Wealth Distribution: Stochastic Aging

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Topics

We have seen that the stochastic life-cycle model goes a long way towards accounting for U.S. wealth inequality.

But fails to account for the concentration of wealth within the top 5% or 1% of the population.

We study two candidate solutions (in one paper)

1. bequests (see also Nardi 2015; De Nardi and Yang 2014)
2. alternative labor earning processes

Stochastic aging

A computational problem: the curse of dimensionality.

The household problem must be solved for all possible combinations of states.

Approximation: put states on a grid.

With many states, the grid gets very large.

Stochastic aging collapses the age dimension into a few phases (e.g. work and retirement)

Key reference: Castaneda et al. (2003)

Castaneda et al. (2003): Model

Main innovations relative to Huggett (1996):

- Households are *altruistic* (additional source of wealth and motive for saving).
- *Earnings process* is chosen to match SCF data on earnings and wealth inequality.
- *Social Security* system modeled in more detail (to give high retirement incomes to low earnings households; helps account for low wealth observations).
- *Progressive* income tax system (found important for wealth distribution).
- *Stochastic aging*.

Main finding: The model accounts for distribution of earnings and wealth.

Environment

There is a continuum of families.

Each family consists of non-overlapping individuals.

In each period, a person:

- draws a stochastic labor endowment e,
- chooses consumption and saving,
- retires with some probability,
- dies with some probability.

New individuals inherit assets and labor endowments from their parents.

Household problem

State variables:

- "age": working or retired (there is no symbol for age).
- labor endowment e.
- wealth a.

The exogenous states are collected in $s = (age, e)$.

s_t evolves according to a transition matrix Γ.

Household problem

$$\max E\left\{\sum_{t=0}^{\infty}\beta^t u(c_t,\ell-l_t)\,|s_0\right\}$$

subject to the budget constraint

$$c+z=y-\tau(y)+a \tag{1}$$

$$y=a\,r+w\,e_s\,l(s,a)+\omega(s) \tag{2}$$

$$a'(z)=\{\begin{array}{c} z\,\text{if survive} \\ (1-\tau_E(z))\,z\ \text{if death}\end{array} \tag{3}$$

Remarks

Households are modeled as infinitely lived.

- This is a reduced form for a sequence of non-overlapping individuals linked by altruistic bequests.
- There is no separate age state variable.

Labor endowments are drawn from $S = \varepsilon \cup \Re$.

- $e \in \varepsilon$ means "working".
- $e \in \Re$ means retired.

Stochastic aging

Individuals are born as working ($e \in \varepsilon, \omega = 0$).

In each period, they draw a new e.

If $e \in \Re$, the household retired.

If retired and household draws $e \in \varepsilon$, he dies and is replaced by a child.

Benefits:

- Small state vector: (s, a).
- Value function must be computed for only 2 "ages"

Drawbacks:

- Some households have very long or short working lives.
- Hard to match life-cycle features (age-earnings profile, mortality rates)

Dynamic program

$$v(s,a) = \max \ u(c, \ell - l) + \beta \sum_{s' \in S} \Gamma_{ss'} \ v\left(s', a'(z)\right) \tag{4}$$

$$c + z = y - \tau(y) + a \tag{5}$$

$$y = a \ r + e(s) \, lw + \omega(s) \tag{6}$$

$$a'(z) = \{ \begin{array}{c} (1 - \tau_E(z)) \, z \text{ if } s \in \Re \text{ and } s' \in \varepsilon \\ z \text{ otherwise} \end{array} \tag{7}$$

Other model agents

Firms maximize period profits.

- Production technology is $F(K,L)$.

Government

- Taxes bequests at rate $\tau_E(z)$, where z is the bequest amount.
- Taxes income at rate $\tau(y)$.
- Provides retirement transfers to households.
- Balances the budget in each period: $G_t + Tr_t = T_t$.

Steady state

Objects:

- Policy functions: $c(s,a), z(s,a), l(s,a)$.
- Government policies: $\tau(y), \tau_E(z), \omega(s), G$.
- A stationary probability distribution over household types: x.
- Aggregate quantities: K, L, T, Tr.

Steady state

These satisfy:

- Policy functions are optimal decision rules.
- Factor market clearing: $K = \int a \, dx, \;\; L = \int e(s) \; l(s,a) \; dx$.
- Goods market clearing:
  $F(K,L) + (1-\delta) \; K = G + \int [c(s,a) + z(s,a)] \; dx$.
- Firm's first-order conditions.
- Government budget constraints.
- Measure of households is stationary.

Calibration

(We omit details)

Income and estate tax schedule mimick U.S. progressive tax system.

Labor endowments are drawn from a Markov chain

Transition matrix matches:

- points on the Lorenz curves for earnings and wealth ($\Gamma_{\varepsilon\varepsilon}, e(s)$).
- intergenerational persistence of labor endowments ($\Gamma_{\Re\varepsilon}$).
- length of working lives ($p_{\varepsilon,\rho}$).
- life expectancy ($p_{\rho,\rho}$).

Total number of parameters: **39** (unusually large [for macro])

Other calibration targets

Various features of U.S. tax schedules.

Aggregate ratios: $K/Y, I/Y, G/Y, Tr/Y, l/\ell$

Ratio of standard deviations for c and l.

Average length of work life: 45 years.

Average length of retirement: 18 years.

Average earnings middle age / young: 1.3

Intergenerational correlation of log lifetime earnings: 0.4

Results

Model economy matches cross-sectional earnings distribution very well.

Wealth distribution match is good, not perfect.

TABLE 7

DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

		Quintile					Top Groups (Percentile)		
Economy	Gini	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
				A. Distributions of Earnings					
United States	.63	−.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	.63	.00	3.74	14.59	15.99	65.68	15.15	17.65	14.93
				B. Distributions of Wealth					
United States	.78	−.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	.79	.21	1.21	1.93	14.68	81.97	16.97	18.21	29.85

Assessment

The model successfully replicates the cross-sectional distribution of wealth.

No departure from standard theory is needed.

Key features for the model's success:

- Intended bequests permit households to accumulate wealth over longer time periods.
- Earnings process consistent with cross-sectional SCF data.

Earnings process

Calibration does not use information on persistence of earnings.

The earnings process is "cooked" to match the wealth distribution.

The lower 3 earnings states "look like" something estimated from the PSID (though persistence is very high).

The top earnings state is totally transitory.

TABLE 5
RELATIVE ENDOWMENTS OF EFFICIENCY LABOR UNITS, $e(s)$, AND THE STATIONARY DISTRIBUTION OF WORKING-AGE HOUSEHOLDS, $\boldsymbol{\gamma}^*_\varepsilon$

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.15	9.78	1,061.00
$\boldsymbol{\gamma}^*_\varepsilon$ (%)	61.11	22.35	16.50	.0389

Earnings process

The top earnings level is very large

TABLE 4

TRANSITION PROBABILITIES OF THE PROCESS ON THE ENDOWMENT OF EFFICIENCY LABOR UNITS FOR WORKING-AGE HOUSEHOLDS THAT REMAIN AT WORKING AGE ONE PERIOD LATER, $\mathbf{\Gamma}_{\varepsilon\varepsilon}$ (%)

	To s'			
From s	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.24	1.14	.39	.006
$s = 2$	3.07	94.33	.37	.000
$s = 3$	1.50	.43	95.82	.020
$s = 4$	10.66	.49	6.11	80.51

Intuition:

- households win the lottery once every 25 years
- lottery winners save everything because the top state is so transitory

Reservations

The paper shoes that it is **possible** to write down a standard life-cycle model that matches wealth concentration based on an earnings process with the right amount of cross-sectional inequality.

It does not show that a life-cycle model generates the right wealth distribution when a "**realistic**" earnings process is imposed.

Could one fix this?

- why not combine info on the process for the bottom 99% from the PSID with info for the cross-sectional distribution for everyone from SCF?
- one solution: Nardi et al. (2016)

References I

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