Review Questions: Search and Matching

Econ720. Fall 2016. Prof. Lutz Hendricks

---

1 McCall Model

- Romer, "Advanced Macro," exercises 9.10, 9.11.
- Ljunqvist & Sargent, "Recursive Macroeconomic Theory," 2nd ed., exercises 26.2, 26.3.
- Work out the McCall model with random terminations (Ljunqvist & Sargent 6.3).

1.1 McCall Model with Uniform Wage Offers

[Based on a question due to Steve Williamson]. Consider a McCall model in which wage offers are drawn from a uniform distribution over the interval $[w_l, w_h]$.

1. Derive the reservation wage.
2. Determine the effects of a higher upper bound w_h on the reservation wage and on the probability that a worker receives an acceptable wage offer. Explain the intuition.
3. Determine the effects of a mean preserving spread in the wage offer distribution: w_l falls and w_h rises. How do the reservation wage and the probability of accepting an offer change? Explain the intuition.

1.1.1 Answer: McCall Model with Uniform Wage Offers

1. The first steps are as in the McCall model. With the uniform distribution

$$E\{w' - \bar{w} | w' \geq \bar{w}\} = \frac{w_h + \bar{w}}{2} \tag{1}$$

$$Pr(w' \geq \bar{w}) = \frac{w_h - \bar{w}}{w_h - w_l} \tag{2}$$

This leads to the quadratic formula for the reservation wage:

$$\bar{w} - c = \beta \frac{w_h + \bar{w}}{2} \frac{w_h - \bar{w}}{w_h - w_l} \tag{3}$$

$$= a(w_h^2 - \bar{w}^2) \tag{4}$$

The solution is

$$\bar{w} = \frac{-1 \pm \sqrt{1 + 4a(c + w_h^2 a)}}{2a} \tag{5}$$

where $a = \frac{\beta}{2(1-\beta)(w_h - w_l)}$. Obviously, of the two solutions only the one with the + is positive.

2. Write the solution as

$$2\bar{w} = -\frac{1}{a} + \sqrt{1/a^2 + 4c/a + 4w_h^2} \tag{6}$$

Since a is decreasing in w_h, the reservation wage is increasing in w_h. The intuition is of course that the option value of waiting becomes more valuable.

3. The intuition is clear: the reservation wage should rise because low offers are irrelevant and rejected anyway. I don't see this in the math? Any solutions?

1.2 Model with two point wage distribution

[Based on a question due to Steve Williamson]

There is a continuum of workers with unit mass. Each worker is risk-neutral and discounts the future at rate $r > 0$. If a worker is unemployed, he or she receives an unemployment insurance benefit of b, and receives a wage offer each period, which is w_1 with probability π_1 and 0 with probability $1 - \pi_1$, where $0 < \pi_1 < 1$.

A worker who is employed earning a wage w_1 will suffer a separation during the period with probability δ_1, and receives the offer of another job with probability π_2, where $0 < \pi_2 < 1$. This new job pays a higher wage $w_2 > w_1$, but is more risky. A worker employed at wage w_2 will experience a separation with probability $\delta_2 > \delta_1$. Assume that $0 < b < w_1 < w_2$, and that separation from any job implies that the worker is unemployed.

1. Determine conditions under which a worker employed at wage w_1 will accept the higher-paying job if it is offered, and when he or she will not. Explain these conditions.

2. Determine the steady state unemployment rate, and the fraction of workers employed at high-paying and low-paying jobs, under conditions where workers employed at low-paying jobs will accept high-paying jobs, and under conditions where they will not. Explain your results.

1.2.1 Answer: Model with two point wage distribution

1. The Bellman equations are

$$\begin{aligned} V_u &= b + \beta\pi_1 V_1 + \beta(1-\pi_1)V_u \\ V_1 &= w_1 + \beta\delta_1 V_u + \beta(1-\delta_1)\left(\pi_2 \max\{V_1, V_2\} + (1-\pi_2)V_1\right) \\ V_2 &= w_2 + \beta\delta_2 V_u + \beta(1-\delta_2)V_2 \end{aligned}$$

Now solve brute force:

$$\begin{aligned} V_u &= \frac{b + \beta\pi_1 V_1}{1 - \beta(1-\pi_1)} \\ V_2 &= \frac{w_2 + \beta\delta_2 \frac{b+\beta\pi_1 V_1}{1-\beta(1-\pi_1)}}{1-\beta(1-\delta_2)} \end{aligned}$$

Next, plug this into the equation for V_1 for the two cases: accept or reject type 2 jobs. Nasty algebra...

2. If employed workers reject type 2 jobs: outflows from unemployment are $\pi_1 U$ while inflows are $\delta_1(1-U)$. Set both equal to find the unemployment rate.

If the employed accept type 2 jobs, the flow equations are

$$\begin{aligned} \pi_1 U &= \delta_1 E_1 + \delta_2 E_2 \\ [\delta_1 + (1-\delta_1)\pi_2 E_1 &= \pi_1 U \\ (1-\delta_1)\pi_2 E_1 &= \delta_2 E_2 \end{aligned}$$

Solve these...

1.3 McCall Model With On-the-job Search

[Final exam 2012] Consider a McCall model where employed workers receive job offers. Time is continuous. The worker lives forever and maximizes the expected present value of income, discounted at rate r.

When unmployed, the arrival rate of job offers is α_0. When employed it is α_1. Offers are drawn from the cdf $F(w)$. Employed workers lose their jobs with probability λ and move into unemployment next period.

Questions:

1. State the Bellman equation for an unemployed worker. Hint: rU = [current payoff] + [expected "capital gain"].
2. State the Bellman equation for an employed worker. Explain it. Hint: With Possion events, the probability of being hit by 2 events at the same time is 0. Therefore, the probability of getting an offer is α_1 and the probability of being layed off is λ.
3. Clearly, unemployed workers set a reservation wage such that $U = W(w_R)$, while employed workers accept any job with $w' > w$. Derive

$$w_R - b = (\alpha_0 - \alpha_1) \int_{w_R}^{\infty} [W(w') - W(w_R)] dF(w') \tag{7}$$

4. Explain (7) in words.
5. Why might a worker accept a job with $w < b$?
6. How would a higher b affect the reservation wage? What is the intuition?
7. Now add the possibility of quits to the model. The worker enters the period with w. Then he observes all the shocks (wage offer, loss of job). Then he decides whether or not to quit and move into unemployment next period. Write down and explain the Bellman equation.

1.3.1 Answer: McCall Model With On-the-job Search[1]

1. Unemployed:

$$rU = b + \alpha_0 \int_{w_R}^{\infty} [W(w') - U] dF(w')] \tag{8}$$

2. Employed:

$$rW(w) = w + \alpha_1 \int_{0}^{\infty} \max\left(W(w') - W(w), 0\right) dF(w') + \lambda(U - W(w)) \tag{9}$$

3. Steps: Eval (9) at $w = w_R$. LHS becomes rU. Difference with (8). Done.
4. In words: The benefit of working today, $w - b$, just equals its cost. The cost is that the arrival rate of wage offers drops to α_1.
5. A: higher job offer rate, if $\alpha_0 < \alpha_1$.
6. Higher b raises reservation wage. Less costly to wait for a great offer.
7. Quits

$$\begin{aligned} rW(w) &= w + \lambda(U - W(w)) + (1-\alpha) \max\{U - W(w), 0\} && (10) \\ &+ \alpha F(w_r) \max\{U - W(w), 0\} && (11) \\ &+ \alpha \int_{w_r}^{\infty} \max\{W(w') - W(w), U - W(w)\} dF(w') && (12) \end{aligned}$$

Explanation: With probability λ lose the job. With probability $1 - \alpha$ get no wage offer and decide between U and staying on. With probability $\alpha F(w_r)$ get a job you won't take and choose between U and staying on. With probability $\alpha(1 - F(w_r))$ get an offer that you prefer over the current job. Take it if it is better than U.

[1] Based on Rogerson, R., Shimer, R. & Wright, R., 2005. Search-theoretic models of the labor market: A survey. Journal of Economic Literature, 43(4), pp.959–988.

2 Mortenson-Pissarides Model

- Romer, "Advanced Macro," exercises 9.13, 9.14, 9.16.
- Ljunqvist & Sargent, "Recursive Methods," exercises 26.7, 26.8, 26.9, 26.10.

2.1 Comparative statics

In the model we studied in class, how do the following affect equilibrium employment:

1. A higher job breakup rate b.
2. A higher interest rate r.
3. More efficient matching (higher K).
4. The firm receives a smaller share of the surplus in bargaining.

Provide intuition. The derivations can be tedious.

2.1.1 Answer: Comparative statics

In the model, we need to find out whether the free entry condition

$$rV_V = -C + \frac{\alpha(E)\ A}{\alpha(E) + a(E) + 2(b+r)} = 0 \tag{13}$$

shifts up or down for any given E. Then we can use Romer's figure 9.6 to find the change in E.

1. Note that the right hand side of (13) is decreasing in E, b, r and increasing in A. Ignore for the moment that $a(E)$ and $\alpha(E)$ also depend on b. Then, a higher b requires a lower E.

The main channel: A higher b reduces the value of creating a vacancy. Vacancies last less long. In addition, the surplus of a filled vacancy declines, and the firm receives half of it.

There are more complicated indirect effects. $a(E) = bE/\left(\bar{L} - E\right)$ increases for given E. The higher breakup rate of matches increases the number of unfilled vacancies (given E), so it takes less time to find a job when unemployed. This reinforces the direct effect (the right hand side rises further).

Recall that $\alpha(E) = K^{1/\gamma}\ (bE)^{(\gamma-1)/\gamma}\ \left(\bar{L} - E\right)^{\beta/\gamma}$ also depends on b. The direction of this effect depends on γ (which determines how the number of vacancies affects the number of matches). For given E, it becomes easier or harder to fill a vacancy (easier because there are more workers who are searching).

2. According to (13), a higher r has the same effect as a higher b, except that the $\alpha(E)$ and $a(E)$ curves do not shift.

For a given E, vacancies are less valuable. They are created in the hope of generating future payoffs, which are discounted more heavily. Equilibrium E falls because firms create fewer vacancies.

3. We need to determine what happens to the value of a vacancy rV_V for given E. Unsurprisingly, vacancies are filled faster (higher α). Surprisingly, workers do not find jobs any faster (a unchanged for given E). This happens because in equilibrium inflows into and outflows from unemployment must be equal. Recall that we derived

$$a = \frac{bE}{\bar{L} - E}. \tag{14}$$

The free entry expression then tells us that rV_V rises for given E. Equilibrium employment rises.

4. We would need to rewrite the model with a variable share, but the intuition is straightforward: for any E creating a vacancy is less valuable. E falls.