Aggregate Uncertainty: Krusell and Smith

Econ720

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November 22, 2016

A Bewley Model of the Wealth Distribution

- We study Krusell and Smith (1998 JPE).
- The problem: In models with aggregate uncertainty, the entire (wealth) distribution is a state variable.
- KS propose an important method for solving models with aggregate uncertainty and heterogeneity.
- It contains an important finding: **approximate aggregation**
  - First moments are often enough to approximate the entire distribution of the state vector.
  - The aggregate law of motion for K looks a lot like an individual's decision rule.

The Question

- Economists commonly use models with representative households.
- Are these **good approximations** for models with heterogeneous agents?

Contributions

1. In the standard Real Business Cycle (RBC) framework, the representative agent is a good approximation.
2. A method for computing models with heterogeneity and aggregate uncertainty.

The approach

- Compute a standard RBC model (representative agent)
- Add uninsured employment risk.
- Compare: how good is the representative agent approximation?

Result: Approximate Aggregation

- Aggregate consumption and saving resemble those of a representative agent.
- Therefore: it is enough to keep track of mean wealth, instead of keeping track of the wealth distribution, in order to forecast future prices.

Intuition

- The distribution of wealth is unimportant, if most agents have the same marginal propensity to consume out of aggregate shocks.
- This is true in the model because agents achieve good self-insurance (consumption policy functions are roughly linear).
- Only for the very poor does self-insurance fail. But the very poor account for only a small fraction of aggregate consumption.
- A point made in passing: **preference heterogeneity** permits the model to match the U.S. wealth distribution.

The Model

Demographics:

- a unit mass of infinitely lived households
- households are ex ante identical (Bewley model)

Preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \; U(c_t)$$

Technologies

$$\bar{k}' = (1-\delta)\,\bar{k} + \bar{y} - \bar{c}$$

$$\bar{y} = z\;\bar{k}^{\alpha}\;\bar{l}^{1-\alpha}$$

- $\bar{k}, \bar{l}$: aggregate capital and labor inputs.
- z is a two-state Markov process.
- takes on values z_g, z_b
- $\Pr(z' = z) = P_z$

Endowments

$\bar{k}_0$ units of capital at $t = 0$

$\varepsilon_{i,t}$ units of labor time

- $\varepsilon_{i,t} = 1$: employed
- $\varepsilon_{i,t} = 0$: unemployed

Unemployment probability depends on aggregate state:

- $\Pr(\varepsilon_{i,t} = 0 | z = z_g) = u_g$ and $\Pr(\varepsilon_{i,t} = 0 | z = z_b) = u_b$

Markets

- Goods (numeraire)
- Capital rental: r
- Labor rental: w
- Households hold capital, but cannot borrow: $k_{i,t} \geq 0$.

Household Problem

- Individual state: k, ε.
- Aggregate state: z, Γ.
- Γ is the distribution of households over (k, ε).
- Bellman equation:

$$v(k, \varepsilon, z, \Gamma) = \max\ u(c) + \beta\ E\ v\left(k', \varepsilon', z', \Gamma'\right)$$

subject to

$$k' \quad = \quad r\left(\bar{k}, \bar{l}, z\right) k + w\left(\bar{k}, \bar{l}, z\right) \varepsilon + (1 - \delta) k - c$$

and law of motion for aggregate state

Law of motion for aggregate state

z: Markov chain, $\Pr(z'|z)$ is a model primitive

ε: Markov chain, $\Pr(\varepsilon'|\varepsilon, z')$ is a model primitive

k': Household decision rule $k' = f(k, \varepsilon, z, \Gamma)$

Notation for Γ:

$$\Gamma' = H(\Gamma, z, z') \tag{1}$$

There are different laws of motion for each z, z' because

- k' depends on z
- ε' depends on z'

Recursive Competitive Equilibrium

Objects:

- Household value function v and decision rule $k' = f(k, \varepsilon, z, \Gamma)$.
- Price functions $r(.)$ and $w(.)$.
- Law of motion for the distribution of (k, ε): H.

These satisfy:

- v,f solve the household problem.
- r, w are consistent with firm profit maximization.
- H is "consistent with" household decision rules f.
  - see Bewley slides

Computation

- Problem: The distribution Γ cannot be described with a finite number of parameters.
- KS's idea: Only keep track of a small number of moments of the distribution: ℓ.
  - e.g.: mean, variance, percentile values, ...
- Guess a law of motion for ℓ:

$$\ell' = h\left(\ell, z, z'\right) \tag{2}$$

.

- Solve the household problem, given h rather than H.
- As long as ℓ contains $\bar{k}$, the household can compute prices.

Approximate Household Problem

Bellman equation:

$$v(k, \varepsilon, z, \ell) = \max u(c) + \beta E v\left(k', \varepsilon', z', \ell'\right)$$

subject to

$$\begin{aligned} k' &= r(z, \ell) k + w(z, \ell) \varepsilon + (1 - \delta) k - c \\ \ell' &= h(\ell, z, z') \end{aligned}$$

Algorithm

- Start from an arbitrary guess for h, such as $\ell' = \ell$.
- Solve the household problem, given h.
- Simulate many household histories.
- Update the guess for h from the household solution.
- Iterate until the guesses for h converge.

Computation

The key problem:

- How to represent the distribution using a small vector ℓ?
- How to find the law of motion h from simulated household histories?

Krusell and Smith approximate Γ using the first J **moments** of the distribution of k_i

- mean, standard deviation, etc.
- they don't keep track of the correlation of k with ε

To check the accuracy of the approximation:

- Verify that the forecast errors are "small."
- Verify that increasing J has little effect on the equilibrium properties.

This is a form of **bounded rationality**: Households only use the first J moments and forecast them using only today's moments.

Details

- Assume that ℓ_j' is a linear function of ℓ, conditional on z.
- Simulate a large number of households from their decision rules.
- Compute a history ℓ_t.
- Estimate the coefficients by running a regression of ℓ' on ℓ (for every z).
- Iterate until regression coefficients converge.
- To check the accuracy of the approximation: Try alternative functional forms for h.

Details

Start with $J = 1$ moments: $\ell = \bar{k}$.

- If $z = z_g$: $\ln\left(\bar{k}'\right) = a_0 + a_1 \ln\left(\bar{k}\right)$.
- If $z = z_b$: $\ln\left(\bar{k}'\right) = b_0 + b_1 \ln\left(\bar{k}\right)$.

Then explore whether adding more moments changes anything.

Parameters

"Standard" RBC parameters.

Preferences: $\beta = 0.99$, $\sigma = 1$.

Technology:

- $\alpha = 0.36$.
- $z_g = 1.01$ and $z_b = 0.99$ based on size of aggregate output fluctuations.

Unemployment rates: $u_g = 0.04$ and $u_b = 0.1$.

P_z: match length of business cycles.

P_ε: Labor endowments match length of unemployment spells.

Results

- Solve for $J = 1$ and $\ell = \bar{k}$.
- Forecasting equations are of the form: $\ln\left(\bar{k}'\right) = a_0 + a_1 \ln\left(\bar{k}\right)$.
- Goodness of fit:
  - $R^2 = 0.999998$.
  - Variance of error term: $\sigma^2 = 0.00003$.
- The log-linear forecasting equation is nearly perfect.
- The welfare gains from better forecasts are negligible.

Approximate aggregation

Individual decision rules are nearly linear.

All have nearly identical marginal propensities to consume.

Redistributing wealth has essentially no effect on aggregate consumption.

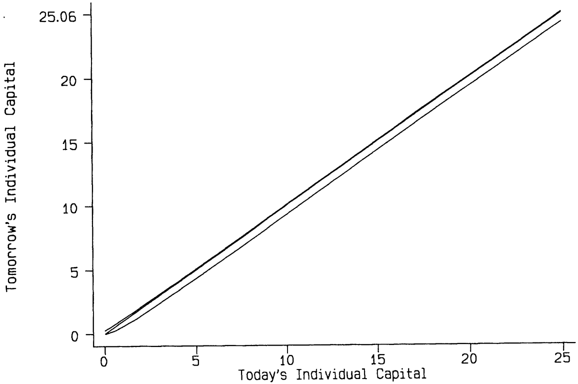

FIG. 2.—An individual agent's decision rules (benchmark model, aggregate capital = 11.7, good aggregate state).

Why approximate aggregation?

- Why are decision rules nearly linear?
- Most agents are rich enough to almost completely smooth shocks.
- One reason: aggregate capital is (by construction) 3 times larger than output.
- Another reason: agents live forever.
- Only a small number of poor agents cannot self-insure. But they account for a tiny fraction of aggregate wealth.

How important is heterogeneity for business cycles?

- The experiment: Compare two identical economies, except that one has complete markets (therefore no heterogeneity).
- Finding: heterogeneity has little effect on the model's business cycle properties.

How important is heterogeneity for business cycles?

AGGREGATE TIME SERIES

Model	Mean(k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic-β:				
Incomplete markets	11.78	.825	.027	.459

Preference heterogeneity

- The question:
  - The baseline model has far too little wealth heterogeneity.
  - Does approximate aggregation still hold when there is a realistic amount of wealth heterogeneity?
- The approach:
  - Add enough preference heterogeneity to the model to roughly replicate the observed distribution of wealth.
  - Check that the mean is enough to forecast prices very accurately.
- Model
  - Allow for 3 arbitrary values of β: 0.986, 0.989, 0.993.
  - Agents switch β values stochastically, on average every 50 years (once per generation).

Results

Approximate aggregation is still very good:

- $R^2 = 0.99999$
- $\sigma^2 = 0.00006$.

Results

The model matches wealth distribution statistics.

Distribution of Wealth: Models and Data

Model	Percentage of Wealth Held by Top 1%	5%	10%	20%	30%	Fraction with Wealth < 0	Gini Coefficient
Benchmark model	3	11	19	35	46	0	.25
Stochastic-β model	24	55	73	88	92	11	.82
Data	30	51	64	79	88	11	.79

But: this does not show that preference heterogeneity is important in the data.

- An open question!

Summary

- The main contribution of Krusell and Smith is the **method** for computing economies with heterogeneity and aggregate uncertainty.
- The finding that **approximate aggregation** holds seems robust for frictionless business cycle models (the RBC type), but we don't know whether it holds more generally.

Reading

Krusell, Per; Anthony A. Smith (1998). "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy* 5: 867-96.