Wealth Distribution: Motivation and Baseline Model

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Econ720

November 21, 2017

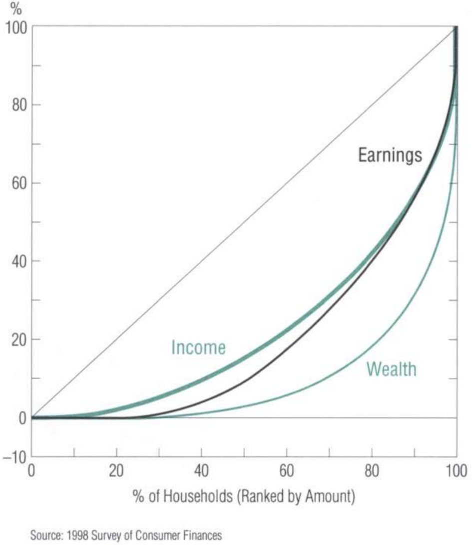
The Question

- We study heterogeneous agent models
- Theoretical objective: learn how to define equilibrium
    - also think a bit about computing equilibrium
- Applications to wealth distribution and earnings distribution

Data: U.S. Wealth Distribution

- Top 1% hold 28% of total wealth
- Top 5% hold half of total wealth
- Bottom 40% hold essentially nothing
- Gini: 0.72

Lorenz curves



Source: Rodríguez et al. (2002)

Baseline model: Huggett (1996)

- We start with a classic paper: Huggett (1996)
- The question
  - to what extent can a standard life-cycle model with idiosyncratic earnings risk account for the observed concentration of wealth?
- Model ingredients:
  - uninsured shocks
  - finite lives
  - ex ante identical agents (Bewley model)

Demographics / Preferences

- Demographics
  - in each period 1 unit mass of agents are born
  - they live at most N periods
  - exogenous survival probabilities s_j
- Preferences

$$\mathbb{E}\sum_{t=1}^{N}\beta^t\left(\prod_{j=1}^{t}s_j\right)u(c_t) \tag{1}$$

Endowments / Technologies

- Endowments
  - an agent of age t is endowed with $e(z,t)$ units of work time
  - z is a Markov productivity shock
- Technology

$$Y = AK^{\alpha}L^{1-\alpha} \tag{2}$$

Markets

- labor rental (wage w)
- capital rental (interest rate r)
- good (price 1)
- risk free bonds (interest rate r)

Government

- taxes income rate rate τ
- social security tax θ pays old age transfers b
- lump-sum transfers T redistribute accidental bequests

Household Problem

Individual state: $x = (a, z)$

Bellman

$$V(x,t) = \max_{c,a'} u(c) + \beta s_{t+1} \mathbb{E} V(a', z', t+1) \tag{3}$$

subject to

$$c + a' = a(1 + r[1-\tau]) + (1-\theta-\tau)e(z,t)w + T + b_t \tag{4}$$

$$a \geq \underline{\mathsf{a}} \tag{5}$$

Terminal value: $V(x, N+1) = 0$

Equilibrium

Focus on stationary equilibria.

Aggregate state:

- joint distribution of (a, z) for each age t
- density for age t: $\psi_t(B)$ where B is a set of states

Transition function: $P(x, t, B) = \Pr(x' \in B | x, t)$.

Stationarity condition

Stationarity of distribution requires

$$\psi_t(B) = \int_X P(x, t-1, B) d\psi_{t-1}(x) \tag{6}$$

In words:

- today's distribution for age $t-1$ is ψ_{t-1}
- agents make choices that induce transitions described by P
- then tomorrow's distribution for age t is ψ_t (for the same ψ)

Stationary Equlibrium

Objects:

- household: $c(x,t), a(x,t), V(x,t)$
- prices: r, w
- policies: τ, θ, b_t, T, G
- aggregates: K, L

All of these are functions of the aggregate state, but that is a constant, so we don't need to worry about it.

Equilibrium conditions

- households "maximize"
- firm first-order conditions
- government budget constraint

$$G = \tau(rK + wL) \tag{7}$$

- social security budget constraint

$$\theta wL = b \sum_{t=R}^{N} \mu_t \tag{8}$$

- market clearing
- stationarity of the aggregate state

Market Clearing

Goods

$$F(K,L)+(1-\delta)K = G+\sum_t \mu_t \int_X [c(x,t)+a(x,t)]d\psi_t(x) \qquad (9)$$

Capital

$$K = \sum_t \mu_t \int_X a(x,t)d\psi_t(x) \qquad (10)$$

Labor

$$L = \sum_t \mu_t \int_X e(z,t)d\psi_t(x) \qquad (11)$$

Perspective

Why did Huggett choose this model?

He was aiming for the simplest, most standard model as a benchmark.

The goal is not to fit the empirical distribution, but starting to understand what it might take to fit it.

Key ingredients of the model:

- finite lives: because a chunk of wealth heterogeneity comes from cross-age variation
- a single source of heterogeneity: earnings shocks (clearly important)

Quantitative Implications

Methods

How to quantify the model's implications?

- set model parameters
- simulate many households
- compute statistics from simulated histories (wealth distribution, ...)

Setting model parameters: 2 approaches

1. calibration
2. estimation

Estimation

Roughly speaking:

- add “error terms” to the model equations
- add covariates to the model equations (e.g. utility depends on family size, marital status, ...)
- simulate households observed in the data (with their covariates)
- search over model parameters that optimize the “fit” of the model somehow

Example: MLE

- maximize the likelihood of the error terms

Calibration

Set some parameters based on outside evidence

- e.g. capital share in production function = 1/3
- tax rates
- stochastic process for earnings

The remaining parameters will be "calibrated"

Set calibration targets

- data moments that seem informative about the calibrated parameters
- e.g.: discount factor affects K/Y
- should not include wealth distribution statistics

Calibration

Simlulate many households (no covariates)

Choose the calibrated parameters to match targets

Simplest case: exactly identified

- the number of calibrated parameters matches the number of moments
- the model matches the moments exactly

More common these days: overidentified

- number of targets > number of calibrated parameters
- the minimize a distance between data moments and model moments

Which Approach Is Better?

Researchers disagree.

Benefits of estimation:

1. discipline (but perhaps more illusion than reality)
2. standard errors

Benefits of calibration:

1. can target moments that matter
2. less expensive
3. more transparent

Methods such as indirect inference and simulated method of moments blur the distinction between estimation and calibration.

Huggett's calibration

Fixed based on outside evidence:

- preference parameters
- technology parameters
- demographics
- taxes
- labor endowment process (some parameters)

Calibrated:

- $Var(y_1)$ and persistence of labor endowment process
- targets: Gini of earnings for young workers and overall

Main Result

Fraction held by top	1%	5%	20%	Gini	% neg. wealth
Huggett (1996)	10.8	32.4	68.9	0.70	19%
U.S. data	**34.7**	57.8	81.7	0.80	11%

The model has too many households without wealth.

Still, wealth inequality is lower than in the data.

Models of this kind fail to account for wealth concentration at the top

The paper spawned a large literature that tries to generate enough rich households.

What Goes Wrong?

1. The rich do not have an **incentive to save**
   Possible solutions: entrepreneurship, bequests
   Quadrini (1999), Cagetti and Nardi (2006)
2. The only **source of income** is earnings
   The rich don't earn enough to accumulate as much wealth as in the data
   Possible solutions: entrepreneurship, bequests
3. Earnings and wealth are too highly **correlated**
   Hendricks (2007)

References I

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