for earnings	--	--	-2.2
Percentage point changes			
Fraction with college	4.8	5.3	5.1
$q = 1$	15.6	16.8	15.4
$q = 2$	5.6	6.1	6.6
$q = 3$	1.2	1.5	1.7
$q = 4$	-3.1	-3.2	-3.5
Fraction among college educated with $q = 1$	12.4	13.2	11.9
with $q = 4$	-11.0	-11.7	-11.5
Pr( $q = 4$ )	0.0	0.0	0.2
Pr( $s = 2 \mid s^P$ ) with $s^P = 1$	4.8	1.3	2.9
with $s^P = 2$	4.8	7.5	2.7

Table 6. Steady state effects of a consumption tax. Job-training not tax-deductible.

Percentage changes	No intergenerational transmission	Only transmission of schooling	Baseline model
Output	14.3	14.3	15.8
Capital stock	36.5	36.5	38.0
Labor input	5.9	5.9	7.4
High school labor input ( $L^1$ )	4.3	4.4	5.8
College labor input ( $L^2$ )	9.4	9.3	10.9
High school wage rate ( $\omega^1$ )	9.1	9.0	8.9
College wage rate ( $\omega^2$ )	5.5	5.6	5.4
Quintile ratio	-2.1	-1.9	-1.9
College premium	80.4	83.1	79.9
Galton coefficient for earnings	--	--	8.7
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Percentage point changes			
Fraction with college	-4.5	-4.6	-4.5
$q = 1$	-8.4	-8.4	-8.4
$q = 2$	-11.5	-11.7	-12.0
$q = 3$	-12.5	-13.3	-13.8
$q = 4$	14.5	14.9	13.5
Fraction among college educated with $q = 1$	-9.1	-9.1	-9.1
with $q = 4$	31.7	32.8	34.5
Pr( $q = 4$ )	0.0	0.0	1.4
Pr( $s = 2 \mid s^P$ ) with $s^P = 1$	-4.5	1.0	-2.2
with $s^P = 2$	-4.5	-19.0	-4.1

### 4. Figures for Technical Appendix

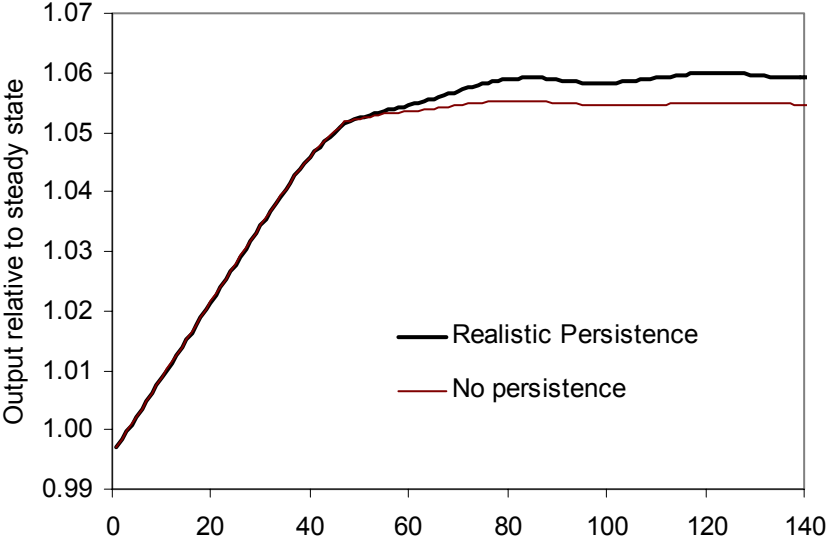


Figure 1. Transition path following the move to consumption taxation

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