Problem Set 4: Cash-in-Advance Model

Econ720. Fall 2023. Prof. Lutz Hendricks. November 2, 2023

1 Shopping time

Demographics: There is a single representative household who lives forever.

Preferences: The household values consumption (c) and leisure (l) according to

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t); \qquad 0 < \beta < 1.$$

Endowments: In each period, the agent is endowed with 1 unit of time that can be used for leisure (l), work (n), and shopping (s):

$$1 = l_t + n_t + s_t$$

The household is endowed with k_0 units of capital and M_0 units of money in period 0.

Technology: The transactions technology is such that s_t units of time are required to purchase c_t given money balances $m_t = M_t/P_t$:

$$s_t = g(c_t, m_t)$$

where P_t is the price of the good. Obviously, $g_c > 0$ and $g_m < 0$.

Goods are produced from capital and labor with the production function $f(k_t, n_t)$, which has nice properties. The resource constraint is $f(k,n) + (1-\delta)k = c + k'$.

Markets: The usual markets for goods, money, capital and labor rental operate. There is no government and the money supply is constant.

Questions:

1. Define a solution to the household problem.
2. Define a competitive equilibrium.
3. Is money neutral in this economy? Prove your answer using the system of equations that define a competitive equilibrium.
4. Would money still be neutral if the transactions technology used nominal money balances i.e., $s_t = g(c_t, M_t)$? Explain the intuition. You need not derive your answer.

2 Answer: Shopping time

1. The household's problem is:

$$\max_{c_t, l_t, n_t, s_t, k_t, m_{t-1}} \sum_{t=1}^{\infty} \beta u(c_t, l_t) \tag{1}$$

subject to:

$$\begin{aligned} s_t &= g(c_t, m_t) && (2) \\ l_t &= 1 - s_t - n_t && (3) \\ c_t + m_{t+1}(1+\pi_{t+1}) + k_{t+1} &= w_t n_t + (1+r_t)k_t + m_t && (4) \end{aligned}$$

where $1+\pi_{t+1} = P_{t+1}/P_t$.

After using the first two constraints to substitute l_t and s_t out of the problem we can write the household's problem as:

$$\begin{aligned} V(k,m) &= \max_{c,n,k',m'} u(c,1-n-g(c,m)) + \beta V(k',m') \\ &+ \lambda(wn + (1+r)k + m - c - m'(1+\pi') - k') \end{aligned} \tag{5}$$

FOCs

$$\begin{aligned} u_c - u_l g_c &= \lambda \\ u_l &= w\,\lambda \\ \beta\, V_k(.') &= \lambda \\ \beta\, V_m(.') &= \lambda\ (1+\pi') \end{aligned}$$

Interpretation:

1. A unit of income can be eaten, but only after spending g_c units of time on shopping.
2. Working one unit of time gives w units of income.
3. One unit of income today can be converted into one unit of capital tomorrow.
4. One unit of income today can be converted into $(1+\pi')^{-1}$ units of (real) money tomorrow (inflation tax).

Envelope:

$$\begin{aligned} V_k &= \lambda(1+r) \\ V_m &= -u_l\ g_m + \lambda \end{aligned}$$

Interpretation:

1. One unit of capital pays income $1+r$
2. One unit of (real) money gives income and conserves g_m units of shopping time.

Define the "total marginal utility from consumption" as

$$v(c,l,m) = u_c(c,l) - g_c(c,m)\ \ u_l(c,l) = \lambda$$

Taking the first order conditions, using the envelope theorem to substitute out $V_k(k',m')$ and $V_m(k',m')$, and manipulating gives:

$$v(c,l,m) = -u_l(c,l)/w \tag{6}$$

which defines the intratemporal trade-off between consumption and work. The Euler equation is

$$v(c,l,m) = \beta(1+r')\ v(c',l',m') \tag{7}$$

The allocation of assets is governed by

$$\begin{aligned} \frac{V_k(.')}{V_m(.')} &= \frac{1}{1+\pi'} = \frac{\lambda'\ \ (1+r')}{\lambda' - u_l(.')\ g_m(.')} \\ 1 - \frac{u_l\ g_m}{v} &= (1+r)\ \ (1+\pi) \\ (1+r)(1+\pi) - 1 &= -\frac{u_l\ g_m}{v} > 0 \end{aligned}$$

Interpretation: If $u_l = 0$ or $g_m = 0$, money and capital must pay the same return. But if $u_l g_m > 0$, money has liquidity value and therefore pays lower return than capital.

A solution to the household problem in sequence form: $\{c_t, m_t, l_t, n_t, s_t, k_t\}$ that satisfy:

- Euler equation and 2 static first-order conditions.
- $s = g(c, m)$.
- Budget constraints.
- Time constraint.
- Transversality. k_0, m_0 given.

2. An equilibrium is a set of sequences of $\{c_t, l_t, s_t, n_t, m_t, k_t, r_t, w_t, \pi_t\}$ such that:

- Household solves its problem (see above).
- Firms choose $\{k_t, n_t\}$ to maximize profits, taking $\{r_t, w_t\}$ as given:

$$f_k - \delta = r_t \qquad \text{and} \qquad f_l = w_t \tag{8}$$

- Markets clear:
  1. Goods: $f(k, n) + (1 - \delta) k = c + k'$.
  2. Capital, labor, money: implicit.
  3. Constant nominal money supply implies $m_{t+1}/m_t = 1/(1 + \pi_{t+1})$.

3. Money is neutral in this economy. A change in the level of the money supply (in all periods) causes a proportional increase in all nominal prices but leaves the equilibrium values of real variables unaffected. We can see this by observing that no nominal variables appear in the equilibrium conditions.

4. Money is not neutral. Intuition: Think about what happens when M and P double in every period. This could not be an equilibrium because the household not needs less time for shopping. Increasing M makes shopping time more "productive."