Money in OLG Models

Prof. Lutz Hendricks

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Money in OLG Models

- We study the value of money in OLG models.
- We develop an important model of money.
- The model can be used to price other assets.

Monetary Economics

The central question of monetary economics:

Why and when is money valued in equilibrium?

By money we mean bits of green paper with pictures of dead presidents.

Monetary Economics

Rate of return dominance is a problem:

- Why would anyone hold money in the presence of other assets that offer a higher rate of return?

The answer of the OLG model:

- Money is a bubble.
- Its value derives solely from the expectation that money will be valued tomorrow.
- It is only valued if no other asset offers a higher rate of return.

Monetary Economics

Can money alleviate dynamic inefficiency?

- Previous models lacked a long-lived asset that would facilitate intergenerational trade.
- Money could solve this problem.

An OLG Model of Money

- Start with the standard two period OLG model without production or bonds.
- The population grows at rate n.
- Money is introduced as follows.
  - In period 1, the initial old are given M_0 bits of green paper
  - In every subsequent period, the government prints additional paper and hands it to the current old *in proportion* to current paper holdings.
  - Effectively, money pays (nominal) interest if held from young to old age.
  - The money growth rate is constant:

$$M_{t+1}/M_t = 1 + \theta$$

Timing

Beginning of t:

- N_t young are born and receive e_1 goods.
- N_{t-1} old
  - receive e_2 goods
  - carry over M_{t-1}/N_{t-1} units of money
  - receive money transfer $\theta M_{t-1}/N_{t-1}$.
  - they now hold M_t/N_{t-1} each

Timing

During t

- the young sell goods to the old
- the old sell money to the young

At the end of t:

- the young hold M_t/N_t each

Household

- This is a standard two period household problem with endowments.
- Preferences are

$$u(c_t^y, c_{t+1}^o)$$

- The household receives endowments e_1, e_2 of (perishable) goods.
- The price of goods in period t is P_t.
- The budget constraints are therefore

$$\begin{aligned} P_t(e_1 - c_t^y) &= P_t\, x_t \\ P_{t+1}(c_{t+1}^o - e_2) &= x_t(1+\theta)P_t \end{aligned}$$

Household

- The lifetime budget constraint is:

$$e_1 - c_t^y = \frac{c_{t+1}^o - e_2}{(1+\theta)P_t/P_{t+1}}$$

- Note that money acts exactly like a bond that pays gross interest

$$R_{t+1} = (1+\theta)P_t/P_{t+1}$$

Household

- The Lagrangian is the same as in previous models:

$$\Gamma = u(c_t^y, c_{t+1}^o) + \lambda_t \left[e_1 - c_t^y + \frac{e_2 - c_{t+1}^o}{R_{t+1}} \right]$$

- FOCs:

$$\begin{aligned} u_1(t) &= \lambda_t \\ u_2(t) &= \lambda_t / R_{t+1} \end{aligned}$$

- Euler equation:

$$u_1(t) = R_{t+1} u_2(t)$$

Household: Solution

A solution to the household problem is a triple (c_t^y, c_{t+1}^o, x_t) which satisfies

- the Euler equation and
- the two budget constraints.

Optimal behavior can be characterized by a savings function (which is now a *money demand function*)

$$x_t = s(R_{t+1}, e_1, e_2) \tag{1}$$

Equilibrium

The government is simply described by a money growth rule:

$$M_{t+1}/M_t = 1 + \theta$$

Market clearing:

Equilibrium Definition

A sequence of prices and quantities

such that

Characterizing Equilibrium

- We look for a difference equation in terms of the economy's state variables.
- State variables are M and P.
- But in this model (and typically) only the ratio $m = M/PN$ matters.

Characterizing Equilibrium

Start from the money market clearing condition

$$m_t = s(R_{t+1}) \tag{2}$$

Subsitute out R using

$$R_{t+1} = (1+\theta)P_t/P_{t+1} \tag{3}$$

We need an expression for inflation. From

$$\frac{M_{t+1}}{M_t} = \frac{m_{t+1}}{m_t}\frac{P_{t+1}}{P_t}\frac{N_{t+1}}{N_t}$$

we have

$$R_{t+1} = (1+\theta)P_t/P_{t+1} = (1+n)m_{t+1}/m_t$$

The law of motion is

$$m_t = s((1+n)m_{t+1}/m_t) \tag{4}$$

Characterizing Equilibrium

A more explicit way of deriving this.

For ease of notation assume

$$u\left(c_t^y, c_{t+1}^o\right) = v\left(c_t^y\right) + \beta v\left(c_{t+1}^o\right) \tag{5}$$

Sub budget constraint into Euler equation:

$$v'(e_1 - x_t) = R_{t+1}\beta v'(e_2 + R_{t+1}x_t) \tag{6}$$

Sub in $m_t = x_t$ and $R_{t+1} = (1+n)m_{t+1}/m_t$:

$$v'(e_1 - m_t) = (1+n)\frac{m_{t+1}}{m_t}\beta v'(e_2 + (1+n)m_{t+1}) \tag{7}$$

Intuition

$$m_t = s((1+n)m_{t+1}/m_t) \tag{8}$$

Why is this true?

- we really have $m_t = s(R_{t+1})$
- fixed: nominal money growth and n
- higher growth in $m = M/PN \implies$ lower inflation $\implies$ higher return

The Offer Curve

The Offer Curve

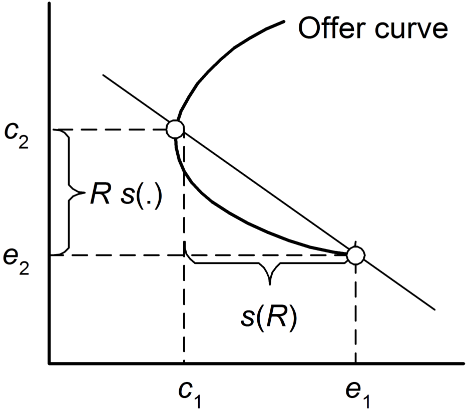
We want to determine the shape of the law of motion.

The key idea is to use the household's intertemporal consumption allocation to figure out how money evolves over time.

Household consumption choice

The lifetime budget constraint has slope $-R_{t+1}$.

Plot the tangencies between budget constraint and indifference curves for all interest rates $\rightarrow$ offer curve.



Offer curve

What do we know about the offer curve?

1. It goes through the endowment point.
2. At low levels of R the household would like to borrow (but cannot).
3. For interest rates where the household saves very little, income effects are small
   $\implies$ savings rise with R_{t+1}
   $\implies$ the offer curve is upward sloping
4. The offer curve intersects each budget line only once.

Money demand

Money demand equals saving of the young:

$$m_t = s(R_{t+1}) = e_1 - c_t^y \tag{9}$$

- Hence: the horizontal axis shows m_t.

Money demand also equals capital income of the old:

$$(1+n)m_{t+1} = R_{t+1}s(R_{t+1}) = c_{t+1}^o - e_2 \tag{10}$$

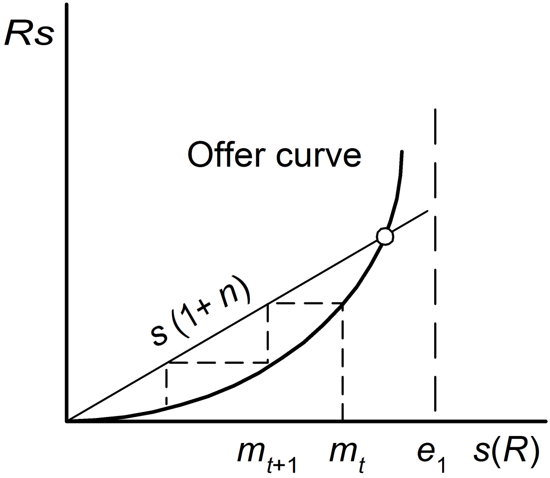
- Hence: the vertical axis shows $(1+n)\ m_{t+1}$.

The offer curve therefore describes the law of motion for m:

$$(1+n)m_{t+1} = F(m_t)$$

where F is the offer curve.

Law of motion



Using a line of slope $(1+n)$ we can find the path of m_t for any start value m_0.

Steady state

- There is a unique monetary **steady state** (intersection of offer curve and ray through origin).
- It is ***unstable***.

Properties of the steady state

m is constant over time.

The gross rate of return on money is

$$\begin{aligned} R_{t+1} &= (1+\theta)P_t/P_{t+1} \\ &= (1+n)m_{t+1}/m_t \end{aligned}$$

Therefore, in steady state, the Golden Rule holds:

$$R = 1+n$$

Steady state inflation is

$$P_{t+1}/P_t = \frac{1+\theta}{1+n}$$

Dynamics

Assumption: the offer curve is not backward bending.

Take m_0 as given for now.

What if $m_0 > m_{ss}$?

- This cannot happen because m_t would blow up towards ∞.
- But then consumption will exceed total output at some point.

Dynamics

If $m_0 < m_{ss}$: m_t collapses towards 0.

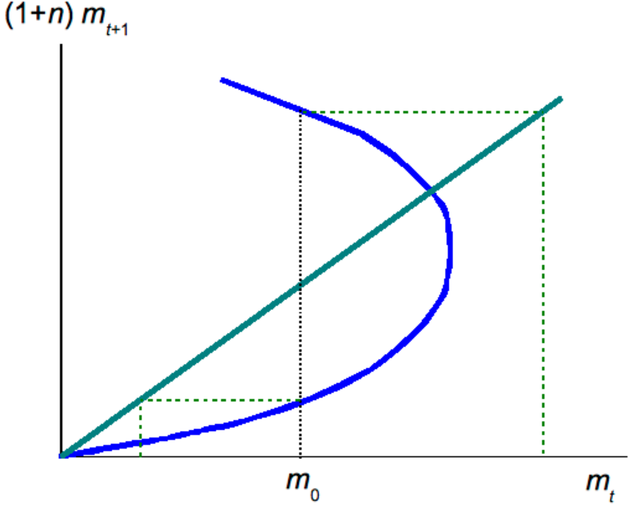
- Because M grows at a constant rate, this must happen through inflation.
- Along this path R falls over time $\Rightarrow$ inflation accelerates.

Intuition:

- If $m_0 = m_{ss}$ people save just enough to keep m constant.
- If m_0 is a bit lower, then R is a bit lower. People save less.
- That requires a lower m_1, hence more inflation.
- That leads people to save less again, etc.

Dynamics: Backward bending Offer Curve

We have **multiple equilibria** and **complex dynamics**.



Initial money stock

- Nothing in the model pins down m_0. Any value below m_{ss} is acceptable.
- There is a continuum of equilibrium paths.
- The reason: money is a bubble.
- As long as expectations are such that people are willing to hold m_0, we have an equilibrium.
- $m_0 = 0$ is also an equilibrium.

Dynamic Efficiency

Does money solve the dynamic inefficiency problem?

- It might because it permits intergenerational trade.

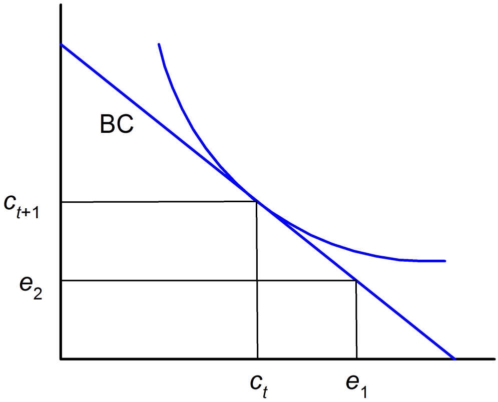
Two cases:

1. Samuelson case: the offer curve at the origin is flatter than $1+n$;
2. Classical case: it is steeper than $1+n$.

Dynamic Efficiency

- Why is the slope of the offer curve at the origin interesting?
- Because it is the interest rate in the non-monetary economy.
- Therefore:
  - Samuelson case: non-monetary economy is dyn. **inefficient**
  - Classical case: it is **efficient**

Samuelson case



Non-monetary interest rate is $< 1 + n$

At $R = 1 + n$ households want to save

Money can be valued in equilibrium

Classical Case

- The non-monetary economy is dynamically **efficient**.
- The offer curve is too steep to intersect the $1+n$ line.
- A monetary equilibrium does not exist.

Result:

Money is valued in equilibrium only in an economy that would be dynamically inefficient without money.

Is this a good theory of money?

Good features of the OLG model of money are:

1. The outcome that money is valued in equilibrium is not assumed (e.g. because money yields utility or is simply required for transactions).
2. The value of money depends on expectations and is fragile.

The problem:

1. The model does not generate rate of return dominance.
2. A key feature of money seems to be missing: liquidity.

How to construct a theory of money that resolves the problems without introducing new ones is an open question.

Fiscal Theory of the Price Level

Model With Government Spending

- We add government spending to the model and get an odd result.
- Preferences, endowments, demographics are unchanged
- Government
  - buys $G_t = g_t N_t$ goods
  - prints money to finance the purchases.
- Markets: There are markets for goods and for money.

Government Budget Constraint

The government budget constraint is

$$M_{t+1} - M_t = P_{t+1} G_{t+1}$$

Divide both sides by $P_{t+1} N_{t+1}$:

$$m_{t+1} = m_t / [(1+n)(1+\pi_{t+1})] + g_{t+1}$$

Or:

$$P_t / P_{t+1} = (1+n)(m_{t+1} - g_{t+1}) / m_t$$

where $m = M/(PN)$

Household

The household problem is exactly the same as in the OLG model with one period bonds.

- Young budget constraint: $c_t^y = e_1 - x_t$.
- Old budget constraint: $c_{t+1}^o = e_2 + x_t P_t / P_{t+1}$
- Lifetime budget constraint:

$$c_t^y + c_{t+1}^o / R_{t+1} = e_1 + e_2 / R_{t+1}$$

where the real interest rate is

$$R_{t+1} = P_t / P_{t+1}.$$

We get the saving function $s(R_{t+1}; e_1, e_2)$ as usual from the Euler equation and the budget constraints.

Equilibrium

A CE consists of sequences $\left\{c_t^y, c_t^o, x_t, m_t, P_t\right\}$ that satisfy

- 3 household conditions (2 b.c. and saving function);
- government budget constraint;
- Goods market clearing:

$$c_t^y + c_t^o/(1+n) + g_t = e_1 + e_2/(1+n)$$

- Money market clearing:

$$x_t \quad = m_t = \quad s(P_t/P_{t+1})$$

Offer Curve

$$m_t = s\left((1+n)(m_{t+1} - g_{t+1})/m_t\right), \tag{11}$$

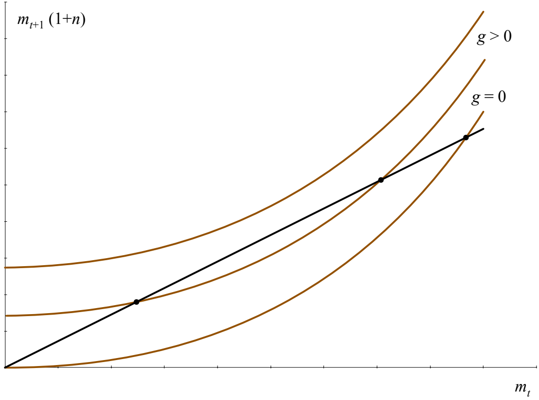
With $g = 0$ this is the model we studied earlier.

The offer curve now relates $m_{t+1} - g_{t+1}$ to m_t.

Assume

- g is constant over time
- $s(1+n) > 0$ (Samuelson case).
- the offer curve is convex, but not backward bending.

Offer Curve: Varying g



$g \uparrow$ shifts the offer curve up
There is a continuum of equilibria indexed by $m_1 \in (0, m^*)$

Multiple Steady States

- There are two steady states.
- How do the 2 steady states differ?
- The lower steady state is stable, while the higher one is not.
- From the government budget constraint

$$1 = 1/[(1+n)(1+\pi)] + g/m$$

- A higher m implies a lower π (given g).

No Non-monetary Equilibrium

- The odd finding:
  With $g > 0$ the non-monetary equilibria have disappeared!
- The reason: the government promises to violate its budget constraint in equilibria it does not like
- This is the essence of the "Fiscal Theory of the Price Level."
- Government spending, via the budget constraint, determines the value of money in equilibrium.

Reading

- Blanchard and Fischer (1989), ch. 4.1 [A clear exposition.]
- Krueger, "Macroeconomic Theory," ch. 8 discusses offer curves (can be found online).
- Ljungqvist and Sargent (2004), ch. 9 [Detailed.]
- McCandless and Wallace (1991)

References I

Blanchard, O. J. and S. Fischer (1989): *Lectures on macroeconomics*, MIT press.

Ljungqvist, L. and T. J. Sargent (2004): *Recursive macroeconomic theory*, 2nd ed.

McCandless, G. T. and N. Wallace (1991): *Introduction to dynamic macroeconomic theory: an overlapping generations approach*, Harvard University Press.