Asset Pricing

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Pricing Assets: An Example

The infinite horizon model can be used to price long-lived assets.

- This is more interesting in stochastic economies.
- It then yields the famous β measure of risk and the CAPM.

Environment

Demographics:

- a representative, infinitely lived household
- mass 1

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$

Endowments:

- in each period: N units of labor time
- at $t = 0$: L units of land

Environment

Technologies: $F(N_t, L_t; A_t) = c_t$

- constant returns to (N, L)
- the productivity sequence $\{A_t\}$ is given.

Markets:

- goods (numeraire)
- land rental: r_t
- labor: w_t
- land purchase: p_t

Firm's problem

The firm's problem is standard:

$$\max F(N_t, L_t; A_t) - w_t N_t - r_t L_t \tag{1}$$

FOCs:

$$r = F_L \tag{2}$$
$$w = F_N \tag{3}$$

Solution: N_t, L_t that satisfy the 2 FOCs.

Household

Budget constraint:

Bellman equation:

First-order conditions:

Household

The Euler equation is standard

$$u'(c) = \beta\, u'(c') \frac{r' + p'}{p}$$

Solution: $\{c_t, l_t\}$ that solve the Euler equation and budget constraint.

Equilibrium

A competitive equilibrium is a set of sequences

that satisfy:

- household: Euler equation and budget constraint;
- firm: 2 FOCs:
- market clearing for land:
- market clearing for goods:

The price of land

- We find a difference equation for p_t.
- Substitute the goods market clearing condition and the first-order condition for r into the Euler equation to obtain

$$u'(F(N,L)) = \beta\, u'(F(N',L')) \frac{F_L(N',L') + p'}{p} \tag{4}$$

- Note that this difference equation only contains p and exogenous variables.

The price of land

- We solve the difference equation for p_t by **forward iteration**.

$$\begin{aligned} p_t &= \beta \frac{u'(c_{t+1})}{u'(c_t)} \left\{ F_L(t+1) + \beta \frac{u'(c_{t+2})}{u'(c_{t+1})} \left[F_L(t+2) + p_{t+2} \right] \right\} \\ &= \sum_{j=0}^{\infty} F_L(t+j+1) \frac{\beta^{j+1} u'\left(F(N,L,A_{t+j+1})\right)}{u'\left(F(N,L,A_t)\right)} \qquad (5) \end{aligned}$$

- In deriving (5) I made use of the fact that $c_t = F(N,L,A_t)$ and that

$$\frac{u'(c_{t+1})}{u'(c_t)} \frac{u'(c_{t+2})}{u'(c_{t+1})} \dots \frac{u'(c_{t+1+T})}{u'(c_{t+T})} = \frac{u'(c_{t+1+T})}{u'(c_t)}$$

The price of land

The asset price equals the discounted present value of "divdends."

$$p_t = \sum_{j=0}^{\infty} F(t+j+1) \; MRS(t, t+j+1) \tag{6}$$

The discount factor is the Marginal Rate of Substitution

$$MRS(t, t+j) = \frac{\beta^j u'(t+j)}{u'(t)} \tag{7}$$

This is a fairly general result.

The price of land: Intuition

- Start from the equilibrium price.
- Add ε to the date $t+j$ payoff.
- The household gains $\beta^j u'(t+j)\varepsilon$.
- The household's willingness to pay for this: $u'(t)\varepsilon$.
- The derivative of the price:

$$\partial p_t / \partial F(t+j) = \frac{\beta^j u'(t+j)}{u'(t)} \tag{8}$$

The price of land: stationary economy

- We calculate the price of land for a stationary economy, in which $A_t = A^S$.

$$p_t^S = F_L(N, L, A^S)\,\frac{\beta}{1-\beta}$$

The price of land: fluctuating economy

We calculate p_t for an economy which is subject to deterministic fluctuations.

In even periods $A_t = A^H$

In odd periods $A_t = A^L \leq A^H$.

Simplifying assumptions:

- the marginal product of land is independent of A_t.
- $2F^S = F^H + F^L$, where $F^j = F(N, L; A^j)$.

The price of land: fluctuating economy

The trick is to break the sum in (5) into two parts, one for even and one for odd periods:

$$
\begin{aligned}
p_t^R &= \beta F_L \left\{ \sum_{j=0}^{\infty} \beta^{2j} \frac{u'(c_{t+1+2j})}{u'(c_t)} + \beta \sum_{j=0}^{\infty} \beta^{2j} \frac{u'(c_{t+2+2j})}{u'(c_t)} \right\} \\
&= \beta F_L \left\{ \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{1-\beta^2} + \frac{u'(c_{t+2})}{u'(c_t)} \frac{\beta}{1-\beta^2} \right\}
\end{aligned}
$$

Denote the marginal rate of substition between odd and even periods by

$$\alpha = u'(F^H)/u'(F^L) < 1$$

The price of land: fluctuating economy

If t is even, then

$$p_t^{even} = (1/\alpha + \beta)\, F_L \, \frac{\beta}{1-\beta^2}$$

If t is odd, then

$$p_t^{odd} = (\alpha + \beta)\, F_L \, \frac{\beta}{1-\beta^2}$$

Intuition:

- we get alternating payoffs that are either $(1/\alpha, 1)$ or $(\alpha, 1)$ times F_L
- discounting at β^2 produces the present value factor $1/\left(1-\beta^2\right)$

The price of land: fluctuating economy

Since $1-\beta^2 = (1+\beta)(1-\beta)$,

$$\begin{aligned} \frac{p_t^{even}}{p_t^S} &= \frac{1/\alpha+\beta}{1+\beta} > 1 \\ \frac{p_t^{odd}}{p_t^S} &= \frac{\alpha+\beta}{1+\beta} < 1 \end{aligned}$$

The price of the asset fluctuates around its price in the stationary economy.

The price of land: Intuition

Consider an even period.

- Times are good, so that saving is easy.
- And the return tomorrow is worth a lot because times will be bad.
- Hence, the demand for land is high and so is the price.

In odd periods, saving is painful and the return won't be worth much tomorrow. So the price is low.

Summary

The standard growth model is also the standard framework for pricing assets.

The price of an asset equals the present value of "dividends."

The discount factors are the Marginal Rates of Substitution.

This survives in stochastic environments. Just add $E[.]$.