Problem Set 3: Growth Model in Discrete Time

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1 Bonds Of Different Maturities

This question examines Ricardian Equivalence when the government has bonds of different maturities to finance spending. Consider a standard growth model in discrete time where the government issues two types of bonds:

- b_{t+1} one-period bonds are issued at date t; each has a price of 1 and pay R_{t+1} units of consumption at $t+1$.
- $B_{t+1} - B_t$ infinitely lived bonds are issued at date t; each costs p_t and pays one unit of consumption at dates $s \geq t+1$.

The government also imposes a lump-sum tax τ_t and spends g_t units of the good on a useless purpose.

Firms are standard with first-order conditions $r = f'(k)$ and $w = f(k) - f'(k)\,k$.

(a) The household maximizes

$$\sum_{t=0}^{\infty} \beta^t \, u(c_t)$$

subject to the budget constraint

$$k_{t+1} + c_t + \tau_t + b_{t+1} + p_t \, (B_{t+1} - B_t) = (r_t + 1 - \delta) \, k_t + w_t + R_t \, b_t + B_t$$

with initial endowments (k_0, b_0, B_0) given. Solve the household problem using Dynamic Programming.

(b) The government budget constraint is

$$g_t + R_t \, b_t + B_t = \tau_t + b_{t+1} + p_t (B_{t+1} - B_t)$$

Show that the present value budget constraint of the government can be written as

$$b_0 + (1 + p_0) \, B_0 / R_0 = \sum_{t=0}^{\infty} \frac{\tau_t - g_t}{D_t}$$

where $D_t = R_0 \cdot ... \cdot R_t$ is a cumulative discount factor.

(c) Show that the household's present value budget constraint is given by

$$b_0 + \frac{(1 + p_0) \, B_0}{R_0} + k_0 = \sum_{t=0}^{\infty} \frac{c_t + \tau_t - w_t}{D_t}$$

(d) Show that Ricardian Equivalence holds in this economy. That is, a change in the timing of taxation does not affect the equilibrium allocation (for a given sequence g_t). The best way of answering this part is to define a competitive equilibrium in such a way that a set of equations that does not depend on $\tau's$ determines the allocation.

2 Wealth in the utility function

Consider the following modification of the standard growth model where the households derives utility from holding wealth.

Demographics: There is a representative household of unit mass who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t, k_{t-1})$ where c_t is consumption and k_{t-1} is last period's capital (wealth). The utility function is strictly concave and increasing in both arguments.

Endowments: At $t = 0$ the household is endowed with capital K_0. In each period the household works 1 unit of time.

Technologies:

$$K_{t+1} = A\,F(K_t, L_t) + (1 - \delta)\,K_t - c_t \tag{1}$$

The production function has constant returns to scale.

Markets: Production takes place in a representative firm which rents capital and labor from households. There are competitive markets for goods (price 1), capital rental (r_t), and labor rental (w_t).

1. State the household's dynamic program.
2. Derive and explain the conditions that characterize a solution to the household problem (in sequence language).
3. Define a competitive equilibrium.
4. Derive a single equation that determines the steady state capital stock.
5. Is the steady state unique? Explain the intuition why the steady state is or is not unique.