Problem Set 5: Growth Model in Continuous Time

Econ720. Fall 2016. Prof. Lutz Hendricks

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1 Capital income tax

[Romer 2.9] Consider a Ramsey economy on its balanced growth path. At time 0 the government starts to tax capital income, so that the interest rate facing the household is

$$r(t) = (1 - \tau)\ f'(k_t)$$

where I have assumed that $\delta = 0$. Tax revenues are rebated to the household in a lump-sum fashion. The change in the policy is unanticipated.

1. How does the tax affect the $\dot{k} = 0$ and the $\dot{c} = 0$ loci?
2. How do the balanced growth values of c and k change?
3. Describe the changes at time 0 and the transition path thereafter.
4. Show that the saving rate on the balanced growth path ($[y - c]/y$) is decreasing in τ.
5. Imagine there are two countries that differ only in τ. Do the residents of the high τ countries have an incentive to invest in the low τ country or vice versa?
6. How do your answers change if the tax revenues are used to pay for government purchases instead of being rebated?

2 Continuous Time CIA Model. Cash and Credit Goods.

Crusoe solves the following problem:

$$\max \int_0^\infty e^{-\rho t}\, u(c_t, g_t)\ dt$$

subject to the budget constraint

$$\dot{k}_t + c_t + g_t + \dot{M}_t/p_t = f(k_t) + x_t$$

and the CIA constraint

$$c_t \leq M_t/p_t$$

The notation is standard. There are two consumption goods, which are perfect substitutes in production but not in consumption. The cash good c is subject to the CIA constraint, while the credit good g is not. Denote real balances by $m_t = M_t/p_t$. x_t is a money transfer from the government.

(a) Write down the household's Hamiltonian. Which are his states and controls? Derive first-order conditions for two cases: either the CIA constraint always binds or it never binds.

(b) Define a competitive equilibrium. Assume that the government lets the money stock grow at the constant rate $g(M)$.

(c) Derive a set of equations that characterize the steady state. Show that the nominal interest rate equals zero, if the CIA constraint does not bind.

(d) Determine the effects of a higher money growth rate on the steady state allocation. Assume that the utility function takes the form $u(c, g) = U(c) + V(g)$, where U and V are strictly concave functions.