

# Mortenson Pissarides Model

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# Mortenson / Pissarides Model

- ▶ Search models are popular in many contexts: labor markets, monetary theory, etc.
- ▶ They are distinguished by
  1. how agents meet
  2. how the payoffs are determined when agents meet.
- ▶ The MP model has
  1. a matching function
  2. Nash bargaining.

# Model

- ▶ Time is continuous.
- ▶ Demographics:
  - ▶ There are  $\bar{L}$  identical workers.
  - ▶ They live forever (or they could die stochastically).
- ▶ Preferences:
  - ▶ Utility = consumption (one good).
  - ▶ Discount rate  $r$ .

# Technology

- ▶ Output is produced from labor only.
- ▶ Production can take place only in a worker-job match.
- ▶ Each match consists of exactly one job / one worker.
- ▶ When matched, a match produces a flow output of  $A$ .

## Model: The logic

- ▶ Enter the "period" with
  - ▶  $U$  unemployed workers
  - ▶  $F = \bar{L} - U$  job matches.
  - ▶  $E = F$  employed workers
- ▶  $bE$  matches break up (exogenously)
- ▶ Firms post  $V$  vacancies, paying a cost.

## Model: The logic

- ▶ Unemployed workers and vacancies meet at random.
- ▶ Workers who don't meet a firm stay unemployed, consume 0.
- ▶ **In a match:**
  - ▶ Firm and worker **bargain** over the wage (no contracts!).
  - ▶ If no agreement is reached, the job becomes vacant and the worker becomes unemployed.
  - ▶ If agreement is reached, the pair produces until exogenous breakup occurs.

# Workers

- ▶ Workers live forever and maximize the expected present value of earnings.
- ▶ The discount rate is  $r$  (exogenous).
- ▶ The only decisions: in wage negotiation.

# Firms

- ▶ Firms can create jobs (vacancies) at a flow cost of  $C$  per unit of time.
- ▶ A filled job produces  $A$  and pays  $w$  (endogenous) to the worker.
- ▶ The firm keeps the profit:  $A - w - C$ .



# Matching

- ▶ A **matching function** describes how workers are matched to vacancies.
- ▶ The number of matches per period is

$$M(U, V) = K U^\beta V^\gamma \quad (1)$$

- ▶ We take  $M(U, V)$  as given.
- ▶ Matching functions can be derived from micro-foundations.
- ▶ More vacancies or more unemployed workers result in more matches.

## Steady state restrictions

- ▶ Focus on situations where  $E, U, V$  are constant.
- ▶ The number of employed workers changes according to

$$\dot{E} = M(U, V) - bE \quad (2)$$

where  $b$  is the exogenous rate of match dissolution.

- ▶ In steady state  $\dot{E} = 0$ :

$$M(U, V) = bE \quad (3)$$

## Steady state restrictions

- ▶ The number of unemployed follows

$$\dot{U} = bE - M(U, V) \quad (4)$$

$$= -\dot{E} \quad (5)$$

- ▶  $\dot{U} = 0$  is implied by  $\dot{E} = 0$ .

# Definitions

- ▶ Define the rate of **exit from unemployment**

$$a = \frac{M(U, V)}{U} \quad (6)$$

- ▶ Define the rate at which **vacancies are filled**:

$$\alpha = \frac{M(U, V)}{V} \quad (7)$$

## Solution method

Assume that all workers receive the same wage  $w$  when matched (verify this later).

For a given wage, there is only one decision to be made: **how many vacancies** to create.

- ▶ Assume that vacancies are created until they yield zero profit (**free entry**).
- ▶ We need to find the value of an open vacancy ( $V_V$ ).

Then we need to find the bargained **wage**.

For this we need to know the values

- ▶ of being employed ( $V_E$ ) or unemployed ( $V_U$ ).
- ▶ of a filled vacancy ( $V_F$ ).

## Workers: Employed

The value of being employed is

$$rV_E = w + b(V_U - V_E) \quad (8)$$

Or:

$$V_E = \frac{w + bV_U + (1 - b)V_E}{1 + r}$$

Intuition:

- ▶ Receive a flow benefit  $w$ .
- ▶ With probability  $b$  switch to unemployment and lose  $V_U - V_E$ .

## Employed worker: Derivation

Consider the value of being employed for a short period  $\Delta t$ .

Receive flow benefit  $w$ , discounted at  $r$ .

- ▶ Probability of remaining in the match:  $e^{-bt}$ .
- ▶ Value:  $\int_0^{\Delta t} e^{-(r+b)t} w dt = \frac{1 - e^{-(r+b)\Delta t}}{r+b} w$ .

At the end, at  $t + \Delta t$ :

- ▶ continue as unemployed with probability  $1 - e^{-b\Delta t}$ .
- ▶ continue in match with probability  $e^{-b\Delta t}$ .
- ▶ Value:  $e^{-r\Delta t} [e^{-b\Delta t} V_E(\Delta t) + (1 - e^{-b\Delta t}) V_U(\Delta t)]$ .

## Employed worker: Derivation

Value of being employed is then:

$$V_E(\Delta t) = \underbrace{\frac{1 - e^{-(r+b)\Delta t}}{r+b} w}_{\text{flow payoff}} + \underbrace{e^{-r\Delta t} \left[ e^{-b\Delta t} V_E(\Delta t) + (1 - e^{-b\Delta t}) V_U(\Delta t) \right]}_{\text{continuation value}}$$

Simplify

$$V_E(\Delta t) = \frac{w}{r+b} + \frac{(1 - e^{-b\Delta t}) e^{-r\Delta t}}{1 - e^{-(r+b)\Delta t}} V_U(\Delta t).$$

Take the limit as  $\Delta t \rightarrow 0$ .

Use l'Hopital's rule to evaluate the ratio in front of  $V_U$ .

It becomes  $\frac{b}{r+b}$ . Therefore

$$V_E = \frac{w}{r+b} + \frac{b}{r+b} V_U$$

Rearrange. Done.



## Unemployed Worker

$$rV_U = 0 + a(V_E - V_U)$$

Or

$$V_U = \frac{0 + aV_E + (1 - a)V_U}{1 + r}$$

Receive nothing right now.

With probability  $a$  switch to "employed."

## Unfilled Vacancies

$$rV_V = -C + \alpha(V_F - V_V)$$

Or

$$V_V = \frac{-C + \alpha V_F + (1 - \alpha)V_V}{1 + r}$$

Pay the vacancy cost  $C$ .

With probability  $\alpha$  fill it and receive  $V_F$ .

## Filled vacancies

$$rV_F = A - w - C + b(V_V - V_F)$$

Or

$$V_F = \frac{A - w - C + bV_V + (1 - b)V_F}{1 + r}$$

Receive the profit  $A - w - C$ .

With probability  $b$  lose the match and receive  $V_V$ .

## Stationary equilibrium

A stationary equilibrium determines  $(V_U, V_E, V_V, V_F, E, U, V, w)$  such that:

- ▶ the values  $V_x$  are determined as above.
- ▶ the labor market "clears:"  $\bar{L} = E + U$ .
- ▶ the number of employed is constant:  $M(U, V) = bE$ .
- ▶ creating new vacancies yields zero profit:  $V_V = 0$
- ▶ wages are somehow determined (this is where  $V_U, V_E$  come in).
- ▶ In addition:  $a, \alpha$  are defined above as functions of  $U, V$ .

## Wage determination

- ▶ What happens when firms and workers meet?
- ▶ The worker accepts any wage such that  $V_E \geq V_U$ .
- ▶ The firm accepts any wage such that  $V_F \geq V_V$ .
- ▶ Bargaining pins down the exact distribution of the surplus.
- ▶ We make an assumption: the surplus is evenly divided:

$$V_E - V_U = V_F - V_V \quad (9)$$

- ▶ Note: there is no good theory that would pin down how the surplus is split.

## Model summary I

Objects:  $(V_U, V_E, V_V, V_F, E, U, V, w)$ .

Flow equations:

$$\bar{L} = E + U \quad (10)$$

$$M(U, V) = bE \quad (11)$$

Values:

$$rV_E = w + b(V_U - V_E) \quad (12)$$

$$rV_U = a(V_E - V_U) \quad (13)$$

$$rV_V = -C + \alpha(V_F - V_V) = 0 \quad (14)$$

$$rV_F = A - w - C - b(V_F - V_V) \quad (15)$$

## Model summary II

Bargaining:

$$V_E - V_U = V_F - V_V \quad (16)$$

Definitions:

$$a = \frac{M(U, V)}{U} \quad (17)$$

$$\alpha = \frac{M(U, V)}{V} \quad (18)$$

## Solving the model

- ▶ This is just algebra: solve the 8 equations for the 8 unknowns.
- ▶ Step 1: substitute out the value functions.
- ▶ Start from bargaining:

$$V_E - V_U = V_F - V_V \quad (19)$$

- ▶ From the definitions:

$$V_E - V_U = \frac{w}{a + b + r} \quad (20)$$

$$V_F - V_V = \frac{A - w}{\alpha + b + r} \quad (21)$$



## Solving the model

Solve for the wage:

$$w = \frac{(a + b + r)A}{a + \alpha + 2b + 2r} \quad (22)$$

Intuition:

- ▶  $A$  is the flow “surplus” generated by filling the vacancy
- ▶ the “surplus” ( $A$ ) is equally divided when  $\alpha = a$ .
- ▶ if workers have a harder time finding jobs (low  $a$ ), their surplus share shrinks.

The next step: express everything in terms of  $E$ .

## Job Finding Rate

Find  $a$  in terms of  $E$ .

$$\begin{aligned} a(E) &= \frac{M(U, V)}{U} \\ &= \frac{bE}{\bar{L} - E} \end{aligned}$$

$a$  is increasing in  $E$ .

- ▶ Higher employment  $\rightarrow$  faster exit from unemployment.

## Vacancy Filling Rate

Find  $\alpha$  in terms of  $E$ .

$$\begin{aligned}\alpha &= \frac{M(U, V)}{V} \\ &= \frac{bE}{V}\end{aligned}$$

$\alpha$  is increasing in  $E$ , but only for given  $V$ .

## Vacancy Filling Rate

Solve the matching function for  $V(E)$ :

$$\begin{aligned} V &= \left( \frac{bE}{KU^\beta} \right)^{1/\gamma} \\ &= \left( \frac{bE}{K[\bar{L} - E]^\beta} \right)^{1/\gamma} \end{aligned}$$

Therefore

$$\alpha(E) = K^{1/\gamma} (bE)^{(\gamma-1)/\gamma} (\bar{L} - E)^{\beta/\gamma} \quad (23)$$

$\alpha$  is decreasing in  $E$ .

Higher employment  $\rightarrow$  vacancies are filled more slowly.

## Free Entry

Express free entry as a function of  $E$ :

$$rV_V = -C + \alpha(V_F - V_V) = 0$$

Substitute (20) and the solution for  $w$ :

$$\begin{aligned} rV_V &= -C + \alpha \frac{A - \frac{(a+b+r)A}{a+\alpha+2b+2r}}{\alpha+b+r} \\ rV_V &= -C + \frac{\alpha A}{a+\alpha+2b+2r} = 0 \end{aligned} \tag{24}$$

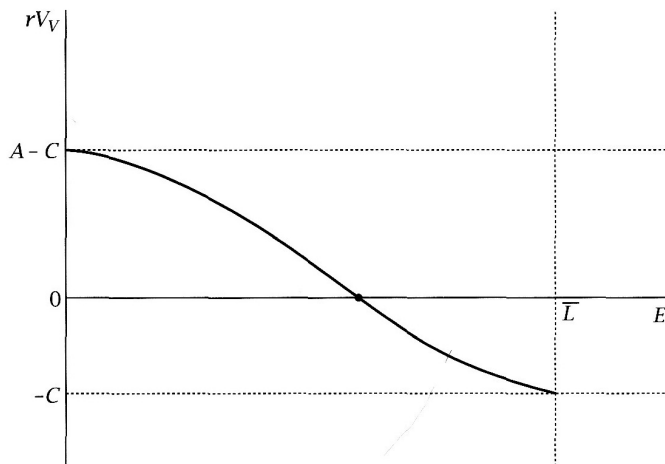
## Solving the model

- ▶ Write free entry as

$$rV_V = -C + \frac{\alpha(E)A}{a(E) + \alpha(E) + 2b + 2r} = 0 \quad (25)$$

- ▶ Recall  $a'(E) > 0$  and  $\alpha'(E) < 0$ .
- ▶ The fraction term is falling in  $E$ .
- ▶ There is a unique solution  $E$  with zero profits.

## Equilibrium Illustration



**FIGURE 9.6** The determination of equilibrium employment in the search and matching model

Source: Romer, *Advanced Macroeconomics*

## Model summary

The model determines  $w, E, a, \alpha$ .

Free entry:

$$rV_V = -C + \frac{\alpha(E)A}{a(E) + \alpha(E) + 2b + 2r} = 0 \quad (26)$$

Higher employment means faster job finding

$$a'(E) > 0 \quad (27)$$

and slower filling of vacancies

$$\alpha'(E) < 0 \quad (28)$$

Wages are determined from

$$w = \frac{(a(E) + b + r)}{a(E) + \alpha(E) + 2b + 2r} A \quad (29)$$



# Implications

## Long-run productivity growth

The model generates a sensible **balanced growth path** with wage growth and no trend in unemployment.

- ▶ Assume: productivity  $A$  and the cost of vacancies  $C$  rise in proportion.
- ▶ Then: no effect on employment ( $E$ ).
- ▶ Therefore  $\alpha, a$  unchanged.
- ▶ Wages rise in proportion with  $A$ .

# Fluctuations in productivity

Example: Recession.  $A/C$  drops.

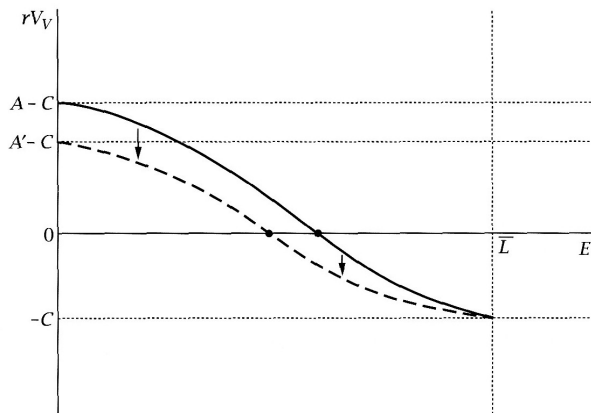


FIGURE 9.7 The effects of a fall in labor demand in the search and matching model

Source: Romer, *Advanced Macroeconomics*

# Fluctuations in productivity

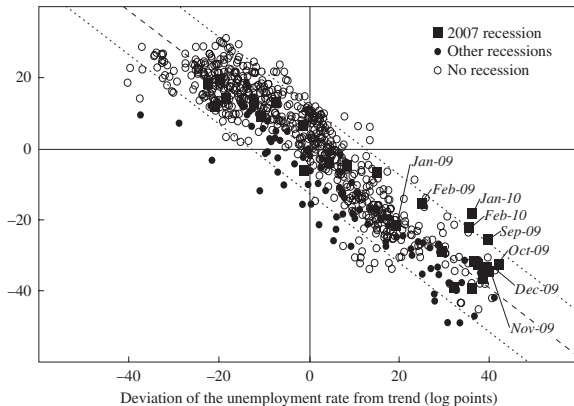
Intuition: Think of higher  $C$ .

- ▶ Post fewer vacancies.
- ▶ It also turns out that equilibrium vacancies drop.
- ▶ Employment declines.
- ▶ The comovement of vacancies and unemployment is observed in the data (the **Beveridge curve**).

# Beveridge curve

Figure 12. The Beveridge Curve, 1951–2010<sup>a</sup>

Deviation of job vacancies from trend (log points)



Source: ELSBY and HOBIJN (2010)

The cyclical behavior of vacancies.

# Fluctuations in productivity

The model does not imply wage rigidity:

- ▶  $A/C$  drops  $\rightarrow E$  drops.
- ▶  $a(E) \downarrow$  and  $\alpha(E) \uparrow$ .
- ▶ Wages are given by (22):

$$w = \frac{(a(E) + b + r)A}{a(E) + \alpha(E) + 2b + 2r}$$

- ▶ Wages may fall **more** than  $A$ .

# Strongly procyclical wages

## Intuition:

- ▶ The current surplus from matching  $(A - C)$  drops by more than  $A$ .
- ▶ Firm surplus shrinks even more because vacancies are easily filled.
- ▶ Worker surplus, however, shrinks less because jobs are hard to find.

## Caveat:

- ▶ Cyclical behavior of wages depends on bargaining solution.
- ▶ If bargaining weights vary over the cycle, wages could be less cyclical.

# Propagation of Shocks

The model implies that **transitory shocks have persistent effects**:

- ▶ When  $A$  drops, employment does not jump: firms have no incentive to fire workers (unless the shock is large enough).
- ▶ Unemployment only rises b/c vacancies decline and dissolved matches are filled more slowly.
- ▶ When  $A$  returns to normal, it will take time to fill the new vacancies.

This is perhaps the main contribution of the matching model: a propagation mechanism for shocks that is lacking in Walrasian models.

# Efficiency

- ▶ The equilibrium is generally not efficient.
- ▶ There are pecuniary externalities:
  - ▶ Posting a new vacancy raises the surplus for workers / reduces it for other firms.
- ▶ Under somewhat general conditions, the **Hosios condition** is necessary and sufficient for efficiency:
  - ▶ The worker's share of the surplus must equal the elasticity of the matching function with respect to unemployment.



## Is unemployment mostly frictional?

In the matching model, there is unemployment even without shocks. This is useful unemployment: it produces matches.

Even separations can be useful:

- ▶ imagine that workers are heterogeneous.
- ▶ when a worker finds a job, she does not know whether it is a good match.
- ▶ it may be optimal to quit after some time b/c a better match comes along.

## How large is frictional unemployment?

The data suggest it may be large.

- ▶ 3% of workers leave their jobs each month in U.S. manufacturing.
- ▶ 10% of jobs are destroyed each year.

But there is also long-term unemployment which is most likely not frictional.

# Summary

- ▶ Search models capture the idea that finding jobs takes time.
- ▶ They are useful for studying labor market regulation.
- ▶ A key shortcoming: Assumptions about bargaining determine the equilibrium.

# Applications

## Business cycle models

- ▶ Shimer (2005). [The MP model has problems accounting for labor market fluctuations.]
- ▶ Hall (2005)

## Analysis of labor market policies:

- ▶ Pries and Rogerson (2005)

## Theories of the wage distribution:

- ▶ Moscarini (2005)

# Reading

- ▶ Romer (2011)
- ▶ Ljungqvist and Sargent (2004) [Their model is easier b/c it has constant returns in the matching function.]
- ▶ Williamson (2006), "Notes on macroeconomic theory," ch. 7.
- ▶ Rogerson et al. (2005) [A survey of search models.]

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