Problem Set 3: Ramsey Model in Discrete Time

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1 Land Prices with Capital Accumulation

Consider the following economy with land and capital.

Demographics: There is a representative household of unit mass who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$

Endowments: At $t = 0$ the household is endowed with capital K_0 and land L. The aggregate endowment of land is fixed.

Technologies:

$$K_{t+1} = A\,F(K_t, L_t) + (1-\delta)\,K_t - c_t \tag{1}$$

where A is an exogenous productivity factor, δ is the depreciation rate of capital, and c is consumption. The production function has constant returns to scale.

Markets: Production takes place in a representative firm which rents capital and land from households. There are competitive markets for goods (price 1), land (p_t), capital rental (r_t), and land rental (q_t).

Questions:

1. Set up the household's Bellman equation. Define a solution to the household problem.
2. Define a competitive equilibrium.
3. Determine the effects of the following changes on steady state prices and quantities. A qualitative characterization is sufficient (which variables increase/decrease?): L increases, A increases.

2 Education Costs

Consider the following version of a standard growth model with human capital. The planner solves

$$\max \sum_{t=1}^{\infty} \beta^t u\left(c_t\right) \tag{2}$$

s.t.

$$\begin{aligned} k_{t+1} &= (1-\delta)\,k_t + x_{kt} && (3)\\ h_{t+1} &= (1-\delta)\,h_t + x_{ht} && (4)\\ c_t + x_{kt} + \eta x_{ht} &= f\left(k_t, h_t\right) && (5) \end{aligned}$$

with k_1 and h_1 given. Here c is consumption, k is physical capital, h is human capital, and η is a constant representing education costs. Assume that the production function is Cobb-Douglas:

$$f(k,h) = zk^{\alpha}h^{\varepsilon} \tag{6}$$

where z is a constant technology parameter and $\alpha + \varepsilon < 1$.

Questions:

1. Derive the first-order condition for the planner's problem using Dynamic Programming. Define a solution in sequence language and in functional language.

2. Solve for the steady state levels of k/h and k.

3. Characterize the impact of cross-country differences in education costs (η) on output per worker in steady state. In particular, calculate the ratio of outputs per worker for two countries that only differ in their η's.