Overlapping Generations Model
Bequests and Altruism

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Topics

We introduce intergenerational links into the OLG model:

- parents leave bequests to their children

The main **goal** is to learn the model setup.

We study whether bequests solve the **dynamic inefficiency** problem

- The answer is no
- Bequests can only increase the capital stock

A key result

A key result:

- when parents leave bequests, they behave as if they lived forever
- some view this as micro-foundation for models where households live forever (though that seems misguided to me)

Bequest Motives

Why do parents leave bequests to their children?

Theoretically, there are various ways of modeling bequests:

1. **Altruism**: parents value their children's utility.
   1.1 **Warm glow**: parents value the bequest itself (a reduced form).
   1.2 **Strategic**: parents promise bequests so kids behave well.

Empirically, we don't know (a possible research question).

OLG Model With Altruism

Model Elements

- We study the standard endowment economy, just with different preferences.
- Demographics: Each household has $(1+n)$ children when old.
- Endowments: e_1 when young, e_2 when old.
- Technology: none.
- Markets: goods, bonds

Preferences

The household values own consumption according to

$$u(c_t^y, c_{t+1}^o)$$

The household also values the utility of the child.

Preferences are defined recursively:

$$V(t) = u(c_t^y, c_{t+1}^o) + \omega V(t+1)$$

$\omega > 0$ governs the strength of altruism.

Household

Expanding this we find that the parent values utility of all future generations:

$$
\begin{aligned}
V(t) &= u(c_t^y, c_{t+1}^o) + \omega[u(c_{t+1}^y, c_{t+2}^o) + \omega V(t+2)] \\
&= u(c_t^y, c_{t+1}^o) + \omega u(c_{t+1}^y, c_{t+2}^o) \\
&\quad + \omega^2[u(c_{t+2}^y, c_{t+3}^o) + \omega V(t+3)]
\end{aligned}
$$

and therefore

$$V(t) = \sum_{j=0}^{\infty} \omega^j u(c_{t+j}^y, c_{t+j+1}^o) \qquad (1)$$

Household

This looks like

- the **planner**'s welfare function,
- the utility function of a household who **lives forever**.

Next, we write the sequence of budget constraints to look like a single budget constraint.

Household problem

Period budget constraints are

$$
\begin{aligned}
c_t^y + s_t &= e_1 + b_t && (2) \\
c_{t+1}^o + (1+n)\,b_{t+1} &= e_2 + R_{t+1} s_t && (3)
\end{aligned}
$$

b_{t+1} is the bequest left to each child by cohort t.

Present value budget constraint (set $n = 0$ for simplicity):

$$
\begin{aligned}
b_t &= \underbrace{c_t^y - e_1 + (c_{t+1}^o - e_2)/R_{t+1}}_{z_t} + b_{t+1}/R_{t+1} && (4) \\
&= z_t + b_{t+1}/R_{t+1} && (5) \\
&= z_t + (z_{t+1} + b_{t+2}/R_{t+2})\,/R_{t+1} && (6) \\
&= z_t + \frac{z_{t+1}}{R_{t+1}} + \frac{b_{t+2}}{R_{t+1}R_{t+2}} && (7)
\end{aligned}
$$

Budget constraint

Successively replace the b_{t+j} with $z_{t+j} + b_{t+j+1}/R_{t+j+1}$ to obtain

$$b_t = \sum_{j=0}^{J} \frac{z_{t+j}}{D_{t,j}} + \frac{b_{t+J+1}}{D_{t,t+J+1}}$$

where

$$D_{t,j} = \prod_{i=1}^{j} R_{t+i}$$

is a discount factor.

Budget constraint

Take $J \to \infty$ and assume that

$$\lim_{J \to \infty} \frac{b_{t+J}}{D_{t,t+J}} = 0$$

We discuss (much) later why we might want to assume this.

- see transversality conditions

Then the present value budget constraint becomes

$$\underbrace{\sum_{j=0}^{\infty} \frac{c^y_{t+j} + c^o_{t+j+1}/R_{t+j+1}}{D_{t,j}}}_{\text{pv of consumption}} = \underbrace{\sum_{j=0}^{\infty} \frac{e_1 + e_2/R_{t+j+1}}{D_{t,j}}}_{\text{pv of "earnings"}} + \underbrace{b_t}_{\text{initial assets}}$$

Budget constraint

This is a common result:

Present value of spending = [Present value of income] + [Initial assets]

This looks like the budget constraint of an infinitely lived household.

Infinitely lived dynasty

The parent therefore behaves exactly like an infinitely lived individual

- maximizing a single utility function over an infinite horizon
- subject to a single present value budget constraint.

This only works if

- households can borrow and lend at the same interest rate;
- bequests can be negative or are always intended to be positive
- parents are altruistic (not warm glow etc)

Exercise

Show that the equilibrium allocation is the same as the planner's allocation.

Implications

Why is this important?

- If we think bequests are positive, we can ignore finite lifetimes and write down models with a single, infinitely lived household.

One potential problem:

- We set up the parent's problem as if he could choose the child's actions.
- Later, we talk about why this is correct (see Dynamic Programming)

When Are Bequests Positive?

And do they help with dynamic inefficiency?

When are bequests positive? I

Bequests are positive, if a small bequests raises parental utility.

Consider the following perturbation of the optimal plan with $b = 0$:

1. Reduce old age consumption by ε. The utility loss is $-u_2(t)\,\varepsilon$.
2. Give $\varepsilon/(1+n)$ to each child as a bequest.
3. Assume the child eats the bequest when young [what if not?] and gains
$$\omega u_1(t+1) \cdot \varepsilon/(1+n) \tag{8}$$
4. The household wants to leave a bequest if
$$\omega u_1(t+1)/(1+n) > u_2(t) \tag{9}$$

Does this expression look familiar?

When are bequests positive? II

5. Apply the parent's FOC to express both gain and loss in terms of u_1. The FOC is

$$u_1(t) = (1 + r_{t+1})u_2(t)$$

Thus the parent increases his bequest if

$$\omega u_1(t+1)/(1+n) > u_1(t)/(1+r_{t+1})$$

or

$$u_1(t) < \frac{1+r_{t+1}}{1+n}\omega u_1(t+1) \tag{10}$$

6. In steady state this reduces to $\omega(1+r) > (1+n)$.

$\omega(1+r) = (1+n)$ is the **modified golden rule** (the planner's FOC).

Dynamic inefficiency

This means:

- A situation where $\omega R = 1 + r > (1+n)$ can never be a steady state.
  - Every parent would want to increase his bequest until the MGR holds with equality
  - Then the economy is dynamically *efficient*.
- If without bequests $\omega R < (1+n)$, households don't want to leave bequests and the bequest motive is irrelevant.
  - Dynamic inefficiency remains.

The same holds in a production economy (the household does problem is the same).

Summary

If the bequest motive is operative $(b > 0)$, then:

- The economy attains the modified golden rule.
- Therefore it is dynamically efficient.
- The market equilibrium coincides with the planner's solution (show this!).
- Ricardian equivalence holds even across generations. (We haven't shown that, but it follows directly from the fact that there is a present value budget constraint that holds across generations.)

If the bequest motive is not operative, it does not matter.

- This happens when the economy is initially dynamically inefficient.

Applications of OLG Models

Two main reasons for using OLG models:

1. Demographic structure matters:
   1.1 Social security and tax analysis (pioneered by Auerbach and Kotlikoff 1987)
   1.2 Human capital: schooling followed by on-the-job learning (e.g., many papers by Heckman and his students)
   1.3 Income or wealth inequality (e.g., Huggett 1996; Huggett et al. 2011)

   These are usually computational many-period models.
2. Analytical tractability:
   With log utility consumption becomes independent of r_{t+1}.
   Easy dynamics because agents behave as if not foward looking.
   E.g., Aghion et al. (2002), Krueger and Ludwig (2007)

Reading

- Acemoglu (2009), ch. 5.3, 9.

References I

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