Money in the Utility Function

Prof. Lutz Hendricks

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Money in the utility function

- A shortcut for getting money valued in equilibrium: assume that households gain utility from holding money.
- "Sidrauski" model.
- Benefits: Tractability.
- Drawbacks: Arbitrary specification of utility affects results.

The Economic Environment

- Much of the model is a standard growth model.
- The government prints paper (costlessly).
- Households gain utility from holding paper.

Environment

- Demographics: 1 representative household
- Endowments:
  - 1 unit of work time at each instant
  - k_0 units of the good
  - M_0 bits of paper
- Technology:
  - $F(K,L) - \delta K = c + \dot{K}$

Environment

- Preferences:

$$\int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \tag{1}$$

  $m_t = M_t / p_t$
- Government:
  - prints $\dot{M}_t$ and hands it to households
- Markets:
  - goods, labor, capital rental, money

Household

Households solve:

$$\max \int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \tag{2}$$

subject to k_0, m_0 given and

$$p(c + \dot{k}) + \dot{M} = p(w + rk + x) \tag{3}$$

x are lump-sum transfers (of money).

Budget constraint in real terms

$$\dot{k} + \dot{M}/p = w + rk + x - c \tag{4}$$

Note that

$$\begin{aligned} \dot{m} &= \dot{M}/p - (M/p^2)\dot{p} \\ &= \dot{M}/p - m\pi \end{aligned}$$

where π is the inflation rate $(\pi = \dot{p}/p)$.

Therefore

$$\dot{k} + \dot{m} = w + rk + x - c - \pi m \tag{5}$$

Budget constraint

- We seem to have 2 state variables (k, m) but only one law of motion.
- The reason: the correct state variable is wealth: $A = k + m$.
- To transform the budget constraint into a law of motion for A, write it as

$$\dot{A} = w + rA + x - c - (r + \pi)m \tag{6}$$

- Every unit of wealth held in money reduces income by the nominal interest rate $(r + \pi)$.

An Equivalence

- The household problem is exactly the same as in a real two-good economy.
- Money is like a consumption good with price $r + \pi$.

Solving the household problem

$$H = u(c,m) + \lambda[w + rA + x - c - (r+\pi)m] \tag{7}$$

FOC:

$$\begin{aligned} u_c &= \lambda \\ u_m &= \lambda(r+\pi) \\ \dot{\lambda} &= (\rho - r)\lambda \end{aligned}$$

TVC:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t A_t = 0$$

Household optimality

Static condition:

$$u_c = u_m/(r+\pi) \tag{8}$$

Intuition?

Intertemporal condition:

$$\dot{\lambda}/\lambda = g(u_c) = -(r-\rho) \tag{9}$$

where $g(z) \equiv \dot{z}/z$ denotes a growth rate.

Household: Separable utility

If the utility function is **separable**,

$$u(c,m) = v(c) + \bar{v}(m) \tag{10}$$

then

$$u_c = v'(c) \tag{11}$$

and

$$g(u_c) = v''(c)\dot{c}/v'(c) = -\sigma g_c \tag{12}$$

Then a very common expression emerges:

$$g(c) = (r - \rho)/\sigma \tag{13}$$

Equilibrium

Firms solve the standard static profit maximization problem:

$$r = f'(k) - \delta \tag{14}$$

$$w = f(k) - f'(k)k \tag{15}$$

Government

- The government grows the money supply at the constant rate $\mu = g(M)$.
- Implied lump-sum transfers are

$$x = \dot{M}/p = \mu m \tag{16}$$

Market clearing

- Money and factor market clearing are implicit in the notation.
- Goods market clearing is feasibility:

$$\dot{k} + c = f(k) - \delta k \tag{17}$$

Equilibrium

An equilibrium is a set of functions of time that satisfy

These are 9 variables and 10 equations.

The boundary conditions are initial values for M and k and the TVC.

Characterization

- We reduce the CE to 4 equations in (c, p, k, m).
- Household first-order conditions:

$$
\begin{aligned}
g(u_c[c,m]) &= -(f'(k) - \delta - \rho) \\
u_c(c,m) &= u_m(c,m)/(f'(k) - \delta + \pi)
\end{aligned}
$$

- Goods market clearing:

$$\dot{k} + c = f(k) - \delta k$$

- Money growth rule:

$$\dot{m} = (\mu - \pi)m$$

Monetary Neutrality

- Assume: the utility function is additively separable

$$u(c,m) = \bar{u}(c) + v(m) \tag{18}$$

- Then money has absolutely no effect on the real sector.
- The evolution of c and k is determined by the Euler equation and the goods market clearing condition alone.
- Intuition?

Steady state

In steady state c, k, m are constant.

The Euler equation then determines the steady state capital stock:

$$r = f'(k) - \delta = \rho \tag{19}$$

Goods market clearing then yields consumption:

$$c = f(k) - \delta k$$

Constant real balances require $\pi = \mu$.

The static optimality condition yields an implicit equation for m:

$$u_m(c_{SS}, m_{SS}) = (\rho + \mu) u_c(c_{SS}, m_{SS}) \tag{20}$$

$\Rightarrow$

$$m_{SS} = m^d(c_{SS}, \rho + \mu) \tag{21}$$

Super-neutral money

- Changes in money growth (μ) only affect the inflation rate, but not real variables (k_{ss}, c_{ss}).
- Intuition: inflation does not alter the intertemporal tradeoff between consumption today and tomorrow.
- Inflation only affects the relative levels of goods and money consumed

Inflation and welfare

- What is the effect of inflation on real money balances? Differentiate (20) to obtain

$$u_{mm}dm = (\rho + \mu)u_{cm}dm + u_c d\mu \tag{22}$$

$\Rightarrow$

$$dm/d\mu = u_c/[u_{mm} - (\rho + \mu)u_{cm}] \tag{23}$$

- Unless money and consumption are too strong complements (u_{cm} large and positive), higher inflation is associated with lower real money balances and thus lower steady state utility.

The Friedman Rule

- Which money growth rate maximizes steady state utility?
- Since μ does not affect c_{ss}, we only need to know how to maximize m_{ss}.
- If we set $\rho + \mu = 0$, then $u_m = 0$, which is the best we can do: satiate the household with money.
- If $u_m > 0$ even asymptotically, the problem does not have a solution.
- The intuition is quite general:
  - If money provides some kind of benefit, the best we can do is to make it costless to hold money.
  - That will be the case when money pays the same rate of return as capital (the Friedman rule).

Is This a Good Theory of Money?

Pros:

- tractable

Cons:

- the value of money is assumed
  therefore: no non-monetary equilibrium / hyperinflation
- money is not used in transactions
  it's really a consumption good

Where Is this Used?

Models of the financial sector, where the details why households hold money play a minor role

- Van den Heuvel, Skander J. "The welfare cost of bank capital requirements." Journal of Monetary Economics 55.2 (2008): 298-320.

Reading

- Blanchard and Fischer (1989), ch. 4.5

References I

Blanchard, O. J. and S. Fischer (1989): *Lectures on macroeconomics*, MIT press.