

# Growth Through Product Creation

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# Issues

- ▶ We study a GE model of growth driven by innovation.
- ▶ Innovation takes the form of inventing new goods.
- ▶ Alternative: Quality ladders.

# A Model of Product Innovation

- ▶ A representative household supplies capital and labor to firms
- ▶ Final goods firms use labor and intermediate inputs
- ▶ Intermediate inputs are produced from final goods
- ▶ Final goods can be used to create new varieties of intermediate inputs
- ▶ Innovators receive permanent monopolies

Note: Now that models get more complicated, it really pays off to be pedantic about details.

# Demographics and Preferences

Demographics:

- ▶ A representative household.

Preferences:

$$\int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt \quad (1)$$

- ▶  $C$ : the final good

## Technology: Final Goods

Resource constraint:

$$C_t + X_t + Z_t = Y_t$$

Final goods  $Y$  are used for

- ▶  $Z$ : R&D investment.
- ▶  $X$ : Inputs into the production of intermediates  $x$ .
- ▶  $C$ : consumption

## Technology: Final Goods

Production of **final goods** from intermediates and labor:

$$Y_t = (1 - \beta)^{-1} \left[ \int_0^{N_t} x(v, t)^{1-\beta} dv \right] L^\beta \quad (2)$$

This is of the Dixit-Stiglitz form:

Write  $\left[ \int x^{1-\beta} dv \right]^{\frac{1-\beta}{1-\beta}}$  to see that this is a CES aggregator of  $x$ .

## Technology: Intermediate Inputs

Each unit of  $x$  requires  $\psi$  units of  $Y$ :

$$X = \psi \int_0^{N_t} x(v, t) dv \quad (3)$$

Intermediate inputs fully depreciate in use.

## Technology: Innovation

Investing the final good yields a flow of new patents:

$$\dot{N} = \eta Z_t \quad (4)$$

Think of this as the aggregate (deterministic) outcome of the (stochastic) innovation efforts of many firms.



# Market arrangements

- ▶ Final goods and labor markets are competitive.
- ▶ Intermediates are sold by **monopolists** (the innovators).
  - ▶ Monopolies are permanent.
  - ▶ What the monopolists do with their profits is not clear.
- ▶ Free entry into innovation
  - ▶ ensures zero present value of profits
- ▶ The household owns the innovating firms.
- ▶ Asset markets are complicated
  - ▶ there is often no need to spell out the details

# Notes

Production is cyclical:

- ▶ today's  $Y$  is used to make  $X$  which makes  $Y$
- ▶ the alternative: durable  $X$  (more complicated)
- ▶ implication: the efficient allocation maximizes  $Y - X = C + Z$

The only long-lived object is a patent

- ▶ this keeps the model simple

Assuming that intermediates are made from final goods fixes marginal costs (and prices)

Solving Each Agent's Problem

## Final goods producers

- ▶ Maximize period profits by choosing  $L$  and  $x(v, t)$ .
- ▶ Normalize the price  $Y$  to 1.
- ▶ Profits

$$Y_t - w_t L_t - \int_0^{N_t} p^x(v, t) x(v, t) dv \quad (5)$$

where

$$Y_t = (1 - \beta)^{-1} \left[ \int_0^{N_t} x(v, t)^{1-\beta} dv \right] L^\beta \quad (6)$$

## Final goods producers

FOCs:

- ▶  $\partial Y / \partial x(v) = L^\beta x(v)^{-\beta} = p^x(v)$
- ▶  $\partial Y / \partial L = \beta Y / L = w$

Demand function (cf. the Dixit Stiglitz discussion):

$$x(v, t) = L p^x(v, t)^{-1/\beta} \quad (7)$$

**Solution** to the firm's problem:  $L_t, x(v, t)$  that satisfy the "2" first-order conditions.

## Intermediate input producers

Problem after inventing a variety.

$x$  is produced at constant marginal cost  $\psi$ .

Maximize present value of profits

$$V(v, t) = \int_t^{\infty} e^{-rs} \pi(v, s) ds \quad (8)$$

Instantaneous profits are

$$\pi(v, t) = (p^x(v, t) - \psi) x(v, t) \quad (9)$$

where  $x(v, t) = Lp^x(v, t)^{-1/\beta}$

This is a sequence of static problems

## Intermediate input producers

- ▶ First order condition (standard monopoly pricing formula):

$$p^x = \psi / (1 - \beta) \quad (10)$$

- ▶ Profits are

$$\pi(v, t) = \psi \frac{\beta}{1 - \beta} x(v, t) \quad (11)$$

- ▶ Solution: A constant  $p^x$ .

# Household

- ▶ The household holds shares of all intermediate input firms.
- ▶ Each firm produces a stream of profits.
- ▶ New firms issue new shares.
- ▶ But: the details don't matter to the household.
- ▶ There simply is an asset with rate of return  $r$ .
- ▶ Euler equation is standard:

$$g(C) = \frac{r - \rho}{\theta} \quad (12)$$

- ▶ Invoke Walras' law - so you never have to write down the budget constraint!



# Equilibrium

- ▶ Objects:  $C_t, X_t, Z_t, x(v, t), V(v, t), N_t$  and prices  $p^x(v, t), r(t), w(t)$ .
- ▶ Conditions:
  - ▶ "Everybody maximizes." (see above)
  - ▶ Markets clear.
    1. Goods: resource constraint.
    2. Shares: omitted b/c I did not write out the household budget constraint.
    3. Intermediates: implicit in notation.
  - ▶ Innovation effort satisfies a **free entry** condition: present value of profits equals 0.

# Symmetric Equilibrium

We assume (and then show) that all varieties  $v$  share the same  $x, V$  and  $p^x$ .

## Intuition:

- ▶  $p^x$ : monopoly pricing with a constant elasticity
- ▶  $x$ : varieties enter final goods production symmetrically
- ▶  $V$ : the age of a variety does not matter  
(no stock of  $x$  to build; permanent patents)

# Equilibrium: Characterization

The growth rate follows from the Euler equation:

$$g(C) = (r - \rho) / \theta.$$

- ▶ We need to solve for  $r$ .

## Simplifications:

- ▶ Normalize marginal cost  $\psi = 1 - \beta$ 
  - ▶ so that  $p^x = 1$ .
  - ▶ Why can I do that?
- ▶ Focus on balanced growth paths.

## Equilibrium: Characterization

Free entry will determine the interest rate

Spend 1 to obtain  $\eta$  new patents, each valued (initially) at  $V(v, t)$

$$\eta V(v, t) = 1 \quad (13)$$

- ▶ Then  $V$  is constant over time.
- ▶ This assumes that innovation takes place.

With balanced growth and constant profits (to be shown):

$$V = \pi/r \quad (14)$$

# Profits

With a fixed markup, profits are a multiple of revenues:

$$\pi(t) = \psi \frac{\beta}{1-\beta} x(t) \quad (15)$$

$$= \beta x(t) \quad (16)$$

Demand for intermediates:

$$\begin{aligned} x(t) &= L p^x(t)^{-1/\beta} \\ &= L \end{aligned}$$

Profits:  $\pi = \beta L$ .

## Free Entry

- ▶ Free entry:

$$\eta V = \eta \beta L / r = 1 \quad (17)$$

- ▶ This is the closed form solution for  $r$ .
- ▶ Balanced **growth** rate then follows from the Euler equation.

$$g(C) = \frac{\eta \beta L - \rho}{\theta} \quad (18)$$

## Equilibrium: Characterization

Production function for final goods with  $x = L$ :

$$Y = \frac{N_t L}{1 - \beta} \quad (19)$$

Wage (from firm's FOC):

$$w_t = \beta \frac{Y_t}{L_t} = \frac{\beta}{1 - \beta} N_t \quad (20)$$

Total expenditure on intermediates:

$$X_t = \psi N_t x_t = (1 - \beta) N_t L \quad (21)$$

# Summary of Equilibrium

Prices and quantities of intermediate inputs are constant.

- ▶ the model is rigged to deliver this
- ▶ for tractability

Growth comes from rising  $N$



# No Transition Dynamics

The equilibrium looks like an AK model with production function

$$Y_t = \frac{L}{1-\beta} N_t$$
$$\dot{N}_t = \eta s_z Y_t$$

Intuition:

- ▶ Period profits  $\pi$  are constant at  $\beta L$ .
- ▶ At any moment we need  $\eta V = 1$ .
- ▶  $V$  is the present value of (constant) profits.
- ▶ Constant  $V$  is only possible with constant  $r$ .
- ▶ Intuition: There is a reduced form AK structure.

## Scale Effects

$$g(C) = \frac{\eta\beta L - \rho}{\theta}$$

Larger economies ( $L$ ) grow faster.

Population growth implies exploding income growth (!)

Mechanical reason:

- ▶ Innovation technology is linear in goods.
- ▶ Larger economy  $\rightarrow$  higher  $Y \rightarrow$  higher  $Z \rightarrow$  faster growth.

We will return to this later.

# Pareto Efficient Allocation

# Efficiency

Two distortions prevent efficiency of equilibrium:

1. Monopoly pricing.
2. Inefficient innovation due to aggregate demand externality.

# Planner's Problem

Solve in two stages:

1. Given  $N$ , find optimal static allocation  $x(v, t)$ .
  - ▶ That is: maximize  $Y - X$  which is available for consumption and investment.
  - ▶ An odd feature of the model: goods are produced from goods without delay.
2. Given the reduced form production function from #1, find optimal  $Z$ .

## Static Allocation

Given  $N$ , choose  $x(v, t)$  to maximize  $Y - X$ :

$$\max (1 - \beta)^{-1} L^\beta \int_0^{N_t} x(v, t)^{1-\beta} dv - \int_0^{N_t} \psi x(v, t) dv \quad (22)$$

First-order condition

$$L^\beta x^{-\beta} = \psi \quad (23)$$

with  $\psi = 1 - \beta$ :

$$x = (1 - \beta)^{-1/\beta} L \quad (24)$$

The planner's  $x$  is larger than the equilibrium  $x$  (Intuition?)

## Static Allocation

Next: find  $Y - X$ .

$$X = \psi N x = (1 - \beta) N (1 - \beta)^{-1/\beta} L \quad (25)$$

Reduced form production function:

$$Y_t = (1 - \beta)^{-1} L^\beta N [(1 - \beta)^{1-1/\beta} L]^{1-\beta} \quad (26)$$

$$= (1 - \beta)^{-1/\beta} L N_t \quad (27)$$

Net output

$$\begin{aligned} Y - X &= (1 - \beta)^{-1/\beta} L N - (1 - \beta)^{1-1/\beta} L N \\ &= (1 - \beta)^{-1/\beta} \beta L N \end{aligned} \quad (28)$$

## Planner: Dynamic Optimization

$$\max \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

subject to

$$\dot{N} = \eta Z$$

$$Y = (1-\beta)^{-1/\beta} \beta L N = C + Z$$

Or

$$\dot{N} = A N - \eta C \tag{29}$$

$$A = \eta (1-\beta)^{-1/\beta} \beta L \tag{30}$$



# Hamiltonian

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \mu [AN - \eta C] \quad (31)$$

FOC

$$\partial H / \partial C = C^{-\theta} - \mu \eta = 0 \quad (32)$$

$$\partial H / \partial N = \rho \mu - \dot{\mu} = \mu A \quad (33)$$

## Optimal growth

The same as in an AK model with

$$A = \eta (1 - \beta)^{-1/\beta} \beta L \quad (34)$$

we have

$$\dot{C}/C = \frac{A - \rho}{\theta} \quad (35)$$

## Comparison with CE

- ▶ CE interest rate:  $\eta\beta L$ .
- ▶ Planner's "interest rate:"  $(1 - \beta)^{-1/\beta} \eta\beta L$ .
- ▶ The planner chooses faster growth.
- ▶ Intuition:
  - ▶ CE under-utilizes the fruits of innovation:  $x$  is too low.
  - ▶ This reduces the value of innovation.

## Policy Implications

- ▶ One might be tempted to reduce monopoly power.
- ▶ A policy that encourages competition (e.g. less patent protection, forcing lower  $p^x$ ) reduces the static price distortion.
- ▶ But it also reduces growth: innovation is less valuable.
- ▶ Similar result for shorter patents.
- ▶ Policy trades off static efficiency and incentives for innovation.

# Reading

- ▶ Acemoglu (2009), ch. 13.
- ▶ Krusell (2014), ch. 9
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)

## References I

- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Jones, C. I. (2005): “Growth and ideas,” *Handbook of economic growth*, 1, 1063–1111.
- Krusell, P. (2014): “Real Macroeconomic Theory,” Unpublished.
- Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.