Growth Through Product Creation

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Econ720

October 30, 2017

Issues

- We study a GE model of growth driven by innovation.
- Innovation takes the form of inventing new goods.
- Alternative: Quality ladders.

A Model of Product Innovation

A representative **household** supplies labor to firms

Final goods firms use labor and intermediate inputs

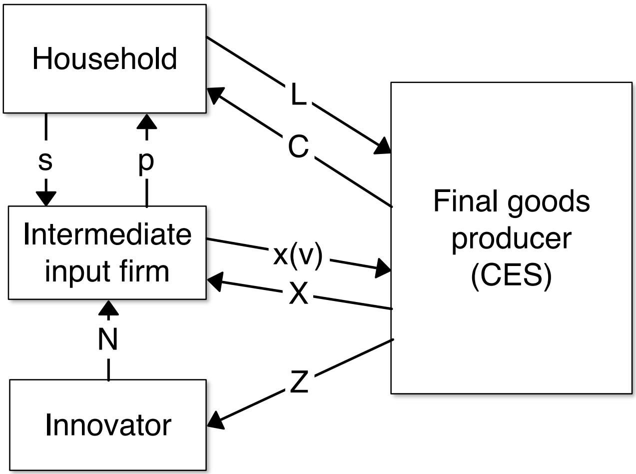
Intermediate inputs are produced from final goods

Final goods can be used to create new varieties of intermediate inputs

Innovators receive permanent monopolies

Note: Now that models get more complicated, it really pays off to be pedantic about details.

Model structure



Demographics and Preferences

Demographics:

- A representative household.

Preferences:

$$\int_0^\infty e^{-\rho t} \, \frac{C_t^{1-\theta} - 1}{1-\theta} dt \tag{1}$$

- C: the final good

Technology: Final Goods

Resource constraint:

$$C_t + X_t + Z_t = Y_t$$

Final goods Y are used for

- Z: R&D investment.
- X: Inputs into the production of intermediates x.
- C: consumption

Technology: Final Goods

Production of **final goods** from intermediates and labor:

$$Y_t = (1-\beta)^{-1} \left[\int_0^{N_t} x(v,t)^{1-\beta}\, dv \right] L^{\beta} \tag{2}$$

This is of the Dixit-Stiglitz form:

Write $\left[\int x^{1-\beta} dv\right]^{\frac{1-\beta}{1-\beta}}$ to see that this is a CES aggregator of x.

Technology: Intermediate Inputs

Each unit of x requires ψ units of Y:

$$X = \psi \int_0^{N_t} x(v,t)\, dv \tag{3}$$

Intermediate inputs fully depreciate in use.

Technology: Innovation

Investing the final good yields a flow of new patents:

$$\dot{N} = \eta Z_t \tag{4}$$

Think of this as the aggregate (deterministic) outcome of the (stochastic) innovation efforts of many firms.

Market arrangements

- Final goods and labor markets are competitive.
- Intermediates are sold by **monopolists** (the innovators).
  - Monopolies are permanent.
  - What the monopolists do with their profits is not clear.
- Free entry into innovation
  - ensures zero present value of profits
- The household owns the innovating firms.
- Asset markets are complicated
  - there is often no need to spell out the details

Notes

Production is cyclical:

- today's Y is used to make X which makes Y
- the alternative: durable X (more complicated)
- implication: the efficient allocation maximizes $Y - X = C + Z$

The only long-lived object is a patent

- this keeps the model simple

Assuming that intermediates are made from final goods fixes marginal costs (and prices)

Solving Each Agent's Problem

Final goods producers

- Maximize period profits by choosing L and $x(v,t)$.
- Normalize the price Y to 1.
- Profits

$$Y_t - w_t L_t - \int_0^{N_t} p^x(v,t)\ x(v,t)\, dv \tag{5}$$

where

$$Y_t = (1-\beta)^{-1} \left[\int_0^{N_t} x(v,t)^{1-\beta}\, dv \right] L^{\beta} \tag{6}$$

Final goods producers

FOCs:

- $\partial Y/\partial x(v) = L^{\beta} x(v)^{-\beta} = p^{x}(v)$
- $\partial Y/\partial L = \beta Y/L = w$

Demand function (cf. the Dixit Stiglitz discussion):

$$x(v,t) = L\, p^{x}(v,t)^{-1/\beta} \tag{7}$$

Solution to the firm's problem: $L_t, x(v,t)$ that satisfy the "2" first-order conditions.

Intermediate input producers

Problem after inventing a variety.

x is produced at constant marginal cost ψ.

Maximize present value of profits

$$V(v,t) = \int_t^{\infty} e^{-rs} \pi(v,s)\, ds \tag{8}$$

Instantaneous profits are

$$\pi(v,t) = (p^x(v,t) - \psi)\; x(v,t) \tag{9}$$

where $x(v,t) = Lp^x(v,t)^{-1/\beta}$

This is a sequence of static problems

Intermediate input producers

- First order condition (standard monopoly pricing formula):

$$p^x = \psi/(1-\beta) \tag{10}$$

- Profits are

$$\pi(v,t) = \psi\frac{\beta}{1-\beta}x(v,t) \tag{11}$$

- Solution: A constant p^x.

Household

- The household holds shares of all intermediate input firms.
- Each firm produces a stream of profits.
- New firms issue new shares.
- But: the details don't matter to the household.
- There simply is an asset with rate of return r.
- Euler equation is standard:

$$g(C) = \frac{r - \rho}{\theta} \tag{12}$$

- Invoke Walras' law - so you never have to write down the budget constraint!

Equilibrium

- Objects: $C_t, X_t, Z_t, x(v,t), V(v,t), N_t$ and prices $p^x(v,t), r(t), w(t)$.
- Conditions:
  - "Everybody maximizes." (see above)
  - Markets clear.
    1. Goods: resource constraint.
    2. Shares: omitted b/c I did not write out the household budget constraint.
    3. Intermediates: implicit in notation.
  - Innovation effort satisfies a **free entry** condition: present value of profits equals 0.

Symmetric Equilibrium

We assume (and then show) that all varieties v share the same x, V and p^x.

Intuition:

- p^x: monopoly pricing with a constant elasticity
- x: varieties enter final goods production symmetrically
- V: the age of a variety does not matter
  (no stock of x to build; permanent patents)

Equilibrium: Characterization

The growth rate follows from the Euler equation:
$g(C) = (r - \rho)/\theta$.

- We need to solve for r.

Simplifications:

- Normalize marginal cost $\psi = 1 - \beta$
  - so that $p^x = 1$.
  - Why can I do that?
- Focus on balanced growth paths.

Equilibrium: Characterization

Free entry will determine the interest rate

Spend 1 to obtain η new patents, each valued (initially) at $V(v,t)$

$$\eta V(v,t) = 1 \tag{13}$$

- Then V is constant over time.
- This assumes that innovation takes place.

With balanced growth and constant profits (to be shown):

$$V = \pi/r \tag{14}$$

Profits

With a fixed markup, profits are a multiple of revenues:

$$
\begin{aligned}
\pi(t) &= \psi \frac{\beta}{1-\beta} x(t) && (15) \\
&= \beta x(t) && (16)
\end{aligned}
$$

Demand for intermediates:

$$
\begin{aligned}
x(t) &= L\, p^x(t)^{-1/\beta} \\
&= L
\end{aligned}
$$

Profits: $\pi = \beta L$.

Free Entry

- Free entry:

$$\eta V = \eta \beta L / r = 1 \tag{17}$$

- This is the closed form solution for r.
- Balanced **growth** rate then follows from the Euler equation.

$$g(C) = \frac{\eta \beta L - \rho}{\theta} \tag{18}$$

Equilibrium: Characterization

Production function for final goods with $x = L$:

$$Y = \frac{N_t L}{1-\beta} \tag{19}$$

Wage (from firm's FOC):

$$w_t = \beta \frac{Y_t}{L_t} = \frac{\beta}{1-\beta} N_t \tag{20}$$

Total expenditure on intermediates:

$$X_t = \psi N_t x_t = (1-\beta) N_t L \tag{21}$$

Summary of Equilibrium

Prices and quantities of intermediate inputs are constant.

- the model is rigged to deliver this
- for tractability

Growth comes from rising N

No Transition Dynamics

The equilibrium looks like an AK model with production function

$$\begin{aligned} Y_t &= \frac{L}{1-\beta} N_t \\ \dot{N}_t &= \eta \; s_z \; Y_t \end{aligned}$$

Intuition:

- Period profits π are constant at βL.
- At any moment we need $\eta V = 1$.
- V is the present value of (constant) profits.
- Constant V is only possible with constant r.
- Intuition: There is a reduced form AK structure.

Scale Effects

$$g(C) = \frac{\eta\beta L - \rho}{\theta}$$

Larger economies (L) grow faster.

Population growth implies exploding income growth (!)

Mechanical reason:

- Innovation technology is linear in goods.
- Larger economy $\rightarrow$ higher Y $\rightarrow$ higher Z $\rightarrow$ faster growth.

We will return to this later.

Pareto Efficient Allocation

Efficiency

Two distortions prevent efficiency of equilibrium:

1. Monopoly pricing.
2. Inefficient innovation due to aggregate demand externality.

Planner's Problem

Solve in two stages:

1. Given N, find optimal static allocation $x(v,t)$.
   - That is: maximize $Y - X$ which is available for consumption and investment.
   - An odd feature of the model: goods are produced from goods without delay.
2. Given the reduced from production function from #1, find optimal Z.

Static Allocation

Given N, choose $x(v,t)$ to maximize $Y - X$:

$$\max\,(1-\beta)^{-1} L^{\beta} \int_0^{N_t} x(v,t)^{1-\beta}\, dv - \int_0^{N_t} \psi x(v,t)\, dv \tag{22}$$

First-order condition

$$L^{\beta} x^{-\beta} = \psi \tag{23}$$

with $\psi = 1 - \beta$:

$$x = (1-\beta)^{-1/\beta}\; L \tag{24}$$

The planner's x is larger than the equilibrium x (Intuition?)

Static Allocation

Next: find $Y - X$.

$$X = \psi N x = (1-\beta) N (1-\beta)^{-1/\beta} L \tag{25}$$

Reduced form production function:

$$\begin{aligned} Y_t &= (1-\beta)^{-1} L^{\beta} N [(1-\beta)^{1-1/\beta} L]^{1-\beta} && (26) \\ &= (1-\beta)^{-1/\beta} L N_t && (27) \end{aligned}$$

Net output

$$\begin{aligned} Y - X &= (1-\beta)^{-1/\beta} LN - (1-\beta)^{1-1/\beta} LN \\ &= (1-\beta)^{-1/\beta} \beta \; L \; N && (28) \end{aligned}$$

Planner: Dynamic Optimization

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

subject to

$$\begin{aligned} \dot{N} &= \eta Z \\ Y &= (1-\beta)^{-1/\beta} \; \beta \; L \, N = C + Z \end{aligned}$$

Or

$$\begin{aligned} \dot{N} &= A \; N - \eta C && (29) \\ A &= \eta \, (1-\beta)^{-1/\beta} \; \beta \; L && (30) \end{aligned}$$

Hamiltonian

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \mu\left[AN - \eta C\right] \tag{31}$$

FOC

$$\begin{aligned} \partial H/\partial C &= C^{-\theta} - \mu\eta = 0 & (32)\\ \partial H/\partial N &= \rho\mu - \dot{\mu} = \mu A & (33) \end{aligned}$$

Optimal growth

The same as in an AK model with

$$A = \eta \, (1-\beta)^{-1/\beta} \;\; \beta \; L \tag{34}$$

we have

$$\dot{C}/C = \frac{A-\rho}{\theta} \tag{35}$$

Comparison with CE

- CE interest rate: $\eta\beta L$.
- Planner's "interest rate:" $(1-\beta)^{-1/\beta}\eta\beta L$.
- The planner chooses faster growth.
- Intuition:
  - CE under-utilizes the fruits of innovation: x is too low.
  - This reduces the value of innovation.

Policy Implications

- One might be tempted to reduce monopoly power.
- A policy that encourages competition (e.g. less patent protection, forcing lower p^x) reduces the static price distortion.
- But it also reduces growth: innovation is less valuable.
- Similar result for shorter patents.
- Policy trades off static efficiency and incentives for innovation.

Reading

- Acemoglu (2009), ch. 13.
- Krusell (2014), ch. 9
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

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