

# Wealth Distribution: Stochastic Aging and Entrepreneurship

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# Topics

We have seen that the stochastic life-cycle model goes a long way towards accounting for U.S. wealth inequality.

But fails to account for the concentration of wealth within the top 5% or 1% of the population.

We study two candidate solutions (in one paper)

1. bequests
2. alternative labor earning processes

## Stochastic aging

A computational problem: the curse of dimensionality.

The household problem must be solved for all possible combinations of states.

Approximation: put states on a grid.

With many states, the grid gets very large.

Stochastic aging collapses the age dimension into a few phases (e.g. work and retirement)

Key reference: Castaneda et al. (2003)

## Castaneda et al. (2003): Model

### Main innovations relative to Huggett (1996):

- ▶ Households are *altruistic* (additional source of wealth and motive for saving).
- ▶ *Earnings process* is chosen to match SCF data on earnings and wealth inequality.
- ▶ *Social Security* system modeled in more detail (to give high retirement incomes to low earnings households; helps account for low wealth observations).
- ▶ *Progressive* income tax system (found important for wealth distribution).
- ▶ *Stochastic aging*.

**Main finding:** The model accounts for distribution of earnings and wealth.

# Environment

There is a continuum of families.

Each family consists of non-overlapping individuals.

In each period, a person:

- ▶ draws a stochastic labor endowment  $e$ ,
- ▶ chooses consumption and saving,
- ▶ retires with some probability,
- ▶ dies with some probability.

New individuals inherit assets and labor endowments from their parents.

# Household problem

## State variables:

- ▶ "age": working or retired (there is no symbol for age).
- ▶ labor endowment  $e$ .
- ▶ wealth  $a$ .

The exogenous states are collected in  $s = (age, e)$ .

$s_t$  evolves according to a transition matrix  $\Gamma$ .

## Household problem

$$\max E \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell - l_t) \mid s_0 \right\}$$

subject to the budget constraint

$$c + z = y - \tau(y) + a \quad (1)$$

$$y = ar + we_s l(s, a) + \omega(s) \quad (2)$$

$$a'(z) = \begin{cases} z & \text{if survive} \\ (1 - \tau_E(z)) z & \text{if death} \end{cases} \quad (3)$$

## Remarks

Households are modeled as infinitely lived.

- ▶ This is a reduced form for a sequence of non-overlapping individuals linked by altruistic bequests.
- ▶ There is no separate age state variable.

Labor endowments are drawn from  $S = \varepsilon \cup \mathfrak{R}$ .

- ▶  $e \in \varepsilon$  means "working".
- ▶  $e \in \mathfrak{R}$  means retired.



# Stochastic aging

Individuals are born as working ( $e \in \mathcal{E}, \omega = 0$ ).

In each period, they draw a new  $e$ .

If  $e \in \mathcal{R}$ , the household retired.

If retired and household draws  $e \in \mathcal{E}$ , he dies and is replaced by a child.

Benefits:

- ▶ Small state vector:  $(s, a)$ .
- ▶ Value function must be computed for only 2 "ages"

Drawbacks:

- ▶ Some households have very long or short working lives.
- ▶ Hard to match life-cycle features (age-earnings profile, mortality rates)

## Dynamic program

$$v(s, a) = \max u(c, \ell - l) + \beta \sum_{s' \in \mathcal{S}} \Gamma_{ss'} v(s', a'(z)) \quad (4)$$

$$c + z = y - \tau(y) + a \quad (5)$$

$$y = a r + e(s) l w + \omega(s) \quad (6)$$

$$a'(z) = \begin{cases} (1 - \tau_E(z)) z & \text{if } s \in \mathfrak{R} \text{ and } s' \in \mathcal{E} \\ z & \text{otherwise} \end{cases} \quad (7)$$

## Other model agents

**Firms** maximize period profits.

- ▶ Production technology is  $F(K, L)$ .

**Government**

- ▶ Taxes bequests at rate  $\tau_E(z)$ , where  $z$  is the bequest amount.
- ▶ Taxes income at rate  $\tau(y)$ .
- ▶ Provides retirement transfers to households.
- ▶ Balances the budget in each period:  $G_t + Tr_t = T_t$ .

# Steady state

Objects:

- ▶ Policy functions:  $c(s, a), z(s, a), l(s, a)$ .
- ▶ Government policies:  $\tau(y), \tau_E(z), \omega(s), G$ .
- ▶ A stationary probability distribution over household types:  $x$ .
- ▶ Aggregate quantities:  $K, L, T, Tr$ .

## Steady state

These satisfy:

- ▶ Policy functions are optimal decision rules.
- ▶ Factor market clearing:  $K = \int a \, dx$ ,  $L = \int e(s) l(s, a) \, dx$ .
- ▶ Goods market clearing:  
 $F(K, L) + (1 - \delta) K = G + \int [c(s, a) + z(s, a)] \, dx$ .
- ▶ Firm's first-order conditions.
- ▶ Government budget constraints.
- ▶ Measure of households is stationary.

# Calibration

(We omit details)

Income and estate tax schedule mimic U.S. progressive tax system. Labor endowments are drawn from a Markov chain

Transition matrix matches:

- ▶ points on the Lorenz curves for earnings and wealth ( $\Gamma_{\varepsilon\varepsilon}, e(s)$ ).
- ▶ intergenerational persistence of labor endowments ( $\Gamma_{\mathcal{R}\varepsilon}$ ).
- ▶ length of working lives ( $p_{\varepsilon,\rho}$ ).
- ▶ life expectancy ( $p_{\rho,\rho}$ ).

Total number of parameters: **39** (unusually large [for macro])

## Other calibration targets

Various features of U.S. tax schedules.

Aggregate ratios:  $K/Y, I/Y, G/Y, Tr/Y, l/\ell$

Ratio of standard deviations for  $c$  and  $l$ .

Average length of work life: 45 years.

Average length of retirement: 18 years.

Average earnings middle age / young: 1.3

Intergenerational correlation of log lifetime earnings: 0.4

# Results

Model economy matches cross-sectional earnings distribution very well.

Wealth distribution match is good, not perfect.

TABLE 7  
DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN THE  
BENCHMARK MODEL ECONOMIES (%)

ECONOMY	GINI	QUINTILE					TOP GROUPS (Percentile)		
		First	Second	Third	Fourth	Fifth	90th– 95th	95th– 99th	99th– 100th
A. Distributions of Earnings									
United States	.63	-.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	.63	.00	3.74	14.59	15.99	65.68	15.15	17.65	14.93
B. Distributions of Wealth									
United States	.78	-.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	.79	.21	1.21	1.93	14.68	81.97	16.97	18.21	29.85



# Assessment

The model successfully replicates the cross-sectional distribution of wealth.

No departure from standard theory is needed.

Key features for the model's success:

- ▶ Intended bequests permit households to accumulate wealth over longer time periods.
- ▶ Earnings process consistent with cross-sectional SCF data.

## Earnings process

Calibration does not use information on persistence of earnings.

The earnings process is "cooked" to match the wealth distribution.

The lower 3 earnings states "look like" something estimated from the PSID (though persistence is very high).

The top earnings state is totally transitory.

TABLE 5  
RELATIVE ENDOWMENTS OF EFFICIENCY LABOR UNITS,  $e(s)$ , AND THE  
STATIONARY DISTRIBUTION OF WORKING-AGE HOUSEHOLDS,  $\gamma_\epsilon^*$

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.15	9.78	1,061.00
$\gamma_\epsilon^*$ (%)	61.11	22.35	16.50	.0389

# Earnings process

The top earnings level is very large

TABLE 4  
TRANSITION PROBABILITIES OF THE PROCESS ON THE ENDOWMENT OF EFFICIENCY LABOR  
UNITS FOR WORKING-AGE HOUSEHOLDS THAT REMAIN AT WORKING AGE ONE PERIOD  
LATER,  $\Gamma_{\epsilon\epsilon}$  (%)

FROM $s$	To $s'$			
	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.24	1.14	.39	.006
$s = 2$	3.07	94.33	.37	.000
$s = 3$	1.50	.43	95.82	.020
$s = 4$	10.66	.49	6.11	80.51

Intuition:

- ▶ households win the lottery once every 25 years
- ▶ lottery winners save everything because the top state is so transitory

## Reservations

The paper shows that it is **possible** to write down a standard life-cycle model that matches wealth concentration based on an earnings process with the right amount of cross-sectional inequality.

It does not show that a life-cycle model generates the right wealth distribution when a "**realistic**" earnings process is imposed.

Could one fix this?

- ▶ why not combine info on the process for the bottom 99% from the PSID with info for the cross-sectional distribution for everyone from SCF?
- ▶ one solution: Nardi et al. (2016)

## References I

- Castaneda, A., J. Diaz-Gimenez, and J. V. Rios-Rull (2003): “Accounting for the US earnings and wealth inequality,” *Journal of political economy*, 111, 818–857.
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- Nardi, M. D., G. Fella, and G. P. Pardo (2016): “The Implications of Richer Earnings Dynamics for Consumption, Wealth, and Welfare,” Working Paper 21917, National Bureau of Economic Research.