Skilled Labor Productivity and Cross-country Income Differences*

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Abstract

This paper extends development accounting to an environment that features imperfectly substitutable skills and cross-country variation in the relative human capital and techological productivity of skilled labor. We obtain two main results. First, human capital accounts for between one-half and three-fourths of cross-country income gaps. Second, human capital accounts for only modest variation in the relative productivity of skilled versus unskilled labor. These findings remain robust when we consider alternative shifters of skilled labor productivity, alternative definitions of skilled and unskilled labor, and alternative values for the elasticity of substitution between skilled and unskilled labor. Based on analytical solutions, we provide precise intuition about the data features that give rise to our main results.

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1 Introduction

This paper studies two questions that have featured prominently in development accounting research. How large are the contributions of human capital and technology for understanding cross-country income differences? Are differences in the relative productivities of skilled versus unskilled workers mainly due to human capital or technological skill bias? We revisit these questions in an environment with imperfect substitution between skill types. We depart from the literature by allowing for multiple margins of relative supply and demand to affect relative labor productivity and the skilled wage premium, including the quantity and quality of labor supply, skill-biased technology, and capital-skill complementarity.

Motivation Our work builds on a sizable development accounting literature. Development accounting decomposes cross-country differences in output per worker into the contributions of factor inputs and total factor productivity. Its objective is to shed light on the proximate sources of cross-country income differences.[1]

Most of the early development accounting literature focuses on the case where workers with different levels of skills are perfect substitutes (Hall and Jones, 1999; Bils and Klenow, 2000; Caselli, 2005). The typical finding is that the gaps in quality-adjusted units of the single type of labor are not large enough to account for much of cross-country income differences.

The perfect substitutes assumption is at odds with an extensive literature that documents large, systematic movements in relative wages, for example over time in the United States (Katz and Murphy, 1992), and attributes them to an underlying race between relative labor supply and relative labor demand (Goldin and Katz, 2008).[2]

This literature has motivated recent work that extends development accounting to environments where skills are imperfect substitutes. However, there is substantial disagreement about the implications. While Jones (2014), Hendricks and Schoellman (2018) and Jones (2019) find a much larger role for human capital in development accounting than earlier work, Caselli and Coleman (2006), Caselli and Ciccone (2013), Caselli (2016) and Caselli and Ciccone (2019) do not.

To understand the source of the disagreement it is useful to revisit the empirical argument. With imperfect substitution, the scarcity of skilled workers should drive up skill premiums

[1] See the surveys by Caselli (2005), Hsieh and Klenow (2010), and Caselli (2016). This work complements a parallel literature that develops quantitative theories of human capital formation, including Erosa et al. (2010), Córdoba and Ripoll (2013), and Cubas et al. (2016).

[2] Relative labor supply can be shifted by the number or quality of skilled versus unskilled workers. Relative labor demand can be shifted by skill-biased technical change (see Acemoglu, 1998, 2002; Gancia and Zilibotti, 2009, and Jerzmanowski and Tamura, 2019) or the accumulation of capital that complements skilled workers (see Krusell et al., 2000 and Parro, 2013).

in low income countries. However, empirical evidence indicates that skill premiums are roughly similar in rich and poor countries (Banerjee and Duflo, 2005), suggesting that skilled labor is relatively less productive in low income countries.

The disagreement in the literature relates to the interpretation of these skilled labor productivity differences. Jones (2014) attributes them to variation in the human capital of skilled workers. In this case, poor countries are scarce in the share and average human capital of skilled workers, magnifying cross-country differences in human capital stocks. Caselli and Coleman (2006) and Caselli (2016) attribute skilled labor productivity differences to the skill bias of technology. Empirically, the two interpretations are difficult to distinguish because both have similar effects on observable wages.

The purpose of this paper is to extend development accounting to an environment that allows for both sources of cross-country variation in the relative productivity of skilled labor.

Approach We consider a class of models where the relative productivity of skilled labor is affected not only by human capital but also by factor-augmenting technologies, which can be exogenously given (as in Katz and Murphy, 1992), chosen endogenously from a frontier (as in Caselli and Coleman 2006), or the result of directed technical change by firms (as in Acemoglu 2007). In addition, we allow for capital-skill complementarity as in Krusell et al. (2000), so that the scarcity of equipment in low income countries affects skilled labor productivity.

We calibrate these models to standard data moments plus evidence on the wage gains at migration from Hendricks and Schoellman (2018). We perform development accounting and decompose cross-country variation in relative skilled labor productivity into the contributions of human capital and factor-augmenting technologies. Average migrant wage gains are informative about the importance of country-specific factors (capital, TFP) versus portable migrant human capital for outcomes. The relative wage gains of skilled versus unskilled migrants are informative about whether relative skilled labor productivities are affected by portable relative human capital or by non-portable skill-bias of technology in the birth country.

Results The first model that we study, labeled the *endogenous technology model*, combines elements of the literature's competing views on the cross-country variation in skilled labor productivities. In this model, relative labor productivities vary across countries due to differences in human capital (as in Jones, 2014) and due to the skill bias of technology, which is chosen by firms from a technology frontier (as in Caselli and Coleman, 2006).

We show analytically that this model is equivalent to one where the skill bias of technology is fixed, but with a higher elasticity of substitution between skilled and unskilled labor.

The reason is that technology choice offers firms an additional margin of adjustment when labor supplies change. In addition to the traditional substitution along a fixed production isoquant, firms adjust the optimal skill bias along the technology frontier. We call this higher elasticity the *long-run elasticity* of substitution; it mixes the traditional (short-run) elasticity of substitution with the curvature of the skill bias technology frontier.

We derive a closed form solution for the share of output gaps that is due to human capital in terms of observable data moments. The solution consists of two parts which we label the *perfect substitutes* term and the *imperfect substitutes* term, respectively. The first term is the share of cross-country output gaps due to human capital when skills are perfect substitutes. It is tightly pinned down by the wage gains of immigrants, which isolate the importance of country-specific factors for workers supplying the same human capital in two countries. Since migrant wage gains are much smaller than cross-country wage gaps, the perfect substitutes term is large, accounting for at least 45% of output gaps.

The imperfect substitutes term captures the additional contribution of human capital that arises due to imperfect skill substitution. Its magnitude depends critically on the elasticity of substitution between skill types. Previous work has considered a wide range of values for this elasticity. We instead discipline it using the relative wage gains of skilled versus unskilled migrants. The gap in wage gains is small, which limits the overall plausible size of the imperfect substitutes term. Overall, the model implies that human capital accounts for 58% to 63% of cross-country output gaps.

The decomposition of the human capital share into perfect substitutes and imperfect substitutes terms carries over to the other models that we study. Since all models feature the same perfect substitutes term, it follows that all imply a human capital share above 45%. While the imperfect substitutes terms differ across models, the logic that similar skilled and unskilled migrant wage gains limit its size carries over. We therefore find that the development accounting implications are broadly similar across cases.

Our second objective is to decompose cross-country differences in the relative productivity of skilled labor into the contributions of human capital versus the skill bias of technology. For conventional values of the short-run elasticity of substitution between skilled and unskilled labor, the endogenous technology model implies that human capital accounts for at most one-third of the relative productivity differences.

The intuition relies on the observation that migrant wage gains are broadly similar for skilled and unskilled workers. This finding suggests that skilled labor productivity differences are largely due to non-portable technology rather than portable human capital.

The *exogenous productivity model* abstracts from technology choice and takes the skill bias as given. We again derive a closed form solution for the human capital share that consists of the same perfect substitutes term as in the endogenous technology model plus a positive imperfect substitutes term.

Due to the complementarity of skill biased technology and skilled labor, the imperfect substitutes term increases with technological skill bias and decreases with the short-run elasticity of substitution. The human capital share therefore depends on whether we fix the skill bias of technology at rich or poor country levels. We consider both cases and find that human capital accounts for between one-half and three-fourths of output gaps.

Since the calibrated human capital and skill bias levels do not depend on whether the technology is endogenous or fixed, the implications for decomposing relative skilled labor productivities are the same as in the endogenous technology model.

The final model adds *capital-skill complementarity* to either the endogenous or the exogenous technology model. Across specifications, we again find that human capital accounts for between one-half and three-fourths of output gaps. Compared with the models discussed previously, capital-skill complementarity reduces the inferred differences in technological skill bias. The reason is that rich countries are relatively abundant in equipment, which is complementary with skilled labor. Reconciling observed variation in skilled versus unskilled labor inputs with observed skill premiums therefore requires smaller differences in technological skill bias.

Robustness Following Hendricks and Schoellman (2018), we interpret the wage gains of migrants as measures of the wage gaps between rich and poor countries. As a robustness check, we consider adjustments for the possibility that wage gains at migration may be downward biased by barriers that prevent immigrants from fully transferring their skills to their new country. For skilled migrants, these adjustments reduce the human capital share in development accounting only modestly.

For unskilled migrants, we are able to go further. The human capital share declines only slightly, even if we increase unskilled wage gains to the point where unskilled human capital does not differ across countries (the baseline assumption of Jones 2014). The intuition is that reducing unskilled human capital gaps has two opposing effects on the human capital share. It reduces the perfect substitutes term, but increases the imperfect substitutes term by raising the differences in relative skilled-to-unskilled human capital across countries. This finding is important because it establishes that the development accounting results are not sensitive to the wage gains of the least skilled migrants, which are difficult to estimate in available datasets (see Hendricks and Schoellman 2018 for a discussion of the relevant data issues).

Across the range of models that we study, we obtain two main conclusions. First, human capital accounts for between one-half and three-fourths of cross-country output gaps. This result is much larger than is standard in the literature and is in line with Jones (2014) and Hendricks and Schoellman (2018). The range of plausible results is much narrower. Second, differences in the relative human capital of skilled versus unskilled workers account for only modest variation in relative skilled labor productivity.

Our work adds to a small literature that investigates the sources of cross-country variation in relative skilled labor productivities. Our results broadly agree with Okoye (2016) and Rossi (2019) who show that the returns to schooling of migrants contain useful information. We extend their work by considering additional sources of variation in relative labor productivities (capital-skill complementarity) and by investigating the importance of human capital for development accounting. Our paper is also related to Malmberg (2019), who uses international trade data to provide complementary evidence on the relationship between comparative advantage and the relative supply of skills.

2 Endogenous Technology Model

We perform development accounting in an environment that allows for relative wages to be affected by labor supply factors (relative employment, relative human capital) and labor demand factors (relative skill bias, relative complementarity with other inputs).

2.1 Model Specification

The endogenous technology model combines the production function and human capital structure of Jones (2014) with the technology frontier of Caselli and Coleman (2006). There are two countries, indexed by $c \in \{p, r\}$ (poor and rich). Output per worker Y_c is produced from physical capital K_c and labor L_c according to the production function

$$Y_c = K_c^{\alpha} \left(z_c L_c\right)^{1-\alpha} \tag{1}$$

where the aggregate labor input is a CES aggregator of unskilled $(j = u)$ and skilled $(j = s)$ labor

$$L_c = \left[\sum_j \left(\theta_{j,c} L_{j,c}\right)^{\rho}\right]^{1/\rho}. \tag{2}$$

The elasticity of substitution between skilled and unskilled labor is $\sigma = 1/\left(1-\rho\right) > 1$, so that $0 < \rho < 1$. Labor inputs are the product of human capital and employment: $L_{j,c} = h_{j,c} N_{j,c}$. Their supplies are taken as exogenous. The skill weights $\theta_{j,c}$ are constrained by a technology frontier, similar to Caselli and Coleman (2006) or Acemoglu (2007), given by

$$\left[\sum_j \left(\kappa_j \theta_{j,c}\right)^{\omega}\right]^{1/\omega} \leq B_c^{1/\omega} \tag{3}$$

with $\omega > 0$. We normalize $B_c = 1$ and $\kappa_u = 1$ by choosing z_c appropriately.[3] We also normalize $\kappa_s = 1$ by choosing units for skilled labor. As in Caselli and Coleman (2006), we assume that

$$\omega - \rho - \omega\rho > 0 \tag{4}$$

This condition ensures that firms choose an interior point on the technology frontier.

Equilibrium In line with the development accounting literature, we assume that the economy is in steady state with an interest rate that is equal to the discount rate of the infinitely lived representative agent (e.g., Hsieh and Klenow 2010). This fixes the rental price of capital q_c and therefore K_c/Y_c. The rental prices of labor inputs, $p_{j,c}$, are determined by labor market clearing. The representative firm solves

$$\max_{K_c, L_{j,c}, \theta_{j,c}} Y_c - q_c K_c - \sum_j p_{j,c} L_{j,c} \tag{5}$$

subject to (1), (2), and (3), taking factor prices as given.

Note that observable wage rates per hour are given by $w_{j,c} = h_{j,c} p_{j,c}$. Hence, the total earnings of skill j workers are given by $W_{j,c} = p_{j,c} L_{j,c} = w_{j,c} N_{j,c}$. The values of $p_{j,c}$ are not directly observable in the data.

Our setup nests the model of Caselli and Coleman (2006) as a special case when $h_{j,c} = 1$. It nests the model of Jones (2014) when the choice of skill bias is removed and $\theta_{j,c} = 1$.

2.2 Development Accounting

We discuss how to perform development accounting when the skill bias of technology is endogenous. As is standard in the literature, we start from

$$Y_c = z_c \left(K_c/Y_c\right)^{\alpha/(1-\alpha)} L_c \tag{6}$$

and decompose the output gap into the contributions of TFP, physical capital, and (jointly) labor inputs and skill bias according to

$$\underbrace{\Delta \ln (Y)}_{\text{output gap}} = \underbrace{\Delta \ln (z)}_{\text{TFP}} + \underbrace{\Delta \ln \left((K/Y)^{\alpha/(1-\alpha)} \right)}_{\text{physical capital}} + \underbrace{\Delta \ln (L)}_{\text{labor + skill bias}} \tag{7}$$

[3] This assumption is relaxed in Appendix D where firms are allowed to invest in extending the technology frontier (increasing B_c, as in Acemoglu 2007). We show that the development accounting results remain unchanged if the costs of investing in B_c scale appropriately with output, so that the aggregate production function features constant returns to scale.

where $\Delta \ln(Y) \equiv \ln(Y_r) - \ln(Y_p)$. The share of the output gap accounted for by each input is given by

$$1 = \underbrace{\frac{\Delta \ln(z)}{\Delta \ln(Y)}}_{\text{share}_z} + \underbrace{\frac{\Delta \ln\left((K/Y)^{\alpha/(1-\alpha)}\right)}{\Delta \ln(Y)}}_{\text{share}_k} + \underbrace{\frac{\Delta \ln(L)}{\Delta \ln(Y)}}_{\text{share}_L} \tag{8}$$

The literature typically defines the contribution of each input to cross-country output gaps via a counterfactual experiment. For example, the contribution of human capital is defined as the change in steady state output when human capital is increased from the poor country's to the rich country's level. To the extent that other inputs respond endogenously, their effect is counted as part of human capital's contribution. In particular, the counterfactual holds the capital-output ratio constant. This captures the induced changes in the capital stock when the saving rate is unchanged (see Hsieh and Klenow, 2010).

In line with this approach, the endogenous technology model counts the effects of induced changes in the skill bias of technology as part of the contribution of labor inputs, measured by $share_L$. Alternatively, $share_L$ could be defined as the change in steady state output, holding skill bias fixed. The exogenous skill bias model considers this approach in Section 4. More generally, the skill bias of technology could be partially endogenous and partially due to factors other than labor supplies. The endogenous and the exogenous technology models bound the development accounting results for such cases.

2.3 Reduced Form Labor Aggregator

One challenge for development accounting is the identification of the two elasticities that govern the substitution between skilled and unskilled labor (ρ and ω). Our first result shows that $share_L$ can be estimated without separately identifying both parameters.

Proposition 1. *Solving out the firm's optimal skill bias choices yields the reduced form labor aggregator*

$$L_c = B_c \left[\sum_j \left(\kappa_j^{-1} L_{j,c} \right)^{\Psi} \right]^{1/\Psi} \tag{9}$$

with an elasticity of substitution governed by

$$\Psi = \frac{\omega\rho}{\omega - \rho} > \rho \tag{10}$$

Proof. Section B.2 □

Proposition 1 establishes that allowing for technology choice is equivalent to increasing the elasticity of substitution while holding the skill bias of technology fixed. The reduced form skill bias parameters $\left(\kappa_j^{-1}\right)$ are common across countries and governed by the technology frontier. Variation in the level of the frontier B_c has the same effect as variation in z_c.

In other words, we are back in the world of Jones (2014) with one crucial difference: the "short-run" elasticity of substitution of the original labor aggregator $1/(1-\rho)$ no longer matters by itself. It is replaced by a higher, "long-run" elasticity $1/(1-\Psi)$ that combines the curvatures of the labor aggregator and the technology frontier. The long-run elasticity reflects two equilibrium responses to an increase in skilled labor abundance $L_{s,c}/L_{u,c}$. The first (standard) response is that the lower skilled wage premium induces firms to substitute along the isoquant of the original CES production technology (2). The second effect is that firms choose a more skill-biased technology along the frontier (3). Not having to separately identify ρ and ω greatly simplifies the identification.

It follows directly that allowing for technology choice does not affect the development accounting results. When calibrated to the same data moments, the endogenous technology model implies exactly the same contribution of labor inputs to cross-country output gaps as does the model of Jones (2014), which abstracts from technology choice.

It is useful to place this result into the context of the literature. Previous work has shown that human capital can account for the majority of cross-country output gaps if skilled workers are relatively more productive in rich countries (Jones, 2014). This work assumed that all variation in relative worker productivities is due to human capital, leaving open the possibility that the role of human capital could be much smaller if other sources of relative productivity differences are considered (Caselli and Ciccone, 2019). Proposition 1 implies that this concern is unfounded when the skill bias of technology is fully endogenous.[4]

2.4 Closed Form Solution

Before we proceed to calibrate the model, we derive a closed form solution for the contribution of labor inputs to cross-country output gaps, $share_L$.

Proposition 2. *The share of output gaps due to labor inputs is given by*

$$share_L = \underbrace{1 - \frac{\ln(\pi_u)}{\Delta \ln(Y)}}_{\textit{perfect substitutes}} + \underbrace{\left(\frac{1}{\Psi} - 1\right) \frac{\Delta \ln(W/W_u)}{\Delta \ln(Y)}}_{\textit{imperfect substitutes}} \tag{11}$$

where $W_{j,c}$ is the labor income of workers of skill j in country c and $\pi_j = p_{j,r}/p_{j,p}$ denotes the

[4] The assumption that all cross-country variation in technological skill bias is induced by skilled labor abundance is analytically convenient, but strong. We show in Section 4 that the polar opposite case where technology is fully exogenous yields similar results.

wage gain due to migration. The long-run elasticity parameter Ψ is given by

$$\Psi = \ln\left(\frac{W_{s,r}/W_{u,r}}{W_{s,p}/W_{u,p}}\right) / \ln\left(\frac{L_{s,r}/L_{u,r}}{L_{s,p}/L_{u,p}}\right) \tag{12}$$

Proof. Section B.3 □

Note that all of the terms in (11) can be estimated from data.[5] This allows us to obtain precise intuition about how $share_L$ depends on data moments. Moreover, since the same solution applies to the model of Jones (2014) (except that the substitution elasticity is governed by ρ instead of Ψ), we gain insight into how his development accounting results differ from ours.

The solution for $share_L$ consists of two terms. The perfect substitutes term is the contribution of labor inputs to output gaps with perfect substitution of skills. Intuitively, the wage gain at migration captures the importance of changing country-specific factors (capital, TFP) for a worker's wages. If wage changes are as large as GDP per worker gaps, then country-specific factors account for all of income differences. If not, the remainder of GDP per worker gaps is attributable to the gaps in average human capital between countries. For example, if workers' wages do not change at all when migrating, then we would infer that country-specific factors are irrelevant and human capital accounts for all of cross-country income differences.[6]

With imperfect skill substitution, the role of human capital is magnified when the rich country is skill abundant, so that $h_{s,r}/h_{u,r} > h_{s,p}/h_{u,p}$. This is captured by the imperfect substitutes term in (11).[7] As in Jones (2014), we can sign this term to be positive, meaning that allowing for imperfect substitution and technology choice expands the role of human capital in development accounting. Its magnitude depends on the elasticity of substitution parameter Ψ and the rich country's relative abundance of skilled labor, captured by the poor-to-rich country gap in the unskilled labor income share, $\Delta \ln (W/W_u)$.

3 Quantitative Results

This section presents the development accounting implications of the endogenous technology model, calibrated to match data moments for output gaps, labor income shares and employment by skill, and migrant wage gains.

[5] We show below that the relative abundance of skilled labor $\frac{L_{s,r}/L_{u,r}}{L_{s,p}/L_{u,p}}$ can be estimated using data on migrant wage gains.

[6] Formally, with perfect substitution, the wage gap $\Delta \ln (w)$ equals the output gap $\Delta \ln (Y)$. Since $w = ph$, we have $\Delta \ln (h) = \Delta \ln (w) - \Delta \ln (p) = \Delta \ln (Y) - \ln (\pi)$. Therefore, $1 - \ln(\pi) / \Delta \ln (Y)$ is the contribution of labor inputs to output gaps.

[7] As $\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}} \to 1$, the imperfect substitutesterm vanishes because $\frac{L_{s,r}/L_{u,r}}{L_{s,p}/L_{u,p}} \to \frac{N_{s,r}/N_{u,r}}{N_{s,p}/N_{u,p}} = \frac{W_{s,r}/W_{u,r}}{W_{s,p}/W_{u,p}}$, so that $(1/\Psi - 1) \to 0$.

3.1 Data

This section summarizes key features of the data. Labor inputs in efficiency units are given by $N_{j,c} = \sum_{\iota \in j} e^{\phi t_\iota} N^{BL}_{\iota,c}$ where $N^{BL}_{\iota,c}$ denotes the employment share of school group ι taken from Barro and Lee (2013). ι takes on seven values (no school, some primary, primary completed, some secondary, secondary completed, some tertiary, tertiary completed). We set the school durations to $t_\iota = [0, 3, 6, 9, 12, 14, 16]$.

Based on the evidence collected by Banerjee and Duflo (2005), we assume that skill premiums are the same in rich and poor countries. Specifically, we assume a Mincer return of $\phi = 0.1$. For each country, we normalize $N_c = \sum_j N_{j,c} = 1$.

Labor income shares $IS_{j,c}$ are constructed as follows. The labor income of school group ι (up to an arbitrary, country specific scale factor) is defined as $W_{\iota,c} = e^{\phi t_\iota} N^{BL}_{\iota,c}$. The share of skill group j is then given by: $IS_{j,c} = \frac{\sum_{\iota \in j} W_{\iota,c}}{\sum_\iota W_{\iota,c}} (1 - \alpha)$. With equal Mincer returns in rich countries, relative earnings and relative labor inputs vary across countries by the same amount: $\frac{W_{s,r}/W_{u,r}}{W_{s,p}/W_{u,p}} = \frac{N_{s,r}/N_{u,r}}{N_{s,p}/N_{u,p}}$.

We take the rich country to be the U.S. Consistent with Hendricks and Schoellman (2018), the poor country is the median of 63 countries with $Y_c/Y_r < 1/4$. We consider four different lower bounds on the set of skilled workers: some secondary schooling (SHS), secondary degree (HSG), some college (SC), and college degree (CG). Table 1 shows the data moments for each skill cutoff. Data moments that do not vary across skill cutoffs are shown in Table 2.

While most of the data inputs to our accounting exercise are standard, the wage gains of migrants are less familiar and yet play a central role in our analysis. We draw here on previous work where we use three data sets with data on pre- and post-migration wages of different groups of immigrants to the U.S. (Hendricks and Schoellman, 2018). We document a number of facts about wage gains, particularly for migrants from poor countries (GDP per worker less than one-fourth of the U.S.). Two of these facts are central for our analysis.

First, the average wage gain is roughly a factor of 3, as compared to an average GDP per worker gap of nearly a factor of 11. Second, wage gains vary systematically and negatively with education. College educated migrants have wage gains of roughly a factor of 2, while migrants without high school degrees have gains of roughly a factor of 4. This gap is qualitatively consistent with imperfect substitution across skill groups: through the lens of this framework, more educated workers gain less because they move from a country where they are relatively scarce to a country where they are relatively abundant. The magnitude of the gap allows us to discipline the importance of imperfect substitution.

We use these wage gains as measures of the cross-country skill price ratio, $\pi_j = p_{j,r}/p_{j,p}$. Conceptually, the argument is that since migrants have the same human capital in their birth country and the United States, the change in their labor earnings reflects gaps in the

Table 1: Data Moments

	Skill Cutoff			
	SHS	HSG	SC	CG
Skilled/unskilled employment, N_s/N_u				
rich	26.16	1.13	0.35	0.06
poor	0.95	0.23	0.08	0.02
rich/poor	27.45	4.86	4.45	2.72
Skilled/unskilled wage bill, W_s/W_u				
rich	71.11	3.74	1.43	0.30
poor	2.59	0.77	0.32	0.11
rich/poor	27.45	4.86	4.45	2.72
Migrant wage gain, $\pi = p_r/p_p$				
unskilled	3.71	3.46	2.98	2.84
skilled	2.29	2.21	2.08	2.04
unskilled/skilled	1.62	1.57	1.43	1.39

Table 2: Data Moments Independent of Skill Cutoff

	Y	K/Y	Capital share
Rich	1.00	3.18	0.33
Poor	0.09	2.66	0.33
Ratio	10.70	1.19	1.00

Table 3: Closed Form Solution for share$_L$

	Skill Cutoff			
	SHS	HSG	SC	CG
$share_L$	0.63	0.59	0.60	0.58
Perfect substitutes term	0.45	0.48	0.54	0.56
Imperfect substitutes term	0.19	0.12	0.06	0.02
$1/\Psi - 1$	0.15	0.28	0.24	0.33
$\frac{\Delta \ln(W/W_u)}{\Delta \ln(Y)}$	1.27	0.42	0.26	0.07
$share_K$	0.04	0.04	0.04	0.04
$share_z$	0.33	0.37	0.36	0.38

Notes: The table shows the closed form solution for the human capital share in cross-country output gaps and its components according to equation (11).

market price for their skills. An important concern is that human capital may not transfer fully across countries due to technology differences, discrimination, licensure, and other barriers. In Hendricks and Schoellman (2018) we consider a number of ways to gauge the quantitative importance of these concerns. For example, we show that the wage gains of migrants who enter on employment visas, who work the exact same 3-digit occupation before and after migrating, or who move from English-speaking countries are 10-20 percent larger than the average. Alternatively, we show that the average migrant's post-migration job is 16 percent lower paying than their pre-migration job, based on the mean wage of U.S. natives in each. Below, we assess the robustness of our findings to adjusting up wage gains of all migrants or only skilled migrants by 20 percent.

3.2 Development Accounting

We perform development accounting by applying the data moments shown in Section 3.1 to the closed form solution for $share_L$, equation (11). As shown in Table 3, $share_L$ is close to 60% for all skill cutoffs. These findings align closely with Hendricks and Schoellman (2018). Having a closed form solution for $share_L$ allows us to provide sharp intuition for our results and for why they are very different from Jones (2014).

Table 3 reveals why $share_L$ is approximately constant across skill cutoffs: variation of the perfect substitutes term and of the imperfect substitutes term roughly balance each other. The perfect substitutes term ranges from 0.45 to 0.56 across skill cutoffs. Recall that the this term is equivalent to the contribution of human capital in a single skill model. It depends on the magnitude of unskilled migrant wage gains relative to the output gap. Since unskilled migrant wage gains are small (2.8 to 3.7) relative to the output gap (10.7), the contribution of human capital is large. Higher skill cutoffs are associated with smaller unskilled migrant wage gains and therefore larger perfect substitutes terms.

The imperfect substitutes term depends on the elasticity of substitution and the skilled-to-unskilled earnings ratios. The fact that the reduced form elasticity of substitution is high (for reasons that are discussed in Section 3.5) limits the size of this term. Since differences in relative employment shares $N_{s,c}/N_{u,c}$ and therefore also in unskilled labor income shares are much smaller for higher skill cutoffs, the imperfect substitutes term is smaller for higher skill cutoffs. It is the offsetting variation in the perfect substitutes and the imperfect substitutes term that generates the approximate constancy of share$_L$ across skill cutoffs.

For completeness, Table 3 also shows the fraction of output gaps due to physical capital and TFP. As commonly found in the literature, the contribution of physical capital is small (0.04), leaving more than one third of the output gap unexplained and hence attributed to TFP.

3.3 Comparison to Literature

Our results broadly agree with Jones (2014), who finds that human capital may account for a large fraction of cross-country output gaps. However, there are important differences in the calibration that affect the interpretation and robustness of the results. In particular, Jones assumes that unskilled workers are endowed with the same human capital in rich and poor countries, and he focuses on conventional values for the short-run elasticity of substitution between 1.5 and 2 (while examining a wider range as a robustness check). Table 4 shows the results if we adopt his calibration strategy using our data (which is broadly consistent with his data). Even restricting attention to the elasticity of substitution between 1.5 and 2, $share_L$ ranges from 12% to 270%.

The closed form solution (11) reveals why $share_L$ declines strongly with the skill cutoff. The imperfect substitutes term is determined by the difference in the unskilled wage bill share across countries $\Delta \ln (W/W_u)$, which declines with the skill cutoff. Especially for low substitution elasticities, its variation dominates how $share_L$ differs across cutoffs.

Since the same closed form solution also applies to the endogenous technology model and since we use similar data values for employment shares $N_{j,c}$ and wage bill ratios $W_{s,c}/W_{u,c}$, the differences in development accounting relative to Jones (2014) must stem entirely from the estimation of $\Delta \ln (h_u)$ and ρ. We explore these differences in the following sections.

Table 4: Jones (2014) Calibration

Short-run Elasticity	Skill Cutoff			
	SHS	HSG	SC	CG
1.25	5.22	1.85	1.19	0.32
1.50	2.69	1.02	0.68	0.18
2.00	1.42	0.60	0.42	0.12
3.00	0.79	0.39	0.29	0.08
4.00	0.58	0.32	0.25	0.07
5.00	0.47	0.29	0.23	0.07

Notes: The table shows the human capital share in cross-country output gaps implied by the calibration strategy of Jones (2014).

3.4 Estimating Human Capital Gaps

The first difference between our approach and Jones (2014) is the determination of the unskilled human capital gap $\Delta \ln(h_u) = \ln(h_{u,r}/h_{u,p})$. While Jones sets $\Delta \ln(h_u) \approx 0$, we estimate it using wage gains at migration, which implies that unskilled workers in rich countries have between 2 and 3.3 more human capital than similar workers in poor countries.

We noted in Section 2.4 that, when skills are perfect substitutes, the human capital gap $\Delta \ln(h)$ can be estimated as $\Delta \ln(Y) - \ln(\pi)$. A similar result holds when there are multiple skills. From $w_{j,c} = p_{j,c} h_{j,c}$, we have

$$\Delta \ln(h_j) = \Delta \ln(w_j) - \Delta \ln(p_j) \tag{13}$$

Intuitively, observed wages are higher in rich countries either due to skill price gaps or due human capital gaps: $\Delta \ln(w_j) = \Delta \ln(p_j) + \Delta \ln(h_j)$. Migrant wage gains estimate skill price gaps $(\ln(\pi_j) = \Delta \ln(p_j))$ and therefore allow us to calculate human capital gaps.[8]

Table 5 shows the human capital gaps implied by equation (13). The table also shows the two ratios that determine these values: observable wage gaps $\Delta \ln(w_j)$ and migrant wage gains π_j. We highlight a number of findings:

1. For all skill cutoffs, the fact that cross-country wage gaps exceed migrant wage gains

[8] The wage gap $\Delta \ln(w_j)$ can be estimated from $\Delta \ln(w_j) = \Delta \ln(W_j) - \Delta \ln(N_j)$. Given data for skill premiums $w_{s,c}/w_{u,c}$ and employment shares by skill $N_{j,c}$, we can calculate wage bill ratios $W_{s,c}/W_{u,c}$. Using data for output per worker, and labor income shares $1-\alpha$ we can calculate wage bill levels using $W_{u,c} = (1-\alpha)/(1 + W_{s,c}/W_{u,c})$.

Table 5: Cross-country Human Capital Gaps

	Skill Cutoff			
	SHS	HSG	SC	CG
$h_{u,r}/h_{u,p}$	2.00	2.00	2.45	3.35
$w_{u,r}/w_{u,p}$	7.41	6.90	7.29	9.49
π_u	3.71	3.46	2.98	2.84
$h_{s,r}/h_{s,p}$	3.24	3.12	3.51	4.65
$w_{s,r}/w_{s,p}$	7.41	6.90	7.29	9.49
π_s	2.29	2.21	2.08	2.04
$\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}}$	1.62	1.57	1.43	1.39

Notes: The table shows the rich-to-poor country human capital ratios, $h_{j,r}/h_{j,p}$, and their components according to equation (13). $w_{j,r}/w_{j,p}$ denotes the gap in observable wages of skill group j. π_j is the wage gain at migration.

implies that workers of all skills have more human capital in rich compared with poor countries.

2. Higher skill cutoffs are associated with larger wage gaps $\Delta \ln (w_j)$, smaller migrant wage gains, and therefore larger human capital gaps $\Delta \ln (h)$.

3. In the rich country, skilled workers have relatively more human capital than in the poor country: $\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}} > 1$. Since migrant wage gains are similar for skilled and unskilled workers, $\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}}$ differs at most 1.6 fold across countries.[9] This limits the size of the imperfect substitutes term in (11) (recall that this term vanishes when $h_{s,r}/h_{u,r} = h_{s,p}/h_{u,p} = 1$).

3.5 Elasticity Implications

The second difference between our approach and Jones (2014) is the determination of the elasticity of substitution between skilled and unskilled labor. Jones abstracts from the possibility of technological adaptation in the face of large and persistent differences in

[9] Specifically, with equal skill premiums in rich and poor countries, we have $\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}} = \pi_1/\pi_2 \in [1.4, 1.6]$. This follows from $w_{s,p}/w_{u,p} = p_{s,p}/p_{u,p} \times h_{s,p}/h_{u,p} = w_{s,r}/w_{u,r} = p_{s,r}/p_{u,r} \times h_{s,r}/h_{u,r}$.

the relative supply of skilled labor. This leads him to use conventional estimates of the elasticity of substitution from the literature. We allow for technological adaptation, here modeled as choosing an appropriate technology along a frontier. This leads us to calibrate an alternative, higher long-run elasticity. The higher elasticity of substitution limits the size of the imperfect substitutes term in (11) and rules out very large values of $share_L$.

To understand why we find a high elasticity, consider the firm's first-order condition for labor inputs, which implies

$$\lambda\left(W\right)=\Psi\lambda\left(L\right). \tag{14}$$

This relates the gap in relative wage bills, $\lambda\left(W\right)\equiv\ln\left(\frac{W_{s,r}/W_{u,r}}{W_{s,p}/W_{u,p}}\right)=\Delta\ln\left(W_s\right)-\Delta\ln\left(W_u\right)$, to the relative abundance of skilled labor. Since $\lambda\left(W\right)=\lambda\left(N\right)=\lambda\left(L\right)-\lambda\left(h\right)$, we have

$$\lambda\left(h\right)=\frac{1-\Psi}{\Psi}\lambda\left(N\right) \tag{15}$$

so that the elasticity of substitution between skilled and unskilled labor is given by

$$\frac{1}{1-\Psi}=1+\frac{\lambda\left(N\right)}{\lambda\left(h\right)}. \tag{16}$$

Intuitively, the elasticity must be high, if relative wage bills $\lambda\left(W\right)=\lambda\left(N\right)$ vary greatly across countries, but relative human capital endowments do not. The latter term may be estimated using $\lambda\left(h\right)=\ln\left(\pi_u/\pi_s\right)$. Table 6 shows the corresponding data values for each skill cutoff. We highlight three observations:

1. $\lambda\left(N\right)=\ln\left(\frac{N_{s,r}/N_{u,r}}{N_{s,p}/N_{u,p}}\right)$ is positive for all skill cutoffs, indicating that rich countries are abundant in skilled labor.

2. Cross-country variation in relative skilled employment $\lambda\left(N\right)$ is much larger than cross-country variation in relative skilled human capital $\lambda\left(h\right)$. As a result, the elasticity of substitution is always high (at least 4).

3. As the skill cutoff is increased, $\lambda\left(N\right)$ declines (as previously noted) while $\lambda\left(h\right)$ is fairly stable, causing the elasticity to decline as well.

Intuitively, cross-country variation in relative skill prices is limited because migrant wage gains do not vary greatly across skill groups. Reconciling small variation in relative skill prices with large variation in relative labor inputs requires a high elasticity of substitution. Conversely, a smaller long-run elasticity of substitution would imply much larger differences in migrant wage gains between skilled and unskilled workers than we see in the data.[10]

[10]Specifically, for a long-run elasticity of 2, we find $\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}}$ from equation (15), setting $\Psi=0.5$. Using $\pi_u/\pi_s=\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}}$, we find that unskilled wage gains exceed skilled wage gains by factor 2.8 for the CG skill cutoff and by factor 27 for the SHS skill cutoff. In the data, the ratio is at most 1.6 (see Table 5).

Table 6: Long-run Elasticity of Substitution

	Skill Cutoff			
	SHS	HSG	SC	CG
Elasticity	7.83	4.53	5.15	4.03
$\lambda(N)$	3.31	1.58	1.49	1.00
$\lambda(h)$	0.48	0.45	0.36	0.33

Notes: The table shows the determinants of the long-run elasticity of substitution given by equation (16). $\lambda(N) \equiv \Delta \ln(N_s) - \Delta \ln(N_u)$ denotes the relative abundance of skilled labor in the rich versus poor country. $\lambda(h)$ is the corresponding term for skilled human capital.

We can now summarize why our results differ from those of Jones (2014). First, we use the wage gains of immigrants to discipline human capital gaps for unskilled workers, which increases the contribution of human capital. Second, we calibrate a much larger (long-run) elasticity of substitution. This has two effects. First, it reduces the imperfect substitutes term, which rules out very large results. Second, it implies that our results are much less sensitive to skill cutoffs than are those of Jones, because a larger value of the elasticity of substitution puts less weight on the large and variable term $\Delta \ln(W/W_u)$ in the closed form solution (11).

There are several ways of reconciling a high long-run elasticity with smaller empirical estimates. One possibility is that empirical estimates, which are often based on within-country time-series evidence (e.g., Katz and Murphy, 1992), do not capture the full response of technology to large and persistent cross-country variation in labor supplies. Alternatively, the skill bias of technology may in part be determined by factors that do not respond to variation in labor supplies. In that case, cross-country skill bias differences should be treated as partially exogenous in our analysis. The development accounting results would then be in between those of the endogenous and the exogenous technology models. Finally, we show in Section 5 that a model with capital-skill complementarity implies a long-run elasticity of substitution that is consistent with conventional empirical estimates.

3.6 Robustness

The previous discussion reveals that migrant wage gains play a central role for our results. The discussion in Hendricks and Schoellman (2018) addresses a number of concerns related to the interpretation of migrant wage gains as measures of cross-country skill price differences. Here, we address two of these concerns.

The first concerns relates to the measurement of unskilled wage gains. The New Immigrant Survey and Latin Migration Project data used by Hendricks and Schoellman (2018) contain few migrants with no secondary education. As a result, the migrant wage gains of this group may be understated. We explore the robustness of our findings by increasing the wage gains of this education group to the point where their human capital no longer differs across countries.

The closed form solution for $share_L$ given by (11) reveals that increasing the unskilled wage gain π_1 has two opposing effects on $share_L$. First, the perfect substitutes term declines because higher wage gains indicate larger contributions of "country" to output gaps. Second, the imperfect substitutes term increases because the elasticity of substitution is reduced according to (12). Intuitively, larger unskilled migrant wage gains imply that $\Delta \ln(h_u)$ declines while $\Delta \ln(h_s)$ is held fixed. As a result, the relative abundance of skilled labor increases in the rich country. Matching the observed wage bill ratios then requires a smaller elasticity of substitution.

Quantitatively, the net result is that $share_L$ declines modestly, as shown in Table 7. Depending on the skill cutoff, it ranges from 0.54 to 0.6. The substitution elasticities decrease to values between 3.3 and 4.0. Our findings are now close to the preferred parameterizations of Jones (2014) who assumes $\Delta \ln(h_u) \approx 0$ and sets the elasticity of substitution based on published empirical estimates. Accurately estimating unskilled wage gains is challenging. It is therefore reassuring that our development accounting results are highly robust in this dimension.

The second concern mainly affects the measurement of skilled wage gains. If skills transfer only imperfectly across countries, the assumption that migrant wage gains equal skill price gaps $(\pi_j = p_{j,r}/p_{j,p})$ is violated. Hendricks and Schoellman (2018) consider a variety of data adjustments to address this problem and conclude that imperfect skill transferability reduces skilled migrant wages by 10-20 percent. With constant returns to scale in the aggregate production function, it follows directly that $share_L$ declines by at most 8 percent $(\ln(1.2)/\Delta \ln(Y) = 0.08)$. The adjustment is slightly smaller if only skilled workers are affected by imperfect skill transferability (see the third panel of Table 7). We have performed similar robustness checks for all of the model versions that we consider with very similar results. Details are available upon request.

3.7 Relative Skilled Labor Productivities

One contribution of our work is to allow for both relative human capital $\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}}$ and relative skill bias $\frac{\theta_{s,r}/\theta_{u,r}}{\theta_{s,p}/\theta_{u,p}}$ in the same framework and to disentangle the two. Thus, we can contribute to the ongoing debate on which of these two forces explains the constancy of skill premiums across countries given the enormous differences in skilled labor supplies.

Table 7: Robustness

	Skill Cutoff			
	SHS	HSG	SC	CG
Baseline				
$share_L$	0.63	0.59	0.60	0.58
Perfect substitutes term	0.45	0.48	0.54	0.56
Imperfect substitutes term	0.19	0.12	0.06	0.02
No human capital gaps for least skilled workers				
$share_L$	0.60	0.54	0.57	0.54
Perfect substitutes term	0.15	0.34	0.48	0.51
Imperfect substitutes term	0.45	0.20	0.09	0.03
Skilled wage gain increased by 20 percent				
$share_L$	0.56	0.55	0.57	0.57
Perfect substitutes term	0.45	0.48	0.54	0.56
Imperfect substitutes term	0.12	0.07	0.03	0.01

Notes: The table shows the fraction of cross-country output gaps that is due to human capital, $share_L$, and decomposes it into perfect substitutes and imperfect substitutes terms according to equation (11). The first panel shows the endogenous technology model. The second panel sets the wage gains for migrants with no secondary schooling such that their human capital does not differ across countries. The third panel increases skilled wage gains by 20 percent relative to the estimates of Hendricks and Schoellman (2018).

We estimate relative skill bias gaps based on the firm's first-order condition for labor, which implies

$$\lambda(\theta h) = \frac{1-\rho}{\rho}\lambda(N) \tag{17}$$

Recall that $\lambda(N) \equiv \ln\left(\frac{N_{s,r}/N_{u,r}}{N_{s,p}/N_{u,p}}\right) = \Delta\ln(N_s) - \Delta\ln(N_u)$ denotes the relative abundance of skilled labor in the rich compared with the poor country. Hence, (17) relates differences in the relative abundance of skilled labor $\lambda(N)$ to differences in its relative productivity $\lambda(\theta h)$. The same expression also applies in the case where labor-augmenting technologies are fixed exogenously.

Intuitively, skilled wage premiums appear similar across countries. Given large differences in relative labor supplies $\lambda(N)$ and conventional estimates of the (short-run) elasticity of substitution, some combination of skill bias of technologies ($\lambda(\theta) > 0$) or a relative human capital per worker advantage ($\lambda(h) > 0$) is required.

Previous work explored versions of equation (17) with one of these two possibilities ruled out, eliminating the identification challenge (Caselli and Coleman, 2006; Jones, 2014). Substantial disagreement has ensued (Caselli and Ciccone, 2019; Jones, 2019).

Our approach is to use the new evidence from the wage gains of migrants to discipline $\lambda(h)$. Implicitly, the remainder is attributed to $\lambda(\theta)$. One way to think about the model with endogenous technology choice is that we calibrate the value of Ψ (and implicitly the curvature of the technology frontier ω) to induce firms to choose $\lambda(\theta)$ as an optimal response to $\lambda(h)$ and $\lambda(N)$. However, we can also measure $\lambda(\theta)$ directly without this structure.

Table 8 shows our results. For typical estimates of the (short-run) elasticity of substitution, the relative skill bias $\frac{\theta_{s,r}/\theta_{u,r}}{\theta_{s,p}/\theta_{u,p}}$ exceeds two. It is larger for low skill cutoffs (because they imply larger $\frac{N_{s,r}/N_{u,r}}{N_{s,p}/N_{u,p}}$, which increases the right-hand side of equation (17)) and smaller values of ρ (which also increases the right-hand side of equation (17)).

Table 9 shows the fraction of cross-country variation in the relative productivity of skilled labor $\lambda(\theta h)$ that is due to human capital, defined as $\lambda(h)/\lambda(\theta h)$. Since relative skilled human capital $\lambda(h)$ does not vary with the short-run elasticity of substitution, this fraction varies inversely with the relative skill bias ratios shown in Table 8. For conventional values of the elasticity of substitution (between 1.5 and 2), at most one-third of the cross-country variation in relative skilled labor productivity is due to human capital. However, the fraction rises rapidly as the elasticity increases.

These findings agree with the previous work of Rossi (2019), who also decomposes cross-country variation in the relative productivity of skilled labor into the contributions of relative human capital and technological skill bias. He uses the returns to schooling of foreign-educated immigrants as the extra moment to provide identification (rather than wage gains of immigrants) and concludes that 90% of the variation can be attributed to technology. If

Table 8: Relative Skill Bias, Rich vs. Poor Country

Short-run Elasticity	Skill Cutoff SHS	HSG	SC	CG
1.25	3.50×10^5	355.98	274.06	39.14
1.50	463.99	15.08	13.83	5.30
2.00	16.91	3.10	3.11	1.95
3.00	3.23	1.41	1.47	1.18
4.00	1.86	1.08	1.15	1.00
5.00	1.41	0.95	1.01	0.92

Notes: The table shows cross-country gaps in the relative skill bias of technology, $\frac{\theta_{s,r}/\theta_{u,r}}{\theta_{s,p}/\theta_{u,p}}$.

Table 9: Fraction of Relative Skilled Labor Productivity Differences Due to Human Capital

Short-run Elasticity	Skill Cutoff SHS	HSG	SC	CG
1.25	3.7	7.1	6.0	8.3
1.50	7.3	14.2	12.0	16.5
2.00	14.6	28.3	24.1	33.0
3.00	29.3	56.7	48.2	66.0
4.00	43.9	85.0	72.3	99.1
5.00	58.5	113.4	96.4	132.1

Notes: The table shows $100 \times \lambda(h)/\lambda(\theta h)$ where $\lambda(h) \equiv \ln\left(\frac{h_{s,r}/h_{u,r}}{h_{s,p}/h_{u,p}}\right)$ denotes relative skilled human capital endowments and $\lambda(h\theta)$ denotes relative skilled labor productivity differences between rich and poor countries.

we focus on the same definition of skill (some college or more) and elasticity of substitution (1.5), we find a very similar share of 88%.

4 Exogenous Technology Model

We now consider a model where the skill bias of technology does not respond to the abundance of skilled labor. The *exogenous technology model* shares the production function (1) and the labor aggregator (2) with the endogenous technology model, but it drops the technology frontier. While the endogenous technology model assumes that all cross-country variation in technological skill bias is due to labor endowments, the exogenous technology model assumes that all of the variation is exogenous. The development accounting implications of the two models therefore bound the implications of a more general setup where a fraction of technological skill bias is endogenous.[11]

4.1 Development Accounting Approach

Development accounting assesses how each factor input affects steady state output. As pointed out by Caselli and Ciccone (2019), the effect of changing labor inputs is not uniquely determined when skilled labor and technological skill bias are complements; it depends on the reference country's technology. The endogenous technology model sidesteps this issue because the skill bias of technology is varies with labor endowments. Now that the skill bias of technology is taken as fixed, we confront this issue by considering two definitions of $share_L$:

1. $share_L^{poor}$ fixes the skill bias of technology at the poor country level. Intuitively, this corresponds to the effect of increasing poor country labor inputs to rich country levels.
2. $share_L^{rich}$ fixes the skill bias of technology at the rich country level. Intuitively, this corresponds to the effect of reducing rich country labor inputs to poor country levels.

4.2 Closed Form Solution

We derive a closed form solution for $share_L^{poor}$ and $share_L^{rich}$ in terms of observable data moments.

[11] We continue to refer to $\theta_{j,c}$ as technological skill bias, recognizing that its variation may be due to factors other than technology, as argued by Caselli and Ciccone (2019).

Proposition 3. *The share of labor inputs evaluated at poor country skill bias is given by*

$$share_L^{poor} = \underbrace{1 - \frac{\ln(\pi_u)}{\Delta \ln(Y)}}_{\textit{perfect substitutes}} + \underbrace{\frac{\frac{1}{\rho}\ln\frac{1+W_{s,p}/W_{u,p}\mathcal{RS}(L)^{\rho}}{1+W_{s,p}/W_{u,p}} - \Delta\ln(W/W_u)}{\Delta\ln(Y)}}_{\textit{imperfect substitutes}}. \tag{18}$$

When evaluated at rich country skill bias, the share of labor inputs is given by

$$share_L^{rich} = \underbrace{1 - \frac{\ln(\pi_u)}{\Delta \ln(Y)}}_{\textit{perfect substitutes}} + \underbrace{\frac{\frac{1}{\rho}\ln\frac{1+W_{s,r}/W_{u,r}}{1+W_{s,r}/W_{u,r}\mathcal{RS}(L)^{-\rho}} - \Delta\ln(W/W_u)}{\Delta\ln(Y)}}_{\textit{imperfect substitutes}} \tag{19}$$

where $\mathcal{RS}(L) \equiv \frac{L_{s,r}/L_{u,r}}{L_{s,p}/L_{u,p}}$ denotes the relative abundance of skilled labor in the rich versus the poor country.

Proof. Section C.1. □

Both expressions resemble the closed form solution for $share_L$ (11) obtained from the endogenous technology model.[12] In both cases, the perfect substitutes term is the same and represents the contribution of labor inputs with a single skill.

The imperfect substitutes term depends on the elasticity of substitution between skilled and unskilled labor (now governed by the short-run elasticity parameter ρ) and on the labor income ratios $W_{s,c}/W_{u,c}$. The imperfect substitutes term is small when skilled labor is "unimportant" in the sense of earning little income, so that $W_{s,c}/W_{u,c}$ is small, or when relative labor supplies $\frac{L_{s,r}/L_{u,r}}{L_{s,p}/L_{u,p}}$ are similar across countries.

4.3 Quantitative Results

We calibrate the model to match the same data moments that were used in the calibration of the endogenous technology model. However, $share_L$ now depends on the short-run elasticity of substitution. The data moments are therefore not sufficient to perform development accounting. Following Jones (2014), we explore a range of values for ρ.

Table 10 shows the share of output gaps accounted for by labor inputs, evaluated at poor country skill bias values. For conventional values of the elasticity of substitution between 1.5 and 2 (Ciccone and Peri, 2005), $share_L^{poor}$ ranges from 50% to 57%. For lower elasticities, especially for the SHS skill cutoff, $share_L^{poor}$ can drop below 50%. However, it is worth keeping in mind that these cases imply extremely large cross-country differences in

[12] In fact, when $\rho = \Psi$, given by (12), $share_L^{poor} = share_L = share_L^{rich}$ because $W_{s,p}/W_{u,p}\mathcal{RS}(L)^{\Psi} = W_{s,r}/W_{u,r}$.

Table 10: Development Accounting with Poor Country Skill Bias

Short-run Elasticity	Skill Cutoff			
	SHS	HSG	SC	CG
1.25	0.44	0.48	0.50	0.56
1.50	0.50	0.51	0.52	0.56
2.00	0.56	0.54	0.55	0.57
3.00	0.60	0.57	0.58	0.58
4.00	0.61	0.59	0.59	0.58
5.00	0.62	0.60	0.60	0.58
Endogenous θ	0.63	0.59	0.60	0.58

Notes: The table shows the human capital share in cross-country output gaps $share_L^{poor}$ for selected values of the elasticity of substitution between skilled and unskilled labor (rows) and for selected skill cutoffs (columns). The last row shows the contribution of labor inputs when skill bias is endogenous, $share_L$, taken from Table 3.

skill bias (see Table 8).[13] As the elasticity approaches the value implied by the model with the technology frontier (Ψ), $share_L^{poor} \to share_L$.

Table 11 shows the corresponding results when the contribution of labor inputs is evaluated using rich country skill bias parameters. For substitution elasticities in the conventional range between 1.5 and 2, $share_L^{rich}$ ranges from 59% to 74%. Across all cells, the range is only modestly wider.

To understand the diverging patterns between $share_L^{rich}$ and $share_L^{poor}$, it is useful to remember from Table 1 that the relative abundance of skilled labor $\frac{N_{s,r}/N_{u,r}}{N_{s,p}/N_{u,p}}$ is much larger with lower skill cutoffs such as SHS. In order to fit the targets, our calibration infers much larger gaps between rich and poor countries in labor augmenting technologies $\frac{\theta_{s,r}/\theta_{u,r}}{\theta_{s,p}/\theta_{u,p}}$ in this case. Thus, development accounting results become much more sensitive to whether we use poor or rich country technologies as the benchmark. Equations (18) and (19) show that this effect interacts with ρ, so that the divergence is larger as the elasticity of substitution moves away from the value we calibrated in the endogenous skill bias case.

[13] For any given value of ρ, the model implies the same values for human capital $h_{j,c}$ and relative skill bias gaps $\mathcal{RS}(\theta)$ as the model with the technology frontier. Since changing all $\theta_{j,c}$ by a common factor is equivalent to varying total factor productivity z_c, the skill bias parameters are only identified up to country specific constants.

Table 11: Development Accounting with Rich Country Skill Bias

Short-run Elasticity	Skill Cutoff			
	SHS	HSG	SC	CG
1.25	0.75	0.71	0.71	0.61
1.50	0.74	0.68	0.68	0.60
2.00	0.72	0.65	0.65	0.59
3.00	0.69	0.62	0.62	0.59
4.00	0.67	0.60	0.61	0.58
5.00	0.65	0.59	0.60	0.58
Endogenous θ	0.63	0.59	0.60	0.58

Notes: The table shows the human capital share in cross-country output gaps $share_L^{rich}$ for selected values of the elasticity of substitution between skilled and unskilled labor (rows) and for selected skill cutoffs (columns). The last row shows the contribution of labor inputs when skill bias is endogenous, $share_L$, taken from Table 3.

5 Capital-skill Complementarity

In this section, we consider capital-skill complementarity as an additional source of cross-country variation in skilled labor productivity. In the main text, we only consider the endogenous technology version of the model, leaving the details of the exogenous technology version for Section E.4.

5.1 Model Specification

The specification of the aggregate production function is based on Krusell et al. (2000). Output per worker Y_c is produced from capital and labor inputs according to

$$Y_c = S_c^{\alpha} \left(z_c L_c\right)^{1-\alpha} \tag{20}$$

where

$$L_c = \left[(\theta_{u,c} L_{u,c})^{\rho} + (\theta_{s,c} Z_c)^{\rho}\right]^{1/\rho} \tag{21}$$

and

$$Z_c = \left[(\mu_e E_c)^{\phi} + (\mu_s L_{s,c})^{\phi}\right]^{1/\phi} \tag{22}$$

with parameters $\alpha, \rho \in (0, 1)$, $\phi < 1$, and $\mu_e, \mu_s > 0$.

S_c denotes structures per capita. L_c is given by a CES aggregator of unskilled labor $L_{u,c}$ and a composite input Z_c, which is in turn a CES aggregator of skilled labor $L_{s,c}$ and equipment E_c. The skill bias parameters $\theta_{j,c}$ are constrained by the technology frontier (3) with $B_c = 1$ taken as fixed. The endogenous technology model emerges as a special case when $\mu_e = 0$ so that $Z_c = L_{s,c}$.

As before, we assume that the economy is in steady state with an interest rate that is equal to the discount rate of the infinitely lived representative agent. This fixes the rental prices of equipment $q_{e,c}$ and structures $q_{s,c}$ and therefore also S_c/Y_c. The representative firm solves

$$\max_{S_c, E_c, L_{j,c}, \theta_{j,c}} Y_c - q_{s,c} S_c - q_{e,c} E_c - \sum_j p_{j,c} L_{j,c} \tag{23}$$

subject to (20), (21), (22), and the frontier constraint (3).

5.2 Reduced Form Labor Aggregator

Similar to the endogenous technology model, we are able to derive a reduced form labor aggregator that substitutes out the firm's optimal skill bias choices.

Proposition 4. *Substituting out the firm's optimal skill bias choices yields the reduced form labor aggregator*

$$L_c = B_c \left([L_{u,c}/\kappa_u]^{\Psi} + [Z_c/\kappa_s]^{\Psi} \right)^{1/\Psi} \tag{24}$$

with $\Psi = \frac{\omega\rho}{\omega-\rho}$ *as in the endogenous technology model.*

Proof. Section E.3.1 □

5.3 Development Accounting

Development accounting proceeds analogously to the endogenous technology model. Starting from

$$Y_c = (S_c/Y_c)^{\alpha/(1-\alpha)} z_c L_c \tag{25}$$

the output gap can be additively separated into the contributions of TFP, structures, and labor inputs jointly with equipment:

$$\underbrace{\Delta \ln (Y)}_{\text{output gap}} = \underbrace{\Delta \ln (z)}_{\text{TFP}} + \underbrace{\Delta \ln \left((S/Y)^{\alpha/(1-\alpha)} \right)}_{\text{structures}} + \underbrace{\Delta \ln (L)}_{\text{labor and equipment}} \tag{26}$$

The share of the output gap accounted for by each input is given by

$$1 = \underbrace{\frac{\Delta \ln (z)}{\Delta \ln (Y)}}_{\text{share}_z} + \underbrace{\frac{\Delta \ln \left((S/Y)^{\alpha/(1-\alpha)} \right)}{\Delta \ln (Y)}}_{\text{share}_S} + \underbrace{\frac{\Delta \ln (L)}{\Delta \ln (Y)}}_{share_{L+E}} \tag{27}$$

The joint contribution of labor inputs and equipment has a closed form solution in terms of data moments (see Section E.3.2). It may be subdivided into the separate contributions of its components ($h_{j,c}$, $N_{j,c}$, E_c). These are defined as the changes in steady state output that result from changing each input from its poor country value to its rich country value, holding the rental prices of equipment and structures fixed. The counterfactual output changes depend on the fixed equipment rental prices. We therefore define two versions of each input's share. Superscript "poor" fixes q_E at the poor country's level. Superscript "rich" fixes them at the rich country's level.

As in the endogenous technology model, the development accounting implications depend on the reduced form curvature parameter Ψ, but not on the separate values of ρ and ω.

5.4 Calibration

We calibrate the model using the same data moments that were used for the endogenous technology model. However, we replace the moments related to capital inputs with separate moments for equipment and structures.

Specifically, we construct equipment/output ratios (E_c/Y_c) and structures/output ratios (S_c/Y_c) from Penn World Table 9 (Feenstra et al., 2015) and International Comparison Project data. The income share of equipment $IS_{e,r} = 0.15$ is taken from Valentinyi and Herrendorf (2008). Together with a labor share of 0.33, this implies an income share for structures of $IS_{s,r} = 0.18$, which is consistent with Valentinyi and Herrendorf (2008). We lack data on equipment and structures shares for low income countries. Since we find that S_c/Y_c and the relative price of structures versus consumption are similar for rich and poor countries, we set $IS_{s,c} = 0.18$ for all countries. These data moments are summarized in Table 12.

In total, we have 14 data moments (6 independent factor incomes shares, 2 output levels, 2 wage gains at migration, 4 capital/output ratios). However, choosing units of E to normalize $\kappa_u = 1$ means that we need to replace the data moments E_c/Y_c with E_r/E_p.[14] This leaves us with 13 data moments that can be used to calibrate the model's 13 parameters (z_c; α; $h_{j,c}$, where $h_{u,r} = 1$; E_c and S_c; Ψ, ϕ).

[14]We also normalize $h_{u,r} = 1$ so that $L_{u,r} = N_{u,r}$. We set $\mu_s = 1$ by choosing units of h_s. We may normalize μ_e, κ_s and B_c to 1 as varying them has the same effect as varying z_c.

Table 12: Additional Calibration Targets

	S/Y	E/Y
Rich	2.81	0.37
Poor	2.85	0.14
Ratio	0.98	2.62

Table 13: Development Accounting with Capital-skill Complementarity

	Skill Cutoff			
	SHS	HSG	SC	CG
$share_L^{poor}$	0.65	0.61	0.62	0.58
$share_L^{rich}$	0.68	0.67	0.70	0.65
$share_{L+E}$	0.78	0.75	0.76	0.74
Elasticity	4.77	2.51	2.17	1.37

Notes: The table shows the human capital share in cross-country output gaps for selected skill cutoffs (columns). $share_L^{poor}$ $\left(share_L^{rich}\right)$ uses poor (rich) country equipment prices. $share_{L+E}$ denotes the joint share of human capital and equipment. The last row shows the long-run elasticity of substitution between unskilled labor and the skilled labor/equipment aggregator Z.

5.5 Development Accounting Results

Table 13 summarizes the development accounting implications. Across skill cutoffs, labor inputs and equipment jointly account for around three-quarters of cross-country output gaps. Using poor country equipment prices, human capital accounts for around 60% of output gaps. Using the lower rich country equipment prices, $share_L^{rich}$ is moderately higher, ranging from 65% to 70%. Since skilled labor and equipment are complements, increasing labor inputs has larger effects on output when equipment is abundant.

The reduced form elasticities of substitution $1/(1-\Psi)$ are much smaller than in the model without capital-skill complementarity. The intuition is based on the observation that Z/L_1 varies more across countries than L_s/L_u. At the same time, the relative income share of Z versus L_u varies less than that of skilled versus unskilled labor. Hence, a smaller elasticity reconciles cross-country variation in factor incomes and factor income shares. For the higher skill cutoffs, the elasticities of substitution are in line with conventional estimates for $1/(1-\rho)$.

Table 14: Development Accounting with Poor Country Equipment Prices

	Skill Cutoff			
	SHS	HSG	SC	CG
$share_L$	0.65	0.61	0.62	0.58
$share_E$	0.07	0.06	0.05	0.06
$share_S$	0.00	0.00	0.00	0.00
$share_z$	0.22	0.25	0.24	0.26

For completeness, Table 14 summarizes the shares of output gaps accounted for by other inputs evaluated at poor country equipment prices. Structures make essentially no contribution. Equipment contributes about 8%. The contribution of TFP is given by $1 - share_S - share_{L+E}$ and therefore amounts to about 22%.[15] The complementarity of skilled labor and equipment implies that jointly increasing both inputs has a larger effect on output than increasing each input separately. This explains why $share_{L+E}$ is almost ten percentage points larger than $share_L + share_E$.

We also explore a version of the model where the skill bias of technology is taken as exogenous. For conventional values of the elasticity of substitution between skilled and unskilled labor, we find that $share_L$ ranges from 52% to 74%. Details are relegated to Section E.4.[16]

Finally, Table 15 shows the fraction of the cross-country variation in relative skilled labor productivities that is due to human capital, defined as $\lambda(h)/\lambda(\theta h)$ where $\lambda(h) \equiv \Delta\ln(h_s) - \Delta\ln(h_u)$ denotes the gap in relative human capital endowments. Even if attention is restricted to conventional values of the elasticity, the fraction due to human capital ranges from 8% to 68%.

To understand this result, note that relative human endowments $\lambda(h)$ are the same across all models. Therefore, relative skilled human capital in the rich country is at most 1.6 times larger than in the poor country. The previous conclusion that human capital accounts for modest variation in relative skilled labor productivities remains valid. However, the presence of capital-skill complementarity reduces the variation in relative skill bias needed to account for the observable skill premiums in both countries, so that $\lambda(\theta)$ declines relative to the models without capital-skill complementarity. When the short-run elasticity of substitution is large enough, cross-country variation in skill bias vanishes and human capital

[15] The results are very similar when we use rich country equipment prices instead. The contribution of equipment is defined as the steady state output change induced by changing q_E from $q_{E,p}$ to $q_{E,r}$ holding q_S fixed (its level does not matter).

[16] These findings are robust to reasonable variations in the equipment stocks or equipment income shares that we use as calibration targets. For example, $share_L$ remains above one-half even if we reduce the poor country's equipment stock to one quarter of its estimated value. $share_L$ remains above 0.46 when the the poor country's equipment income share is reduced by half.

Table 15: Fraction of Relative Skilled Labor Productivity Variation Due to Human Capital

Short-run Elasticity	Skill Cutoff SHS	HSG	SC	CG
1.25	3.8	8.7	8.9	33.3
1.50	7.9	19.1	21.2	n/a
2.00	16.9	48.3	67.8	n/a
3.00	38.9	203.7	n/a	n/a
4.00	68.7	n/a	n/a	n/a
5.00	111.3	n/a	n/a	n/a

Notes: The table shows $100 \times \lambda(h) / \lambda(\theta h)$ where $\lambda(h) \equiv \Delta \ln(h_s) - \Delta \ln(h_u)$ denotes the cross-country gap in relative skilled human capital endowments. This is not defined for cases where the rich country's technology is less skill biased than the poor country's technology.

accounts for all of the (modest) variation in relative skilled labor productivities.

6 Conclusion

We evaluate the contribution of human capital for development accounting and the relative productivity of skilled versus unskilled labor. We do so in an environment with imperfect substitution between skill types and a variety of factors that shift relative labor supply or demand. Our approach utilizes new empirical evidence on the average wage gains of migrants and the relative wage gains of skilled versus unskilled migrants from Hendricks and Schoellman (2018).

We find that human capital accounts for at least 45 percent of cross-country income differences, which is disciplined by the fact that the average wage gains for migrants are small relative to income gaps. This figure is further expanded by the fact that poor countries are particularly scarce in skilled labor and their skilled labor has less human capital. The overall figure is between one-half and three-quarters depending on which version of the model we consider. We also find that the main driver of relative productivity is skill biased technology, in line with Okoye (2016) and Rossi (2019). This result is disciplined by the fact that the wage gains of unskilled migrants are not too much larger than those of skilled migrants, which suggests that migration to a skill-abundant country is largely offset by more skill-biased technologies.

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Online Appendix

A Endogenous Technology Model

The following derivations apply to all models without capital-skill complementarity. Skill bias can be exogenous or chosen from a technology frontier.

A.1 Notation

It is useful to define commonly used notation at the outset.

1. $\mathcal{R}\left(x_j\right) = x_{j,r}/x_{j,p}$ denotes the rich-to-poor country ratio of $x_{j,c}$.
2. $\mathcal{S}\left(x_c\right) = x_{s,c}/x_{u,c}$ denotes the skilled-to-unskilled ratio of $x_{j,c}$.
3. $\mathcal{RS}\left(x\right) = \mathcal{R}\left(\mathcal{S}\left(x\right)\right) = \mathcal{S}\left(\mathcal{R}\left(x\right)\right) = \frac{x_{s,r}/x_{u,r}}{x_{s,p}/x_{u,p}}$ denotes the relative abundance of skilled versus unskilled x in the rich compared with the poor country.
4. The income share of an input is denoted by $IS_{a,c} = \text{income}_{a,c}/Y_c$.
5. The income ratio of two inputs is denoted by $IR_{a/b,c} = \text{income}_{a,c}/\text{income}_{b,c}$. In particular, $IR_{L_s/L_u} = \mathcal{S}\left(W\right)$.

A number of useful properties of the rich-to-poor and skilled–to-unskilled ratios are worth noting. For any constant ϕ, we have

1. $\mathcal{R}\left(x^{\phi}\right) = \mathcal{R}\left(x\right)^{\phi}$ and $\mathcal{S}\left(x^{\phi}\right) = \mathcal{S}\left(x\right)^{\phi}$.
2. The order of rich-to-poor and skilled-to-unskilled ratios is interchangeable:

$$\mathcal{R}\left(\mathcal{S}\left(x^{\phi}\right)\right) = \mathcal{S}\left(\mathcal{R}\left(x^{\phi}\right)\right) \tag{28}$$

$$= \left[\frac{x_{s,r}/x_{u,r}}{x_{s,p}/x_{u,p}}\right]^{\phi} \tag{29}$$

A.2 CES Results

It is useful to state a number of known properties of cost minimization with CES production. These results will be used repeatedly in the derivations below.

Consider the generic cost minimization problem

$$\min_{x_j} \sum_{j=1}^{J} p_j x_j + \lambda \left[\bar{y} - \left[\sum_{j=1}^{J} (\gamma_j x_j)^{\rho} \right]^{1/\rho} \right] \tag{30}$$

The cost-minimizing input ratios are given by

$$\left[\frac{x_i}{x_j} \right]^{1-\rho} = \left[\frac{\gamma_i}{\gamma_j} \right]^{\rho} \frac{p_j}{p_i} \tag{31}$$

The ratio of factor incomes is then given by

$$\frac{p_i x_i}{p_j x_j} = \left[\frac{\gamma_i x_i}{\gamma_j x_j} \right]^{\rho} \tag{32}$$

$$= \left[\frac{\gamma_i}{\gamma_j} \right]^{\frac{1}{1-\rho}} \left[\frac{p_j}{p_i} \right]^{\frac{\rho}{1-\rho}} \tag{33}$$

The income share of each input is given by

$$\frac{p_j x_j}{\bar{y}} = \left[\frac{\gamma_j x_j}{\bar{y}} \right]^{\rho} \tag{34}$$

The minimized cost per unit of output is given by

$$p_y = \left[\sum_{j=1}^{J} (\gamma_j p_j)^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} \tag{35}$$

A.3 Firm First-order Conditions

The firm's first-order conditions for labor inputs are given by

$$p_{j,c} = (1-\alpha)\, z_c^{1-\alpha} K_c^{\alpha} L_c^{1-\rho-\alpha} \theta_{j,c}^{\rho} L_{j,c}^{\rho-1} \tag{36}$$

If skill bias is endogenous, the first-order condition for $\theta_{j,c}$ is given by

$$\frac{\partial Y_c}{\partial \theta_{j,c}} = \lambda_c \omega \kappa_j^{\omega} \theta_{j,c}^{\omega-1} \tag{37}$$

where λ_c is the Lagrange multiplier on the technology frontier constraint and

$$\frac{\partial Y_c}{\partial \theta_{j,c}} = (1-\alpha)\, K_c^{\alpha} z_c^{1-\alpha} L_c^{1-\rho-\alpha} L_{j,c}^{\rho} \theta_{j,c}^{\rho-1} \tag{38}$$

From (32), the wage bill ratio is given by

$$W_{s,c}/W_{u,c} = \mathcal{S}\left(p_c L_c\right) = \mathcal{S}\left(\theta_c L_c\right)^{\rho} \tag{39}$$

Since $\rho > 0$, an increase in the relative supply of type j labor increases its income share.

B Endogenous Skill Bias

The derivations in this section apply for the model with endogenous skill bias.

B.1 Optimal Skill Bias Choice

The first-order conditions (37) imply the optimal skill bias ratio

$$\mathcal{S}\left(\theta_c\right)^{\omega-\rho} = \mathcal{S}\left(\kappa^{-\omega} L_c^{\rho}\right) \tag{40}$$

Proposition 5. *Optimal skill bias levels are given by*

$$\theta_{u,c}^{\omega} = \frac{B_c}{\kappa_u^{\omega}\Lambda_c} \tag{41}$$

with

$$\Lambda_c = \sum_j \left(\frac{\kappa_u}{\kappa_j}\frac{L_{j,c}}{L_{u,c}}\right)^{\Psi} \tag{42}$$

This holds whether or not B_c is chosen by firms.

Proof. Starting from the technology frontier, we have

$$B_c^{\omega} = \sum_j \left(\kappa_j \theta_{j,c}\right)^{\omega} \tag{43}$$

$$= \left(\kappa_u \theta_{u,c}\right)^{\omega} \sum_j \left(\frac{\kappa_j}{\kappa_u}\frac{\theta_{j,c}}{\theta_{u,c}}\right)^{\omega} \tag{44}$$

Substituting in the condition for optimal relative skill bias (40) yields

$$B_c^{\omega} = \left(\kappa_u \theta_{u,c}\right)^{\omega} \sum_j \left(\frac{\kappa_j}{\kappa_u}\right)^{\omega} \left[\left(\frac{L_{j,c}}{L_{u,c}}\right)^{\rho}\left(\frac{\kappa_u}{\kappa_j}\right)^{\omega}\right]^{\frac{\omega}{\omega-\rho}} \tag{45}$$

Note that $\omega - \frac{\omega^2}{\omega-\rho} = \frac{-\rho\omega}{\omega-\rho} = -\Psi$. This implies

$$B_c^{\omega} = (\kappa_u \theta_{u,c})^{\omega} \sum_j \left(\frac{\kappa_u}{\kappa_j} \frac{L_{j,c}}{L_{u,c}} \right)^{\Psi} = (\kappa_u \theta_{u,c})^{\omega} \Lambda_c \tag{46}$$

□

Proposition 6. *When skill bias is endogenous, the skill premium is given by*

$$\mathcal{S}(p_c) = (\mathcal{S}(L_c))^{\Psi-1} \mathcal{S}(\kappa)^{-\Psi} \tag{47}$$

Proof. From (31) we have

$$\mathcal{S}(p_c) = \mathcal{S}(L_c)^{\rho-1} \mathcal{S}(\theta)^{\rho} \tag{48}$$

Applying the optimal skill bias ratio (40)

$$\mathcal{S}(\theta_c^{\rho}) = \mathcal{S}\left(\kappa^{-\Psi} L_c^{\frac{\rho^2}{\omega-\rho}} \right) \tag{49}$$

yields

$$\mathcal{S}(p_c) = \mathcal{S}\left(\kappa^{-\Psi}\right) \mathcal{S}(L_c)^{\rho + \frac{\rho^2}{\rho-\omega} - 1} \tag{50}$$

Using $\rho + \frac{\rho^2}{\omega-\rho} = \frac{\rho\omega}{\omega-\rho} = \Psi$ gives (47). □

B.2 Reduced Form Labor Aggregator

Proof. (Proposition 1)

The following hold regardless of how B_c is determined (endogenous or fixed). The definition of the labor aggregator (21) implies

$$L_c = \theta_{u,c} L_{u,c} \left(\sum_j \left[\frac{\theta_{j,c}}{\theta_{u,c}} \frac{L_{j,c}}{L_{u,c}} \right]^{\rho} \right)^{1/\rho} \tag{51}$$

Substituting in the condition for the optimal choice of relative skill bias (49) yields

$$L_c = \theta_{u,c} L_{u,c} \left(\sum_j \left[\frac{L_{j,c}}{L_{u,c}} \right]^{\frac{\rho^2}{\omega-\rho}} \left[\frac{\kappa_j}{\kappa_u} \right]^{-\Psi} \left[\frac{L_{j,c}}{L_{u,c}} \right]^{\rho} \right)^{1/\rho} \tag{52}$$

The exponent on labor inputs is given by

$$\frac{\rho^2}{\omega-\rho} + \rho = \frac{\omega\rho}{\omega-\rho} = \Psi \tag{53}$$

Then the summation term becomes Λ_c, defined in (42), and we have

$$L_c = \theta_{u,c} L_{u,c} \Lambda_c^{1/\rho} \tag{54}$$

Then using (41), we have

$$L_c = B_c \kappa_u^{-1} \Lambda_c^{-1/\omega} L_{u,c} \Lambda_c^{1/\rho} \tag{55}$$

Note that

$$1/\rho - 1/\omega = \frac{\omega - \rho}{\omega\rho} = 1/\Psi \tag{56}$$

so that

$$L_c = B_c \left(1/\kappa_u\right) L_{u,c} \Lambda_c^{1/\Psi} \tag{57}$$

$$= B_c \left(1/\kappa_u\right) L_{u,c} \left[\sum_j \left(\kappa_j^{-1} L_{j,c}\right)^{\Psi}\right]^{1/\Psi} \kappa_u / L_{u,c} \tag{58}$$

$\square$

B.3 Closed Form Solution

Proof. (Proposition 2)

Using the labor aggregator (9) with $\kappa_j = 1$, we have

$$\mathcal{R}\left(L\right) = \frac{L_{u,p}\left[\mathcal{R}\left(L_u\right)^{\Psi} + \mathcal{S}\left(L_r\right)^{\Psi}\mathcal{R}\left(L_u\right)^{\Psi}\right]^{1/\Psi}}{L_{u,p}\left[1 + \mathcal{S}\left(L_p\right)^{\Psi}\right]^{1/\Psi}} \tag{59}$$

$$= \mathcal{R}\left(L_u\right)\mathcal{R}\left(1 + \mathcal{S}\left(L\right)^{\Psi}\right)^{1/\Psi} \tag{60}$$

Since (39) also applies to the reduced form labor aggregator, we have

$$W_{s,c}/W_{u,c} = \mathcal{S}\left(L_c\right)^{\Psi} \tag{61}$$

Using this to replace $\mathcal{S}\left(L\right)^{\Psi}$ in (60) with W_s/W_u yields

$$\mathcal{R}\left(L\right) = \mathcal{R}\left(L_u\right)\mathcal{R}\left(1 + W_s/W_u\right)^{1/\Psi} \tag{62}$$

If $\mathcal{S}\left(L_r\right) > \mathcal{S}\left(L_p\right)$, then $\mathcal{R}\left(1 + W_s/W_u\right) > 1$ and $\mathcal{R}\left(L\right) > \mathcal{R}\left(L_1\right)$. The ratio of unskilled labor inputs is given by

$$\mathcal{R}(L_1) = \frac{\mathcal{R}(W_u)}{\pi_u} \tag{63}$$

$$= \mathcal{R}(Y)\frac{\mathcal{R}(W_u)}{\pi_u \mathcal{R}((1-\alpha)\,y)} \tag{64}$$

$$= \frac{\mathcal{R}(Y)}{\pi_u}\frac{1}{\mathcal{R}(1+W_s/W_u)} \tag{65}$$

Substituting this into (62) and rearranging yields

$$\mathcal{R}(L) = \frac{\mathcal{R}(Y)}{\pi_u}\mathcal{R}(W/W_u)^{1/\Psi-1}. \tag{66}$$

The solution for Ψ follows from (61) which implies $\mathcal{RS}(W) = \mathcal{RS}(L)^{\Psi}$. □

C Exogenous Skill Bias

C.1 Closed Form Solution

Proof. (Proposition 3)

Define $contrib_L^{poor}$ as the increase in L due to replacing $L_{j,p}$ with $L_{j,r}$, holding $\theta_{j,p}$ fixed:

$$contrib_L^{poor} = \frac{\left[\sum_j (\theta_{j,p}L_{j,r})^{\rho}\right]^{1/\rho}}{\left[\sum_j (\theta_{j,p}L_{j,p})^{\rho}\right]^{1/\rho}} \tag{67}$$

$$= \frac{\theta_{u,p}L_{u,p}\left[\mathcal{R}(L_u)^{\rho} + \left(\frac{\theta_{s,p}}{\theta_{u,p}}\frac{L_{s,r}}{L_{u,p}}\right)^{\rho}\right]^{1/\rho}}{\theta_{u,p}L_{u,p}\left[1+W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{68}$$

$$= \frac{\left[\mathcal{R}(L_u)^{\rho} + \left(\frac{\theta_{s,p}}{\theta_{u,p}}\frac{L_{s,p}}{L_{u,p}}\mathcal{R}(L_s)\right)^{\rho}\right]^{1/\rho}}{\left[1+W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{69}$$

$$= \frac{\left[\mathcal{R}(L_u)^{\rho} + W_{s,p}/W_{u,p}\left(\mathcal{R}(L_s)\right)^{\rho}\right]^{1/\rho}}{\left[1+W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{70}$$

This uses (39) to replace $\mathcal{S}(\theta L)^{\rho}$ with W_s/W_u. Pulling out $\mathcal{R}(L_u)$ yields

$$contrib_L^{poor} = \mathcal{R}(L_u)\left[\frac{1+W_{s,p}/W_{u,p}\mathcal{RS}(L)^{\rho}}{1+W_{s,p}/W_{u,p}}\right]^{1/\rho} \tag{71}$$

Replacing $\mathcal{R}(L_u)$ using (65) gives

$$contrib_L^{poor} = \frac{\mathcal{R}(Y)}{\pi_u} \frac{1}{\mathcal{R}(1 + W_s/W_u)} \left[\frac{1 + W_{s,p}/W_{u,p} \mathcal{RS}(L)^\rho}{1 + W_{s,p}/W_{u,p}} \right]^{1/\rho} \tag{72}$$

To see that $contrib_L^{poor} \in (\mathcal{R}(L_u), \mathcal{R}(L_s))$, note that

$$contrib_L^{poor} = \mathcal{R}(L_s) \frac{\left[\mathcal{RS}(L)^{-\rho} + \mathcal{S}(W_p)\right]^{1/\rho}}{\left[1 + W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{73}$$

If $\mathcal{R}(L_s) > \mathcal{R}(L_u)$, then $\mathcal{R}(L_u) < contrib_L^{poor} < \mathcal{R}(L_s)$; otherwise $\mathcal{R}(L_s) < contrib_L^{poor} < \mathcal{R}(L_u)$.

Using rich country skill bias, we have

$$contrib_L^{rich} = \frac{\left[\sum_j (\theta_{j,r} L_{j,r})^\rho\right]^{1/\rho}}{\left[\sum_j (\theta_{j,r} L_{j,p})^\rho\right]^{1/\rho}} \tag{74}$$

$$= \frac{\theta_{u,r} L_{u,r} \left[1 + \left(\frac{\theta_{s,r}}{\theta_{u,r}} \frac{L_{s,r}}{L_{u,r}}\right)^\rho\right]^{1/\rho}}{\theta_{u,r} L_{u,r} \left[\mathcal{R}(L_u)^{-\rho} + \left(\frac{\theta_{s,r}}{\theta_{u,r}} \frac{L_{s,r}}{L_{u,r}} \frac{L_{s,p}}{L_{s,r}}\right)^\rho\right]^{1/\rho}} \tag{75}$$

$$= \frac{\left[1 + W_{s,r}/W_{u,r}\right]^{1/\rho}}{\left[\mathcal{R}(L_u)^{-\rho} + W_{s,r}/W_{u,r} \mathcal{R}(L_s)^{-\rho}\right]^{1/\rho}}$$

Pulling out $\mathcal{R}(L_u)$ and replacing it using (65) yields (19). □

D Investment in the Frontier

We consider a model where firms can expend resources to shift the technology frontier outwards, as in Acemoglu (2007). The representative firm solves

$$\max_{K_c, L_{j,c}, \theta_{j,c}, B_c} Y_c - q_c K_c - \sum_j p_{j,c} L_{j,c} - C(B_c) \tag{76}$$

subject to (1), (2), and (3), taking factor prices as given. We assume the linear cost function $C(B_c) = b_c B_c$ as in Acemoglu (2007)'s example 1. The firm takes $b_c > 0$ as given. We assume $\omega > 1$ to ensure that optimal skill weights are finite. We normalize all $\kappa_j = 1$.

Compared with the fixed frontier case studied in Section 2, the only change is the endogeneity of B_c. Conditional on its value all quantities and prices are the same as in the endogenous technology model.

If we treat b_c as a parameter, the model has increasing returns to scale. We show that, in this case, $share_L$ is magnified by the factor $\frac{\omega}{\omega-1}$ compared with the endogenous technology model. If b_c scales appropriately with Y_c so that the model has constant returns to scale, we show that the development accounting results of the endogenous technology model remain unchanged.

D.1 Reduced Form Labor Aggregator

Proposition 7. *The labor aggregator is given by*

$$\hat{L}_c = \left((1-\alpha)\, z_c^{1-\alpha} K_c^{\alpha} \omega^{-1} b_c^{-1}\right)^{\frac{1}{\omega+\alpha-1}} \tilde{L}_c^{\frac{\omega}{\omega+\alpha-1}} \tag{77}$$

where $\tilde{L}_c$ is the reduced form labor aggregator with a fixed frontier, given by (9).

Proof. The firm's problem may be written as

$$\max_{K_c, L_{j,c}, \theta_{j,c}} Y_c - q_c K_c - \sum_j p_{j,c} L_{j,c} - b_c \sum_j \left(\kappa_j \theta_{j,c}\right)^{\omega} \tag{78}$$

The firm's first-order condition for $\theta_{j,c}$ is again given by (37), except that now $\lambda_c = b_c$ so that

$$\theta_{j,c}^{\omega-\rho} = X_{j,c} L_{j,c}^{\rho} L_c^{1-\alpha-\rho} \tag{79}$$

where

$$X_{j,c} = \frac{(1-\alpha)\, z_c^{1-\alpha} K_c^{\alpha}}{b_c \omega \kappa_j^{\omega}} \tag{80}$$

Together with $1 + \rho/(\omega-\rho) = \omega/(\omega-\rho)$ this implies

$$\theta_{u,c} L_{u,c} = X_{u,c}^{\frac{1}{\omega-\rho}} L_{u,c}^{\frac{\omega}{\omega-\rho}} L_c^{\frac{1-\alpha-\rho}{\omega-\rho}} \tag{81}$$

From (54), we have

$$\theta_{u,c} L_{u,c} = L_c \Lambda_c^{-1/\rho} \tag{82}$$

$$= L_c \left(L_u \kappa_u^{-1} / \tilde{L}_c\right)^{\Psi/\rho} \tag{83}$$

Setting both expressions for $\theta_{u,c} L_{u,c}$ equal and noting that $\Psi/\rho = \omega/(\omega-\rho)$, we have

$$L_c^{1-\frac{1-\alpha-\rho}{\omega-\rho}} = \left(\kappa_u \tilde{L}_c\right)^{\frac{\omega}{\omega-\rho}} X_{u,c}^{\frac{1}{\omega-\rho}} \tag{84}$$

Since

$$1 - \frac{1-\alpha-\rho}{\omega-\rho} = \frac{\omega+\alpha-1}{\omega-\rho} \tag{85}$$

we have

$$L_c = \left(\kappa_u^\omega X_{u,c}\right)^{\frac{1}{\omega+\alpha-1}} \tilde{L}_c^{\frac{\omega}{\omega+\alpha-1}} \tag{86}$$

□

D.2 Reduced Form Production Function

Proposition 8. *The reduced form production function is given by*

$$Y_c = \left(K_c^\alpha \left(\hat{A}_c z_c \tilde{L}_c\right)^{1-\alpha}\right)^{\frac{\omega}{\omega+\alpha-1}} \tag{87}$$

where $\tilde{L}_c$ is given by (9) and $\hat{A}_c = \left(\frac{1-\alpha}{\omega b_c}\right)^{1/\omega}$ is a constant.

Proof. Substituting the reduced form labor aggregator (86) into the production function, we have

$$Y_c = K_c^\alpha \left(z_c L_c\right)^{1-\alpha} \tag{88}$$

$$= K_c^\alpha z_c^{1-\alpha} \left(\kappa_u^\omega X_{u,c}\right)^{\frac{1-\alpha}{\omega+\alpha-1}} \left(\tilde{L}_c\right)^{\frac{\omega(1-\alpha)}{\omega+\alpha-1}} \tag{89}$$

$$= \tilde{A}_c \left(z_c^{1-\alpha} K_c^\alpha\right)^{1+\frac{1-\alpha}{\omega+\alpha-1}} \left(\tilde{L}_c\right)^{\frac{\omega(1-\alpha)}{\omega+\alpha-1}} \tag{90}$$

where

$$\tilde{A}_c = \left(\frac{1-\alpha}{\omega b_c}\right)^{\frac{1-\alpha}{\omega+\alpha-1}} \tag{91}$$

collects all constant terms. Then

$$Y_c = \left(K_c^\alpha \left(\hat{A}_c z_c \tilde{L}_c\right)^{1-\alpha}\right)^{\frac{\omega}{\omega+\alpha-1}} \tag{92}$$

This is true because the exponent on $z_c^{1-\alpha} K_c^\alpha$ is

$$1 + \frac{1-\alpha}{\omega+\alpha-1} = \frac{\omega}{\omega+\alpha-1} \tag{93}$$

□

If b_c is fixed, the model has increasing returns to scale due to scale effects. Increasing any factor input or increasing TFP raises the benefits from investing in B_c, but not the cost. The optimal level of B_c increases, amplifying the effect on output. The imperfect substitutes term is governed by the exponent $\frac{\omega}{\omega+\alpha-1}$.

The scale effect is eliminated if the cost of investing in B_c scales appropriately with output. Specifically, if $b_c = Y_c$, the production function reverts to the one for the fixed frontier, except that the TFP level z_c is multiplied by a constant. In that case, investment in the frontier has no impact on development accounting.

D.3 Development Accounting

Proposition 9. *The reduced form production function (87) satisfies*

$$Y_c = \left[(K_c/Y_c)^{\alpha/(1-\alpha)} \hat{A}_c z_c \tilde{L}_c \right]^{\frac{\omega}{\omega-1}} \tag{94}$$

Proof. Write (87) as

$$Y_c = \left[(K_c/Y_c)^{\alpha} \left(\hat{A}_c z_c \tilde{L}_c \right)^{1-\alpha} \right]^{\frac{\omega}{\omega+\alpha-1}} Y_c^{\frac{\alpha\omega}{\omega+\alpha-1}} \tag{95}$$

and note that the exponent on Y_c becomes

$$1 - \frac{\alpha\omega}{\omega+\alpha-1} = \frac{\omega+\alpha-1-\alpha\omega}{\omega+\alpha-1} \tag{96}$$

$$= (\alpha-1) \frac{1-\omega}{\omega+\alpha-1} \tag{97}$$

Then

$$Y_c = \left[(K_c/Y_c)^{\alpha} \left(\hat{A}_c z_c \tilde{L}_c \right)^{1-\alpha} \right]^{\phi} \tag{98}$$

with $\phi = \frac{\omega}{\omega+\alpha-1} \times \frac{\omega+\alpha-1}{(1-\alpha)(\omega-1)}$. Simplify exponents to arrive at equation (94). □

Now the only difference relative to the case where B_c is fixed is the exponent $\omega/(\omega-1)$. To perform development accounting, it is necessary to know the values of ω and ρ, not just the reduced form elasticity governed by Ψ. Identifying both values requires an additional data moment. Relative to the model with a fixed frontier, the contribution of labor inputs to output gaps is amplified by a constant factor, $\omega/(1-\omega)$.

Proposition 10. *The share of cross-country output gaps accounted for by labor inputs is given by*

$$share_L = \frac{\omega}{\omega-1} \frac{\ln \mathcal{R}\left(\tilde{L}\right)}{\Delta \ln (Y)} \tag{99}$$

where $\tilde{L}_c$ takes on the same value as in the model without investment in the frontier.

Proof. Let $A_c = (K_c/Y_c)^{\alpha/(1-\alpha)} \hat{A}_c z_c$ collect all country specific terms other than labor inputs. Then $Y_c = \left[A_c \tilde{L}_c\right]^{\frac{\omega}{\omega-1}}$ and

$$\frac{\omega-1}{\omega} \Delta \ln (Y) = \Delta \ln (A) + \Delta \ln \left(\tilde{L}\right) \tag{100}$$

This implies (99). Since the calibrated values of $h_{j,c}$ and Ψ do not depend on whether or not B_c is endogenous, the labor aggregator is the same as in the model with fixed B_c. □

E Capital-skill Complementarity

E.1 Equipment and Structures Data

Calibrating the model with capital-skill complementarity requires additional data moments related to equipment and structures that are constructed as follows. All data are constructed for year 2011, which is the latest and most comprehensive benchmark year for the International Comparison Project. From the PWT, we obtain:

1. output per worker Y as `cgdpo/emp`.
2. capital per worker K as `ck/emp`.
3. the price levels of capital `pl_k` and consumption `pl_c`.
4. the value of the equipment stock at local prices as `Kc_Mach + Kc_TraEq`.
5. the value of the structures stock at local prices as `Kc_Struc + Kc_Other` (from the capital detail file).

From ICP we obtain the PPP prices (series S03) of equipment (classification C20 Machinery and equipment) and structures (classification C21 Construction).

We define the stock of equipment as $E_c = \left(\text{Kc\_Mach+KcTraEq}\right)/\text{emp}/PPP_{C20}$ and the stock of structures as $S_c = \left(\text{Kc\_Struc+Kc\_Other}\right)/\text{emp}/PPP_{C21}$.

Before computing the calibration targets, we drop countries with missing output or employment data or with population (`pop`) < 1m. We also drop 6 countries with capital or consumption prices above 10 times the sample median. Finally, we drop 7 countries for which the discrepancy between k and $E + S$ is above 20%.

E.2 Preliminaries

This section contains results that are used in subsequent derivations. They hold for endogenous and exogenous skill bias.

E.2.1 Firm first-order conditions

The firm's first-order conditions are:

$$S : \alpha Y_c / S_c = q_{s,c} \tag{101}$$

$$E : \frac{\partial Y_c}{\partial L_c} \frac{\partial L_c}{\partial Z_c} Z_c^{1-\phi} \mu_e^{\phi} E_c^{\phi-1} = q_{e,c} \tag{102}$$

$$L_s : \frac{\partial Y_c}{\partial L_c} \frac{\partial L_c}{\partial Z_c} Z_c^{1-\phi} \mu_s^{\phi} L_{s,c}^{\phi-1} = p_{s,c} \tag{103}$$

$$L_u : \frac{\partial Y_c}{\partial L_c} L_c^{1-\rho} \theta_{u,c}^{\rho} L_{u,c}^{\rho-1} = p_{u,c} \tag{104}$$

where

$$\frac{\partial Y_c}{\partial L_c} = (1-\alpha)\, Y_c / L_c \tag{105}$$

$$\frac{\partial L_c}{\partial Z_c} = L_c^{1-\rho} \theta_{s,c}^{\rho} Z_c^{\rho-1} \tag{106}$$

If there is a technology frontier, we also have

$$\theta_{u,c} : \frac{\partial Y_c}{\partial L_c} L_c^{1-\rho} \theta_{u,c}^{\rho-1} L_{u,c}^{\rho} = \lambda_c \omega \kappa_u^{\omega} \theta_{u,c}^{\omega-1} \tag{107}$$

$$\theta_{s,c} : \frac{\partial Y_c}{\partial L_c} L_c^{1-\rho} \theta_{s,c}^{\rho-1} Z_c^{\rho} = \lambda_c \omega \kappa_s^{\omega} \theta_{s,c}^{\omega-1} \tag{108}$$

which implies that the optimal skill bias ratio is a constant elasticity function of relative inputs:

$$\mathcal{S}\,(\theta)^{\omega-\rho} = \mathcal{S}\,(\kappa)^{-\omega}\,(Z/L_u)^{\rho} \tag{109}$$

E.2.2 Income ratios and shares

Applying the generic CES expression (32) yields the income ratios of skilled labor to equipment

$$IR_{L_s/e} = \left(\frac{\mu_s L_s}{\mu_e E}\right)^{\phi} \tag{110}$$

and of Z versus L_u

$$IR_{Z/L_u} = \left(\frac{\theta_s Z}{\theta_u L_u}\right)^{\rho} \tag{111}$$

The income ratio of skilled versus unskilled labor is then given by

$$W_s/W_u = IR_{L_s/e} IR_{Z/L_u} = \left(\frac{\mu_s L_s}{Z}\right)^{\phi} \left(\frac{\theta_s Z}{\theta_u L_u}\right)^{\rho} \tag{112}$$

The income share of equipment is given by $IS_e = IS_L IR_{Z/L} IR_{E/Z}$. Again applying the generic CES expressions yields

$$IS_E = (1-\alpha)\left[\frac{\theta_s Z}{L}\right]^{\rho}\left[\frac{\mu_e E}{Z}\right]^{\phi} \tag{113}$$

E.3 Endogenous Skill Bias

E.3.1 Reduced form labor aggregator

Proof. (Proposition 4)

We may think of the firm as solving its problem in two steps. First, the firm chooses $L_{s,c}/E_c$ to minimize the cost of Z. This is a standard CES cost minimization problem with the solution

$$\left[\frac{L_s}{E}\right]^{1-\phi} = \frac{q_E}{p_2}\left[\frac{\mu_s}{\mu_e}\right]^{\phi} \tag{114}$$

and the unit cost

$$p_Z = \left[(\mu_e q_E)^{\frac{\phi}{1-\phi}} + (\mu_s p_s)^{\frac{\phi}{1-\phi}}\right]^{\frac{1-\phi}{\phi}} \tag{115}$$

In the second step, the firm solves

$$\max_{L_{u,c}, Z_c, \theta_{j,c}, S_c} S^{\alpha}\left[z_c L_c\right]^{1-\alpha} - q_S S - p_u L_u - p_Z Z \tag{116}$$

subject to the labor aggregator (21) and the frontier constraint (3). This problem has the same structure as the one solved by the firm in the endogenous technology model, except that the firm chooses structures instead of capital and Z instead of L_2. It follows directly that the labor aggregator takes on the same form as in the endogenous technology model. □

E.3.2 Joint Contribution of Labor Inputs and Equipment

We derive a closed form solution for the joint contribution of labor inputs and equipment to cross-country output gaps, $share_{L+E}$.

Proposition 11. *The joint contribution of labor inputs and equipment to cross-country output gaps is given by*

$$share_{L+E} = \underbrace{1 - \frac{\ln(\pi_u)}{\Delta \ln(Y)}}_{base} + \underbrace{\frac{\frac{1}{\Psi}\Delta \ln\left(1 + IR_{Z/L_u}\right) - \Delta \ln(1 + W_s/W_u)}{\Delta \ln(Y)}}_{amplification} \tag{117}$$

where the reduced form curvature is given by

$$\Psi = \frac{\Delta \ln\left(IR_{Z/L_u}\right)}{\Delta \ln(Z/L_u)} \tag{118}$$

and the curvature of the Z aggregator is given by

$$\phi = \frac{\Delta \ln\left(IR_{L_s/e}\right)}{\Delta \ln(L_s/E)} \tag{119}$$

In terms of observable data moments, the reduced form curvature may be written as

$$\Psi = \frac{\ln \mathcal{RS}(W) + \Delta \ln\left(1 + IR_{e/L_s}\right)}{\ln \mathcal{RS}(L) + \frac{1}{\phi}\Delta \ln\left(1 + IR_{e/L_s}\right)} \tag{120}$$

Throughout, $IR_{a/b,}$ denotes the ratio of incomes received by inputs a and b.

Proof. The labor aggregator may be written as

$$L_c = L_{u,c}\left[1 + (Z_c/L_{u,c})^{\Psi}\right]^{1/\Psi} \tag{121}$$

Applying the generic CES expressions for income shares and income ratios to the reduced form labor aggregator yields

$$\begin{aligned} \left(\frac{Z}{L_u}\frac{\kappa_u}{\kappa_s}\right)^{\Psi} &= W_s/W_u\left(1 + IR_{e/L_s}\right) && (122)\\ &= IR_{Z/L_u} && (123) \end{aligned}$$

where κ_j may be normalized to one. Using (123) we have

$$L_c = L_{u,c}\left[1 + W_{s,c}/W_{u,c}\left(1 + IR_{e/L_s,c}\right)\right]^{1/\Psi} \tag{124}$$

Taking logarithms and replacing $\mathcal{R}(L_u)$ using (66) yields (117).

The solution for Ψ is obtained by taking the rich-to-poor country ratio of (123) in logarithms which yields

$$\Psi = \frac{\Delta \ln \left(W_s/W_u \left(1 + IR_{e/L_s}\right)\right)}{\Delta \ln \left(Z/L_u\right)} \tag{125}$$

$$= \frac{\Delta \ln \left(IR_{Z/L_u}\right)}{\Delta \ln \left(Z/L_u\right)} \tag{126}$$

where $\mathcal{R}(Z)$ follows from

$$\mathcal{R}(Z) = \mathcal{R}(Z/(\mu_e E))\,\mathcal{R}(E) \tag{127}$$

$$= \mathcal{R}\left(\left[1 + IR_{s/e}\right]^{1/\phi}\right)\mathcal{R}(E) \tag{128}$$

$$= \mathcal{R}\left(\left[1 + IR_{e/L_s}\right]^{1/\phi}\right)\mathcal{R}(L_2) \tag{129}$$

The solution for Ψ can be expressed in a form that is closer to the endogenous technology model. From (123), we have

$$\frac{Z}{L_u} = \frac{Z}{L_s}\mathcal{S}(L) = \mathcal{S}(L)\left(1 + IR_{e/L_s}\right)^{1/\phi} \tag{130}$$

Therefore

$$\Psi = \frac{\Delta \ln \left(W_s/W_u\right) + \Delta \ln \left(1 + IR_{e/L_s}\right)}{\ln \mathcal{R}\mathcal{S}(L) + \frac{1}{\phi}\Delta \ln \left(1 + IR_{e/L_s}\right)} \tag{131}$$

□

The data moments used in the calibration imply that skilled labor and equipment are complements ($\phi < 0$).[17] This is consistent with U.S. time series evidence (see Krusell et al. 2000).

Since we assume that the income share of equipment is the same in rich and poor countries, IR_{e/L_s} is lower in rich compared with poor countries. Together with $\phi < 0$, it follows that the long-run elasticity of substitution between skilled and unskilled labor is lower than in the endogenous technology model. This increases the imperfect substitutes term.

The expression for $share_{L+E}$ is similar in structure to the endogenous technology model's (11). The perfect substitutes term is the same, again reflecting the contribution of human capital in a single skill model. The imperfect substitutes term now depends on the ratio of incomes received by Z (by skilled labor and equipment jointly) to unskilled labor. When equipment is "unimportant," so that $IR_{e/L_s} \approx 0$, the values of Ψ and $\mathcal{R}(L)$ approach those of the endogenous technology model.

[17] The numerator in (119) is positive because $\mathcal{R}(IS_e) = 1$ and $\mathcal{R}(IS_{L_s}) > 1$. The denominator is negative because equipment stocks vary across countries more than labor inputs. Hence $\phi < 0$.

E.4 Exogenous Skill Bias

Our final model treats variation in skill bias $\theta_{j,c}$ across countries as exogenous. Except for dropping the technology frontier, the model is identical to the one described in Section 5.1.

E.4.1 Development Accounting

We define the contribution of labor inputs to cross-country output variation as the change in steady state output that results from increasing $L_{j,p}$ to $L_{j,r}$, holding capital rental prices and skill bias $\theta_{j,c}$ constant. It follows that $share_L$ depends on the fixed levels of q_E (but not on q_S) and now also on those of $\theta_{j,c}$. We consider two cases:

1. $share_L^{poor}$ fixes skill bias and q_E at poor country levels. This corresponds to increasing labor inputs in the poor country.
2. $share_L^{rich}$ fixes skill bias and q_E at rich country levels. This corresponds to reducing labor inputs in the rich country.

Relative to the model with the technology frontier, one additional parameter needs to be calibrated because counterfactual output depends on the values of ρ and ω, not only on the reduced form curvature Ψ. The development accounting results therefore require fixed values of ρ.

E.4.2 Quantitative Results

Figure 1 provides a compact visual summary of the results. When poor country $\theta_{j,c}$ and q_E are used, the results are very similar to the endogenous technology model. $share_L^{poor}$ is smaller than $share_L$ when $\rho < \Psi$. It increases with the elasticity of substitution and the skill cutoff. Values below 0.5 are associated with very large cross-country differences in relative skill bias (at least factor 10^5).

With rich country $\theta_{j,c}$ and q_E, $share_L^{rich}$ is higher than $share_L$ when $\rho < \Psi$. Its value decreases with the elasticity of substitution and the skill cutoff. For conventional values of the elasticity, we find $share_L^{rich}$ between 0.64 and 0.74 (compared with 0.59 to 0.74 in the endogenous technology model).

Figure 1: $share_L$: Capital-skill Complementarity