Monetary Policy Regimes & Real Estate Prices

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1 Introduction

Identifying monetary policy regimes is not an easy task. Sims (2001) argues that by checking for parameter instability in single-equation models or reduced-form regressions, economists fail to gain valuable economic insights by attempting to draw inference from a clearly misspecified model. When estimating structural MS-DSGE models that employ the RE assumption the parameters tend to be poorly identified in estimation.

A flurry of papers have attempted to achieve more accurate estimations of deep parameters by disciplining their models with economic indicators that by their nature incorporate future expectations based on an information set that supersedes the state-variables included in the model. For example, Baele et al. (2015) incorporates the Survey of Professional Forecasters estimates of future inflation rates, and Bikbov and Chernov (2013) incorporates the observed yield curve.

We propose that the real estate capitalization rate (henceforth "cap rate"), the ratio of realized net operating income (NOI) to the transactional price of the underlying asset, may be able to provide additional discipline to the class of MSRE New Keynsian models. We also suspect that during periods where the term structure of interest rates is highly persistent, such as the recent zero-lower-bound (ZLB) episode, that cap rate will retain its ability to inform about monetary policy regimes.

The theoretical underpinnings of our hypothesis stems from the Dynamic Gordon model Campbell and Shiller (1988), which posits that the cap rate is a function of the future expected path of returns, or future expected path of rent growth. This is analogous to the dividend-to-price ratio of common stock. Plazzi et al. (2010) find that indeed the cap rate holds some power to predict both future returns and future rent growth, but the predictability is dependent on how returns covary with rental growth.

Our key insight is that assuming the Fisher relationship is present in apartment rents, how returns covary with the growth of rents depends on how monetary policy in enacted, and how economic agents expect monetary policy to be conducted in the future. For example, it is well known in monetary policy regime switching literature that Fed has gone through periods of where they respond aggressively to inflation (henceforth this will be known as the "active" regime), and periods where the Fed has not adjusted nominal rates enough with inflation ("passive"). Bianchi (2013), Clarida et al. (2000), and Bikbov and Chernov (2013) represents only a few of the papers that find evidence consistent with this theory. We assume that the Fisher relationship holds to a degree, and landlords adjust rents with expectations about inflation to the best of their ability. If the Fisher relationship holds, then during periods where the short-rate moves with inflation, we should not see much variability in the cap rate with interest rates, and the ability of the cap rate to predict either future returns of future rental growth is low. During periods where the short-rate does not move with inflation, the variation of the cap rate should increase, and the cap rate should start to predict future returns and future growth again. These oscillating periods of predictability are what we hope allows our model to estimate the path of monetary policy regimes with increased accuracy over the Bikbov and Chernov (2013) model.

We run reduced-form form regressions as a motivational experiment to document that we observe a statistically significant change in the correlation structure between the cap rate, inflation, and the annualized return of the 10-year treasury note (T-note). Our break point is 1999:Q3, a time close to the 2000-2001 financial crisis where evidence supports the hypothesis that monetary policy switched to a "passive" regime Bianchi (2013).

Pre-1993:Q3 we find the relationship between the cap rate and short-term interest rate is not statistically significant and has a very small R-squared. Post-1993:Q3 we find that the short rate has a statistically significant positive relationship with the cap rate, and R-squared increases by close to 40 percentage points. We find opposite results with regard to how inflation predicts interest rates in both periods. Pre-1999:Q3 inflation had a statistically significant relationship with the short-rate with an R-squared of close to 60%. Post-break we find the relationship switches signs, is no longer statistically significant, and R-squared drops to 7%.

The above evidence is consistent with our priors, but we do not consider this valid evidence of our hypothesis. In order to do our question justice, we employ a structural macro-finance models such that a variety of deep structural parameters are allowed to switch independently with regimes. Agents are forward looking in our model and thus take into account the possibility regimes may switch when forming their expectations.

Our paper provides insight into the relationship between monetary policy and the real estate market. What more, our model framework allows to ask additional interesting questions such as "what was monetary policy's role role in the 2000's housing crisis", or "was the recent housing bubble rational"?

The rest of the paper is structured as follows. Section 2 is a brief literature review. Section 3 describes our data, and describes the motivational experiment as well as present results. Section 4 sets up our model framework. Section 5 describes the methodology. Results follow in section 6, and section 7 concludes.

2 Literature Review

There exists a large literature that attempts to answer the question, "did a change in the way monetary policy was conducted during the 1980s contribute to the reduction in volatility of inflation, & economic growth during that same period". Clarida et al. (2000) (hence CGG) test for instability of a forward-looking monetary policy rule in the spirit of Taylor (1993). The authors find that prior to 1979:Q3, the US monetary authority's response to changing inflation expectations to be less than unity. During the the Volcker-Greenspan era, the authors find that the central raised interest rate more than one-for-one with inflation expectations. Their findings suggest that during Arthur Burn's reign of chairman of the Fed was characterized by destabilizing monetary policy that coincides with a period of a high volatility of economic fundamentals. While the Volcker-Greenspan era was stabilizing and coincides with a period of low volatility of economic fundamentals.

This paper is related to studies in the vein of CGG, that opt to use structural multivariate DGSE models to capture monetary policy regime changes, as opposed to the single-equation used by CGG. Early examples include Lubik and Schorfheide (2004) and Sims and Zha (2006). The latter set of authors state:

> It is time to abandon the idea that policy change is best modeled as a once-and-for-all, non-stochastic regime switch. Policy changes, if they have occurred, have not been monotonic, and they have been difficult to detect. Both the rational public in our models and econometricians must treat the changes in policy probabilistically, with a model of how and when the shifts occur and with recognition of the uncertainty about their nature & timing.

One of the biggest concerns in these structural models is poor identification of the parameters caused by a flat likelihood function . Even when imposing a rational expectations (RE) constraint, it is only possible for agents in the model to form expectations based on the variables included in the model. It is typical in the literature for these variables to be a measure of the output gap, inflation, & the federal funds rate (FFR). In actuality, economic agents form their expectations taking into account information outside of the model information set.

Boivin and Giannoni (2006) use a large data set of potentially informative economic indicators to instrument for informative information outside of the model, and estimate a Markov-Switching Rational Expectations DSGE (MSRE-DSGE) model. They find that they are indeed able to achieve much more accurate estimates by exploiting the outside information. However, the drivers of economic fluctuations implied by the model estimates are sensitive to the choice of variables to form the latent variable, suggesting econometricians must be careful in how they choose what outside information is included to discipline their model.

Rudebusch and Wu (2008) as well as Bekaert et al. (2010) connect an affine no-arbitrage model of the term structure of interest rates to a hybird small-scale New-Keynsian model similar to the one used in this paper but without allowing for multiple regimes. The former authors are able to interpret popular latent term-structure factors, "level" and "slope", through expectations of future inflation & economic growth respectively. While not the main questions of the paper, the authors are cognizant of the term structure's ability to be informative of the future path of economic fundamentals:

> ... a joint macro-finance perspective can also illuminate various macroeconomic issues, since the addition of term structure information to a macroeconomic model can help sharpen inference. Specifically, the term structure factors summarize expectations about future short rates, which in turn reflect expectations about the future dynamics of the economy, and with forward-looking economic agents, these expectations should be important determinants of current and future macroeconomic variables.

Bikbov and Chernov (2013) attempts to employ the term structure of interest rates to get sharper estimates on a Markov-Switching New-Keynesian model allowing for the monetary policy rule parameters, the volatility of the output gap & inflation, and the volatility of short-term nominal interest rates to all switch independently of eachother. The authors find the econometrician's ex-post probability of being in a given monetary policy regime is much sharper in the model that includes select terms of the yield curve. Their model is used as a starting point for our model, as it addresses concerns of Sims (2001); that models of monetary policy that ignore heteroskedasticity while allowing for parameter instability are prone to be spurious.

Baele et al. (2015) take a different approach in broadening their model's information set. They use the Survey of Professional Forecasters series on inflation expectation – where survey participants are highly informed economic agents that arguable use a larger information set than the model to form their forecasts – to discipline their estimates. Supporting findings of CGG they find the passive monetary regime to be indeterminate in the mean square stability

(MSS) sense, in contradiction to results from Bikbov and Chernov (2013) which relies on the term structure of interest to include outside information in the model.

Finally, our paper is related to the empirical real estate literature. Plazzi et al. (2010) test predictions about the cap rate implied by the Dynamic Gordon model Campbell and Shiller (1988); that cap rates should be able to predict future returns or future rent growth. The authors find in a cross-sectional analysis that certain classes of cap rates do predict either rental growths or future returns. They also find that the ability of the cap rate to predict future returns of rental growth is determined by the covariance of the two variables.

3 Data & Motivational Experiment

3.1 Data

We employ data from the US from years 1992:Q1 to 2014:Q2. The output gap is constructed from a quadratic detrending of log real GDP per capita from FRED. We use the annual log difference of personal consumption expenditure (PCE) to instrument for inflation which is also obtained from FRED. The continuously compounded annualized Fama-Bliss return on a 3-month ZCB is used for the short-rate.

Regional cap rate data is from the National Counsel of Real Estate Investment Fiduciaries (NCREIF). We choose to focus on the class of commercial real estate properties associated with apartment buildings or apartment complexes. The reason being is that the standard lease for this class of property is one year, meaning the Fisher effect is more likely to hold than say industrial or office properties whose leases can be many years.

The frequency of the data is quarterly, and is separated roughly into the north (N), south (S), mid-west (M), and west (W). For each quarter and region, if we observe at least thirty transactions we calculate the regional cap rate. If there are less than 30 observations, we leave that quarter-region blank, and we have no observation.

The cap rate we construct is value-weighted. Assuming we observe I transactions for region k in quarter t, the cap rate is calculated as

$$q_{t,k} = \frac{\sum_{i=1}^{I} NOI_i}{\sum_{i=1}^{I} P_i} \tag{1}$$

where NOI_i is the realized income net any costs during the period in which the asset was sold, and P_i is the price that property was sold for.

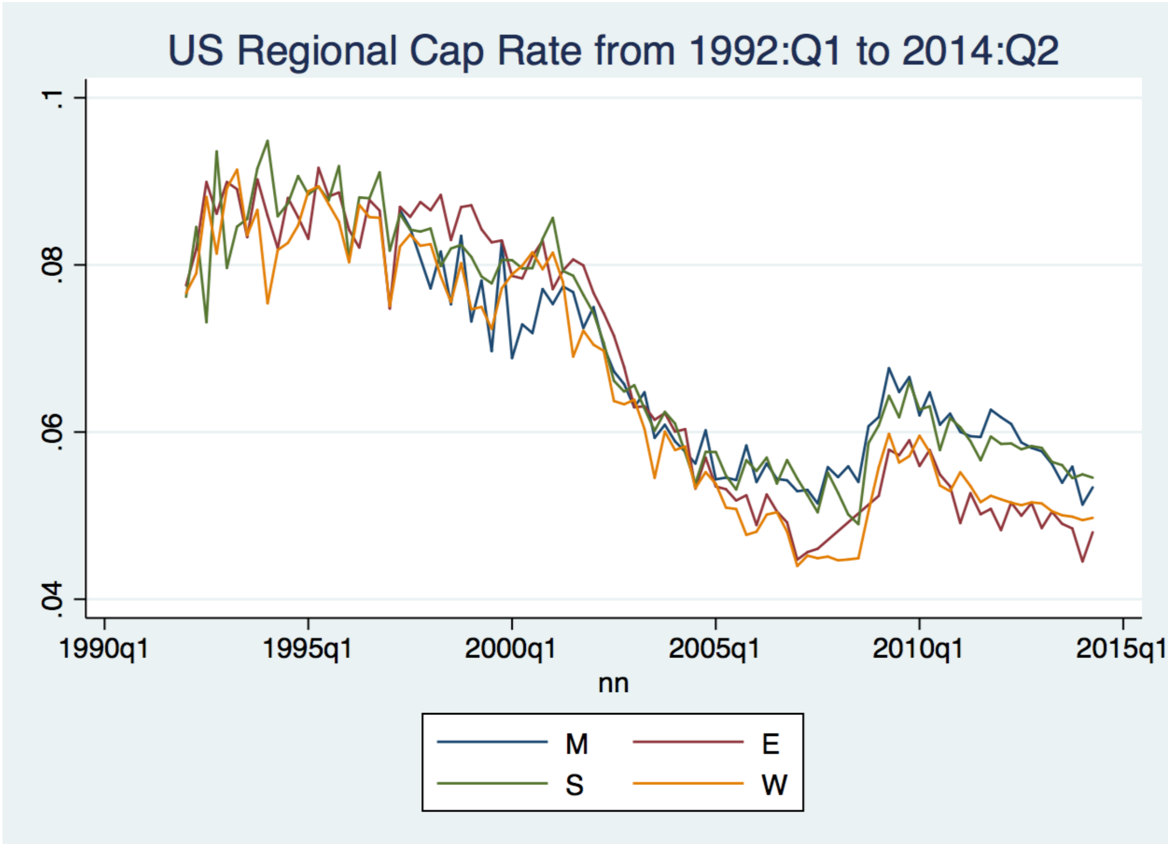
For our motivational experiment, we construct a national cap rate as an arithmetic average across regions. For the sample period there are a total of 22 missing observations at the regional level (4 from the east, and 18 from the mid-west), but zero missing observations at the national level. For each quarter there is never less than 3 observations used to construct the national average.

Table 1 presents the cross-region correlation matrix of cap rates. Each regional cap rate is strongly correlated with each other region. All correlations are significant at the 0.1% significance level. We take this strong positive correlation as evidence of a clear macroeconomic trend in the data.

Table 1: Cross-correlation table

Variables	M	E	S	W
M	1.000			
E	0.934	1.000		
S	0.952	0.953	1.000	
W	0.936	0.972	0.961	1.000

Figure 1 plots the time-series of all regional cap rate. For the east region, we see around 2007-2008 a linear trend which is Stata extrapolating the missing data.



3.2 Motivational Experiment

Based on prior that that there was a change in monetary policy around the 2000-01 financial crisis, we perform a sub-sample analysis to test for test for a change in the correlation structure of the 10-year T-note, inflation, and the national average cap rate. We omit data post-2006:Q4, because we believe another regime change may be present around the start onset of the 2007-09 recession. This leaves us with 60 observations from 1992:Q1-2006:Q4. Splitting the dataset in half, the break-point we arrive at is 1999:Q3. We use the 10-year T-note because real estate investors generally do not turn-over their assets in a few months, we thus attempt to match the holding-period of a typical property.

Table 2 reports post- & pre-break simple regression of the T-10 on cap rate, as well as results from the regression of the inflation rate on the T-10. We find that pre-break interest rates do not move with the cap rate, and that inflation moves with the T-10. After the

Table 2: Regression on T-10 on Cap Rate (right), and Inflation on T-10 (left)

	pre-1999q3	post-
T-10	0.057	0.931***
	(0.08)	(0.22)
constant	0.081***	0.020
	(0.00)	(0.01)
R^2	0.019	0.383
N	30	30

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	pre-1993q3	post
π	1.649***	-0.288
	(0.27)	(0.20)
constant	0.032***	0.055***
	(0.01)	(0.00)
R^2	0.577	0.071
N	30	30

* $p < 0.05$, ** $p < 0.01$, ***, $p < 0.001$

break, interest rates begin to move with cap rates, and the relationship between the T-10 and inflation becomes negative and insignificant. Changes in coefficients are found to be statistically significant in a Wald test for a known structural break point.

4 Model

Here we walk through our employed structural MSRE macro-finance model that connects a small-scale New-Keynsian framework to the price of a general asset for which the ZCB (and thus the term structure) is a special case. As a note, the model is "pulled off the shelf" from Bikbov and Chernov (2013), and much of the credit goes to them and the authors for whom they drew inspiration from.

4.1 Macroeconomic Dynamics

Our specification on the macroeconomic dynamics allow for a simultaneous system of equations that are both forward- and backwards-looking. The variables of interest are the output gap (g_t), the inflation rate (π_t), and short-term interest rate (r_t). Here r_t is chosen to match the time-unit of the model, and is thus the annualized yield on the 3-month risk-free bond.

$$g_t = m_g + (1-\mu_g)g_{t-1} + \mu_g \mathbb{E}_t g_{t+1} - \phi(r_t - \mathbb{E}_t \pi_{t+1}) + \sigma_g(s_t^e)\epsilon_t^g \tag{IS}$$

$$\pi_t = m_\pi + (1-\mu_\pi)\pi_{t-1} + \mu_\pi \mathbb{E}_t \pi_{t+1} + \delta g_t + \sigma_\pi(s_t^e)\epsilon_t^\pi \tag{PC}$$

$$r_t = m_r(s_t^m) + (1-\rho(s_t^m))[\alpha(s_t^m)\mathbb{E}_t \pi_{t+1} + \beta(s_t^m) g_t] + \rho(s_t^m) r_{t-1} + \sigma_r(s_t^d)\epsilon_t^r \tag{MP}$$

We assume $\varepsilon_t^i \sim^{iid} N(0,1)\ \forall i \in \{g, \pi, r\}\ \forall t \in \{1, \ldots, T\}$.

The investment/savings (IS) relationship tells us economic growth today is a function or economic growth yesterday, what agent's expect economic growth to be tomorrow conditional on the information set available to the model's agents today, as well as the expected effective real rate over the next period. This captures the widely held notion from theory, that a high cost of capital decreases economic growth through lower investment.

The Phillip's curve (PC) retains its forwards- and backwards-structure, but also captures the relationship between economic growth and inflation.

The monetary policy rule (MP), is very similar to that used in CGG in that the monetary authority responds only to the current inflation rate in that it affects the expected inflation rate tomorrow.

4.2 Regime Switches

We assume three total regime indicators, s_t^i s.t. $i \in \{m, e, d\}$, allowed to switch between N_i regimes. For the this analysis $N_i = 2\ \forall i$.

The regime that drives changes to the monetary policy response, s_t^m, allows the drift parameter, $m_r(s_t^m)$, the degree of interest-rate smoothing, $\rho(s_t^m)$, the response to inflation expectations, $\alpha(s_t^m)$, and the response to the output gap, $\beta(s_t^m)$, all switch at the same time. Independently, we allow there to be changes in the *discretionary* component to monetary policy, $\sigma(s_t^d)$, to capture the case where the monetary authority uses the same guiding rule to set short-term rates, but exercises more ability to deviate from the rule.

To mitigate concerns of spurious results, we allow the volatility of g_t and π_t to change regime independently to address concerns to heteroskedasticity. To not allow these volatilities puts too much pressure on the monetary response coefficients to drive changes is variation.

We assume each regime variable, s_t^i, follows an independent Markov chain, π^i $\forall i \in \{m, e, d\}$, where π^i is an $(N_i \times N_i)$ matrix such that the row i, column j element is equal to the probability of transitioning from $s_t = i$ to $s_{t+1} = j$. This implies each row sums to unity.

It is well understood that K independent Markov chains can be represented by a single compound Markov chain, $S_t = (s_t^m, s_t^e, s_t^d)$ whose transition dynamics are characterized by the $(N \times N)$ matrix $\Pi = \pi^m \otimes (\pi^e \otimes \pi^d)$, such that $N = \prod_i N_i = 8$, and $\otimes$ is the Kronecker product.

4.3 Reduced-Form

The model assumes that all the dynamics are driven by the state vector $x_t = (g_t, \pi_t, r_t)$, and the compound Markov regime S_t. We employ the Forward solution concept first developed for linear rational expectation (LRE) models by Cho and Moreno (2011), and extended to MSRE models by Cho (2016). In the following we abstract away from constant, but the results can be easily extended and will be later include in the appendix.

Following Cho (2016), equations (IS), (PC), & (MP) can be expressed in a form more general to MSRE models.

$$x_t = \mathbb{E}_t[A(S_t, S_{t+1})x_{t+1}] + B(s_t)x_{t-1} + C(S_t)z_t \tag{2}$$

$$z_t = R(s_t)z_{t-1} + \epsilon_t,\ \epsilon_t \sim (0_{m\times 1}, D) \tag{3}$$

In our specification of the model $R(S_t) = 0_{m\times m}$, $\epsilon_t \sim N(0_{m\times 1}, \mathbb{I}_m)$, $C(S_t) = diag(\sigma_t^g(s_t^e), \sigma_t^\pi(s_t^e), \sigma_t^r(s_t^d))$, $A(S_t, S_{t+1}) = A(S_t)$, and $m = n$, the number of shocks is equal to the number of state variables. Formulations of $A(S_t)$ and $B(S_t)$ are left to the appendix.

Proposition 1 (Cho (2016)) *Any rational expectations solution to equation (1) can be written as a sum of a fundamental solution that depends on the state vectors, x_{t-1}, z_t, and S_t, and a non-fundamental component, w_t, as*

$$x_t = [\Omega(S_t)x_{t-1} + \Gamma(S_t)z_t] + w_t \tag{4}$$

$$w_t = \mathbb{E}_t[F(S_t)w_{t+1}] \tag{5}$$

where $\Sigma(S_t)$, $\Gamma(S_t)$, and $F(S_t)$ have to satisfy the following conditions $\forall s_t, s_{t+1} \in \{1, 2, \ldots, N\}$.

$$\Omega(S_t) = \{\mathbb{I}_n - \mathbb{E}_t[A(S_t)\Omega(S_{t+1})]\}^{-1}B(S_t) \tag{6}$$

$$\Gamma(S_t) = \{\mathbb{I}_n - \mathbb{E}_t[A(S_t)\Omega(S_{t+1})]\}^{-1}C(S_t)^1 \tag{7}$$

$$F(S_t) = \{\mathbb{I}_n - \mathbb{E}_t[A(S_t)\Omega(S_{t+1}])\}^{-1}A(S_t) \tag{8}$$

under the regularity condition that $\mathbb{I}_n - \mathbb{E}_t[A(S_t)\Omega(S_{t+1}]$ is non-singular for all S_t.

For proof see Cho (2016).

Deferring to the original paper for an in-depth description, Cho goes on to define the Forward solution as the unique fundamental solution for which the no bubble condition (NBC) holds. In words, the NBC states that as agents look forward into the future to determine values of the state-vector, eventually the future does not matter for values of the state variables today. This is checked via a convergence condition described in the appendix.

The end result is expressed in the context of our model as

$$x_t = \mu(S_t) + \Phi(S_t)x_{t-1} + \Sigma(S_t)\varepsilon_{t+1} \tag{9}$$

including the constant once again.

It is worth discussing if constraining our analysis to determinant solutions where the NBC holds is too restrictive. One could argue that we observe financial bubbles in reality, and have observed hyperinflation as well. In other words, expectations have gotten out of hand. We believe it is reasonable to think the NBC can still be consistent with these events, as we also observe in each of those situations, a return to stable dynamics. The NBC does not rule out bubbles, it just rules out bubbles that last forever.

4.4 Pricing Kernel

The stochastic discount factor (SDF), $M_{t,t+1}$, is standard in the class no-arbitrage affine term structure models. We define the SDF as

$$logM_{t,t+1} = -r_t - 0.5\Lambda'_{t,t+1}\Lambda_{t,t+1} - \Lambda'_{t,t+1}\epsilon_{t,t+1} \tag{10}$$

where $\Lambda_{t,t+1}$ represents the market price of risk.

$$\Lambda_{t,t+1} = \Sigma'(S_{t+1})\Pi(x_t) \tag{11}$$

$$\Pi(x_t) = \Pi_0 + \Pi_x x_t \tag{12}$$

This specification of risk shows that bond investors need to be compensated in more volatile economic environments. Equation (10) & (11) show that that agents sensitivity to increased expected future volatility is also dependent on the current state-vector, x_t, and regime S_t.

The Radon-Nikodym derivative that allows us to switch from the physical measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}$, as described by Dai et al. (2007) is

$$\Big(\frac{d\mathbb{Q}}{d\mathbb{P}}\Big) = \exp\Big(-\frac{1}{2}\Lambda'_{t,t+1}\Lambda_{t,t+1} - \Lambda'_{t,t+1}\varepsilon_{t+1}\Big) \tag{13}$$

From equation (9) the risk-neutral dynamics can be found be rearranging terms to construct $\varepsilon_t^{\mathbb{Q}}$ such that $\mathbb{E}_t^{\mathbb{Q}}[\varepsilon_t^{\mathbb{Q}}] = \mathbb{E}_t^{\mathbb{P}}[\varepsilon_t^{\mathbb{Q}}(\frac{d\mathbb{Q}}{d\mathbb{P}})] = 0$.

$$x_t = \mu^{\mathbb{Q}}(S_t) + \Phi^{\mathbb{Q}}(S_t)x_{t-1} + \Sigma(S_t)\varepsilon_t^{\mathbb{Q}} \tag{14}$$

such that

$$\mu^{\mathbb{Q}}(S_t) = \mu(S_t) - \Sigma(S_t)\Sigma'(S_t)\Pi_0 \tag{15}$$

$$\Phi^{\mathbb{Q}}(S_t) = \Phi(S_t) - \Sigma(S_t)\Sigma'(S_t)\Pi_x \tag{16}$$

$$\varepsilon_t^{\mathbb{Q}} = \varepsilon_t - \Lambda_{t,t+1} \tag{17}$$

4.5 Pricing Bonds

Defining $M_{t,t+n}$ as the n-period stochastic discount factor

$$M_{t,t+n} = \prod_{i=1}^{n} M_{t+i-1,t+i} \tag{18}$$

and employing the no-arbitrage restriction, the price of an n-period zero-coupon bond (ZCB) is

$$B_t^n(x_t, S_t) = \mathbb{E}[M_{t,t+n}|x_t, S_t] = \mathbb{E}^{\mathbb{Q}}[-\sum_{i=1}^{n} r_{t+i-1}|x_t, S_t] \tag{19}$$

and the corresponding annual yield is

$$y_t^n(x_t, S_t) = -\frac{1}{n}\log(B_t^n) \tag{20}$$

Equation (18) and thus both equations (17) and (18) have no numerically tractable solution, as it involves integrating over N^{n-1} paths making calculations past $n = 9$ computational infeasible with the technology currently available to us.

Bikbov and Chernov (2013) use a quadratic approximation to approximate (19) accurately up until $n = 40$, which is all they need for their analysis. We are undecided as to whether or not we will employ their methodology, an extension of their methodology, or attempt to use a sparse-grid discretization in the spirit of Tauchen (1986).

4.6 Pricing Real Estate

A ZCB is a special case of an asset that has a single deterministic payout. As a result, after controlling for risk preferences that price of a ZCB only depends on expected future path of the short-rate, r_t as can be seen in equation (18).

Consider a typical piece of real estate. When viewed as an asset (if we rent the property out), it pays us a series of undetermined cash flows indefinitely into the future. The price of this property is then the discounted sum of all expected future cash flows under the risk-neutral measure.

Now imagine a special type of ZCB whose price is η_t^n, that is distinct from the risk-free ZCB, whose price is B_t^n, in that the return on the η_t^n bond is tied to expected rent received on the piece of real estate at period n. Not only will η_t^n fluctuate with the expected future path of short rates through the discount channel, but will also fluctuate with expectations about future rental growth rates. Future rental growth rates depend on expected future inflation due to the Fisher effect (landlords adjust rents with expected future inflation), and future income growth which correlated with the output gap. The point we are making is that η_t^n is a function of the entire vector x_t, and not just r_t.

$$\eta_t^n = \mathbb{E}_t^{\mathbb{Q}}[\sum_{i=1}^{n}(\delta x_{t+n-1} + d)|x_t, S_t] \tag{21}$$

such that

$$\delta = (a, b, c)' \tag{22}$$

Notice price of a risk-free ZCB, B_t^n is just a special case of (20) where $a = b = d = 0$, and $c = -1$.

We then define the inverse cap rate to be the infinite sum of all η_t^n for a given property.

$$\hat{Q} = \sum_{i=1}^{\infty} \eta_t^n \tag{23}$$

Since we can observe $\hat{Q}$ in practice, assuming we are able to approximate η_t^n well enough into the future, we should be able to capture cap rates by letting $\{\delta, d\}$ be free parameters in the estimation procedure.

5 Estimation

Due to the nonlinear nature of the likelihood function in MS-DSGE models, we instead take the Bayesian route and attempt to characterize the posterior using MCMC methods.

5.1 Choosing Priors

We follow Bianchi (2013) and Lubik and Schorfheide (2004) for choice of priors, an adopt distributional parameters to the parameter estimates and standard errors of Bikbov and Chernov (2013) and Baele et al. (2015) whose model structure if more closely related to ours. A secondary goal of the choice of priors, is to steer the MCMC algorithm away from the indeterminacy region as described in Baele et al. (2015), without completely foregoing the possibility of an indeterminate solution.

We choose to place asymmetric priors on the monetary policy authority's response to inflation across regimes, but still allow for considerable overlap.

5.2 Calculating the Likelihood

The econometricians data set includes $x_t = (g_t, \pi_t, r_t)$, the vector y_t of longer maturity T-notes (2-year, 5-year, & 10-year), and c_t is either a vector of regional cap rates, or the single national average cap rate. We assume that the short-term interest rate is observed without measurement error, while the higher-term yields and cap rates are observed with standard normal i.i.d. measurement error ϵ_t^y and ϵ_t^c with standard deviation σ_y & σ_c respectively. The higher-term yields can then be modeled as following with the model implied yields being represented by $f^y(x_t, S_t, \tau_i)$ such that τ_i represents the i^{ith} higher term yield. The cap rate is approximated via $f^c(x_t, S_t)$.

$$y_t^{(i)} = f^y(x_t, S_t, \tau_i) + \epsilon_t^y \tag{24}$$

$$c_t = f^c(x_t, S_t; \delta, d) + \epsilon_t^c \tag{25}$$

We proceed to construct the likelihood following similar recursive methods as Dai et al. (2007) and Ang et al. (2008). The recursion begins by putting together the joint-conditional probability of observing $\omega_{t+1} = (x_{t+1}, y_{t+1}, c_{t+1})'$ and the regime tomorrow being regime j given the econometricians's information set at time t, $\Omega_t = \{\omega_1, \ldots, \omega_t\}$, and prior beliefs about the regime in place during period t.

$$\begin{aligned} p(\omega_{t+1}, S_{t+1} = j | \Omega_t) = &\sum_i \pi_{i,j} Q_t^i p(x_{t+1} | x_t, S_t = i, S_{t+1} = j) \\ &\cdot p(y_{t+1} | x_{t+1}, S_t = i, S_{t+1} = j) p(c_{t+1} | x_{t+1}, S_t = i, S_{t+1} = j) \end{aligned} \tag{26}$$

Equation (26) is derived by applying Bayes' rule to the joint-conditional probability $p(x_{t+1}, y_{t+1}, c_{t+1} | S_{t+1} = j, S_t = i)$. Furthermore, given our assumptions on the error term these densities can be easily derived by exploiting properties of the normal distribution.

$Q_t^i \equiv Pr(S_t = i|\Omega_t)$ is the filtered probability of beliefs about the current states variables.

$$p(x_{t+1}|x_t, S_t = i, S_{t+1} = j) = \frac{1}{(2\pi)^{3/2}|\Sigma_j \Sigma_j'|^{1/2}} \cdot \exp\{-\frac{1}{2}(x_{t+1} - \mu_j - \Phi_j x_t)'(\Sigma_j \Sigma_j')^{-1}(x_{t+1} - \mu_j - \Phi_j x_t)\} \tag{27}$$

$$p(y_{t+1}|x_{t+1}, S_t = i, S_{t+1} = j) = \frac{1}{(2\pi\sigma_y^2)^{n/2}} \cdot \exp\{-\frac{1}{2\sigma_y^2}\sum_{k=1}^{n}(y_{t+1}^{(k)} - f(x_{t+1}, j, \tau_k))^2\} \tag{28}$$

$$p(y_{t+1}|x_{t+1}, S_t = i, S_{t+1} = j) = \frac{1}{(2\pi\sigma_c^2)^{1/2}} \cdot \exp\{-\frac{1}{2\sigma_c^2}(c_{t+1} - f(x_{t+1}, j; \delta, d))^2\} \tag{29}$$

Where n is the number of higher-term interest rates included in the model. Once (26) has been calculated we can update the smoothed filter.

$$Q_{t+1}^j = \frac{p(\omega_{t+1}, S_{t+1} = j|\Omega_t)}{p(\omega_{t+1}|\Omega_t)} \tag{30}$$

$$p(\omega_{t+1}|\Omega_t) = \sum_j p(\omega_{t+1}, S_{t+1} = j|\omega_t) \tag{31}$$

The log-likelihood function itself is then constructed as...

$$\mathcal{L} = \frac{1}{T-1}\sum_{t=1}^{T-1} \log p(\omega_{t+1}|\Omega_t) \tag{32}$$

The iteration is initiated by setting $Q_1^i = \pi_i^*$, where π_i^* is the stationary unconditional probability of being in state i, which is obtained via the properties of an ergodic Markov chain.

The posteriors is then proportional to $p(\theta)\mathcal{L}$ where $p(\theta)$ represents our priors.

5.3 Characterizing the Posterior

Largely undecided. Standard choice is a a Multi-Move Gibbs Sampler, but we are also interested in taking advantage of the parallel capabilities of the sequential Monte-Carlo algorithm described in Bognanni and Herbst (2015).

5.4 Estimating the Cap Rate

We perform a conditional MCMC experiment to test the accuracy of the quadratic approximation of bond prices in Bikbov and Chernov (2013), to see if the method can be used to price the cap rate. While the experiment is not fully done running, we find a substantial negative bias in the quadratic approximation as the maturity of the bond goes past 15-years. We finding augmenting the quadratic approximation with a cubic term analogous to the third uncentered moment of the expression $-r_t - \cdots - r_{t+1-n}$ allows for a far better approximation compared to the MCMC estimates even 100 years out. Even though we are hopeful this result extends to the cap rate, we are skeptical of relying on a third moment in practice as the MSS concept of stability used does not guarantee the existence of a third moment. At the moment this is the largest obstacle we must overcome.

6 Results

[insert results]

7 Conclusion

[insert conclusion]

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