Models of Creative Destruction (Quality Ladders)

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Motivation

We study models of process innovation ("quality ladders").

New issues:

1. Innovations replace existing monopolies - creative destruction.
2. Multiple firms can produce the same good - price competition.

Innovation through quality improvements

We study the generic problem of an innovating firm.

The industry faces a **demand curve** $Q = D(p)$.

- Price elasticity: ε_D.

There are infinitely many competitors with marginal cost ψ.

All can make the same good.

No innovation $\implies$ everyone gets zero profit.

Innovation

Spend μ goods

Reduce marginal cost to ψ/λ.

The innovator becomes a **monopolist**

- subject to a fringe of competitors with MC ψ.

Pricing Decision

- Monopoly price:

$$p^M = \frac{\psi/\lambda}{1 - 1/\varepsilon_D} \tag{1}$$

- Monopoly profit:

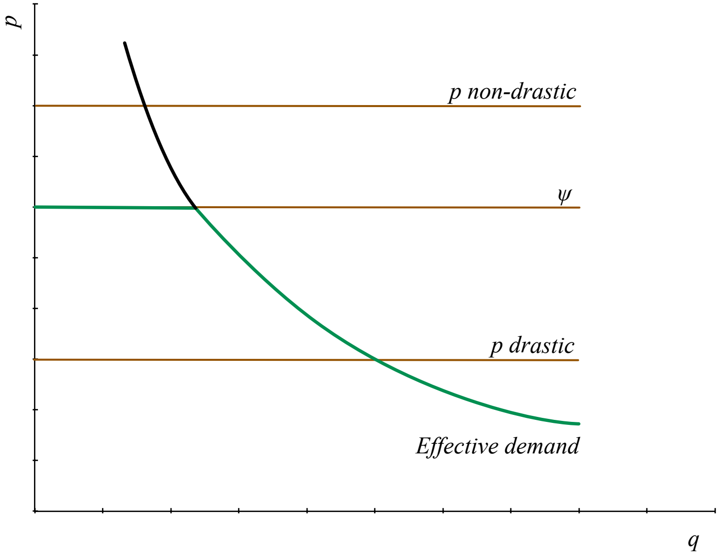
$$\pi_1^I = D\left(p^M\right)\ \left[p^M - \psi/\lambda\right] - \mu \tag{2}$$

- This is also the (private) value of the innovation.

Drastic / non drastic innovation

- A **drastic** innovation gives the innovator a monopoly.
  - The monopoly price is below competitor's marginal cost ψ.
  - It is optimal to set the monopoly price.
- A **non-drastic** innovation leaves the monopoly price above ψ.
  - Charging monopoly prices results in 0 sales.
  - The demand curve facing the firm becomes infinitely elastic at price ψ.
  - It is optimal to set $p = \psi$.

Effective demand curve



Optimality

Is the level of innovation too above or below what maximizes "welfare"?

The answer is usually ambiguous.

1. The innovator does not capture the entire consumer surplus.
2. Business stealing effect: innovation destroys competitor profits.
3. Replacement effect: innovation destroys the innovator's own profit.

The Role of Patents

Innovation only occurs, if the innovator earns monopoly status for some time

- but see Boldrin and Levine (2008)

The trade-off:

- Stronger patents - higher profits - more innovation.
- Stronger patents distort prices.

Baseline Model

A Baseline Model

- Demographics: There is a single, infinitely lived household.
- Preferences:

$$\int_0^\infty e^{-\rho t} u(C_t) dt \tag{3}$$

- Endowments:
  - 1 unit of work time each instant
  - households also own all firms / patents

Commodities

At date t we have:

- 1 final good Y. Used for consumption, R&D, and production of intermediates.
- A unit measure of intermediate inputs, indexed by v.

Each intermediate good can be produced with many different "qualities" $q(v,t)$.

Innovation takes the form of introducing better qualities.

Final Goods Technology

- There is one final good that can be used for consumption, investment in R&D, and production of intermediate inputs:

$$Y_t = C_t + X_t + Z_t \tag{4}$$

- Final goods are produced from labor and intermediates:

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 q(v,t) x(v,t)^{1-\beta} dv \tag{5}$$

- There is a unit measure of intermediates.
- $q(v,t)$ is the best available quality of intermediate v at t.
- Assumption: Only the best quality is used in equilibrium.

Final Goods Technology

- Why is only the best quality used?
- For each good v, a large number of qualities are offered (by monopolists): $q(s,v,t)$.
- They are perfect substitutes in the production of final goods.
- Think of the production function as

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 X(v,t)^{1-\beta} dv \tag{6}$$

- $X(v,t)$ is input of all vintages of good v:

$$X(v,t) = \left[\int_{-\infty}^{t} q(s,v,t)^{1/(1-\beta)} x(s,v,t) ds \right] \tag{7}$$

Final Goods Technology

- When patent owners for all vintages s compete (see Ch. 12), pricing ensures that only the vintage with the highest q is used in equilibrium.

$$X(v,t) = q(v,t)^{1/(1-\beta)} x(v,t) \tag{8}$$

where $q(v,t) = \max_s q(s,v,t)$.

- Exercise: Derive conditions such that this is true. (See end of slides for an answer sketch.)

Technology: Innovation

- Each innovation takes the quality from $q(v,t)$ to $\lambda q(v,t)$.
- The quality step is $\lambda > 1$.
- Innovation takes place separately for each v.
- Investing $Z(v,t)$ for interval Δt creates 1 quality improvement with probability:

$$n(v,t)\Delta t = \eta Z(v,t)\Delta t / q(v,t) \tag{9}$$

- Over a short interval:

$$q(v,t+\Delta t) = \begin{cases} q(v,t) & \text{with probability } 1 - n(v,t)\Delta t \\ \lambda q(v,t) & \text{with probability } n(v,t)\Delta t \end{cases} \tag{10}$$

Technology: Intermediate Goods

- Intermediates perish in production.
- Their marginal cost is $\psi q(v,t)$.

$$\int_0^1 x(v,t)\, q(v,t)\, \psi = X_t \tag{11}$$

- Note: $q(v,t)$ shows up in various places in such a way to ensure balanced growth.

Market Arrangements

- Final goods: perfect competition.
- Innovators received permanent patents for the qualities they create.
  - Other firms can improve on their qualities.
- Intermediate goods firms are the same as innovators (or innovators sell qualities at competitive prices).
  - They are monopolists
  - but there is a competitive fringe of firms offering lower qualities
- Assumption: Current monopolists cannot innovate.
  - not binding: they would not want to innovate b/c their gain in profits is lower than the gain for new entrants.
- Free entry into innovation.
- Households own the innovating firms and receive their profits.

Equilibrium

Allocation: C_t, X_t, Z_t, Y_t and $q(v,t), x(v,t)$.

Prices: $p^x(v,t), V(v,t)$, r_t, w_t.

Such that:

1. Agents "maximize" (below).
   1.1 household: choose C
   1.2 final goods firm: Y, L, x
   1.3 intermediate goods firm: p^x, V
   1.4 innovators: free entry
2. Markets clear.

A wrinkle: $q(v,t)$ is stochastic. So the equilibrium def is slightly wrong.

Assumption: Invoke a law of large numbers to ensure that aggregates are deterministic.

Market clearing

Goods: Resource constraint

Labor: $L = 1$

Intermediates: implicit in notation

Assets: details omitted

Solving agents' problems

Household

- Again: avoid writing out the budget constraint.
- Just note that the household owns a portfolio of assets (shares of intermediate goods firms) with deterministic rate of return $r(t)$.
- Euler equation:

$$g(C(t)) = \frac{r(t) - \rho}{\theta} \tag{12}$$

- Value of assets held:

$$a(t) = \int_0^1 V(v,t)dv \tag{13}$$

- $V(v,t)$ is the value of the intermediate input firm v.
- TVC: $\lim_{t\to\infty} e^{-rt}a(t) = 0$ [with constant interest rate].

Final goods firm

$$\max Y_t - w_t L_t - \int_0^1 p^x(v,t)\, x(v,t) \tag{14}$$

where

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 q(v,t) x(v,t)^{1-\beta} dv \tag{15}$$

FOCs:

$$x(v,t) = \left(\frac{q(v,t)}{p^x(v,t)} \right)^{1/\beta} L \tag{16}$$

$$w_t = \beta Y_t / L_t \tag{17}$$

Solution: $Y_t, L_t, x(v,t)$ that solve 2 FOCs and production function.

Intermediate goods firm

Static profit maximization with constant demand elasticity $1/\beta$

Assume drastic innovation.

Owner of current best quality can set monopoly price:

$$p^x(v,t) = \frac{\psi q(v,t)}{1-\beta} \tag{18}$$

Solution: $p^x(v,t)$

Innovators

Free entry

- increase innovation Z until marginal cost = present value of profits

Suppose current quality is $q(v,t)/\lambda$. (simplifies notation)

Investing $Z(v,t)$ for period Δt yields an innovation with probability $n(v,t)\Delta t$ where

$$n(v,t) = \eta Z(v,t)/[q(v,t)/\lambda] \tag{19}$$

Marginal cost: $Z(v,t)\Delta t$.

Marginal benefit:

- new quality is $q(v,t)$ with value $V(v,t|q)$.
- a patent valued at $V(v,t|q)$ with probability $\eta Z(v,t)/[q(v,t)/\lambda]\Delta t$.

Innovators

- If marginal benefit < marginal cost: no innovation (not interesting).
- Otherwise: innovation continues until

$$\underbrace{Z(v,t)\Delta t}_{\text{marginal cost}} = \underbrace{V(v,t|q)\frac{\lambda\eta}{q(v,t)}Z(v,t)\Delta t}_{\text{marginal benefit}} \tag{20}$$

- Or:

$$\frac{q(v,t)}{\lambda\eta} = V(v,t|q) \tag{21}$$

Value of innovation

General asset pricing equation (which we will derive later...):

$$rp = \dot{p} + d \qquad (22)$$

In words:

- the current payoff of the asset consists of capital gain $\dot{p}$ and dividend d.
- rate of return = [current payoff] / [current price]

Value of innovation

Applying the asset pricing equation to the value of the firm.

Current price: $p = V(v, t, |q)$.

Dividend: Flow profit: $\pi(v, t) = d$.

Lose profit flow at rate $z(v, t|q)$ - endogenous, chosen by competitors.

Capital gain: $\dot{V}(v, t|q) - z(v, t|q)V(v, t|q)$.

Pricing equation:

$$r(t)V(v, t|q) = \dot{V}(v, t|q) + \pi(v, t|q) - z(v, t|q)V(v, t|q) \tag{23}$$

Digression: Capital Gain

- One might expect the capital gain to be

$$(1-z)\dot{V} - zV \tag{24}$$

- Write out payoffs over interval Δt

$$Vr\Delta t = \pi\Delta t + (1 - z\Delta t)\dot{V}\Delta t - z\Delta t V \tag{25}$$

- Take $\Delta t \to 0$ and the term $(1 - z\Delta t) \to 1$.

Equilibrium Characterization

Equilibrium

For simplicity: assume **balanced growth**.

Start from the Euler equation

$$g(C(t)) = \frac{r(t) - \rho}{\theta} \tag{26}$$

We need to find r.

Finding r

Use free entry:

$$\frac{q(v,t)}{\lambda\eta} = V(v,t|q) \tag{27}$$

where

$$r(t)V(v,t|q) = \dot{V}(v,t|q) + \pi(v,t|q) - z(v,t|q)V(v,t|q) \tag{28}$$

We need profits.

Finding profits

$$\pi(v,t) = [p^x(v,t) - \psi q(v,t)]x(v,t) \tag{29}$$

From the intermediate input firm

$$p^x(v,t) = \frac{\psi q(v,t)}{1-\beta} \tag{30}$$

Normalize $\psi = 1 - \beta$, so that $p^x = q$.
Then demand is

$$x(v,t) = L \tag{31}$$

Profits:

$$\pi(v,t) = \beta q(v,t)L$$

Finding V

Assume $\dot{V}(v,t|q) = 0$ (not obvious yet).

- Intuition: Replacement probability and profits are constant over time.

Pricing equation:

$$\begin{aligned} rV(v,t|q) &= \dot{V}(v,t|q) + \pi(v,t|q) - z(v,t|q)V(v,t|q) && (32) \\ &= 0 + \beta q(v,t)L - z(v,t|q)V(v,t|q) && (33) \end{aligned}$$

or, with free entry:

$$V(v,t|q) = \frac{\beta q(v,t)L}{r + z(v,t|q)} = \frac{q(v,t)}{\lambda\eta} \quad (34)$$

This means: $z(v,t|q) = z^*$ in all sectors with innovation.

Could there be sectors without innovation?

- No - V is present value of expected profits.
- Without innovation in sector v: $z(v,t|q)=0$.
- That raises the value of the firm to

$$V(v,t|q)=\frac{\beta q(v,t)L}{r}>\frac{q(v,t)}{\lambda\eta} \tag{35}$$

- There would be strictly positive profits for entrants.

Balanced Growth

We have almost found r, except that we still need to know z^*:

$$r = \lambda \eta \beta L - z^* \tag{36}$$

We get z^* from the balanced growth condition $g(C) = g(Y)$.

Intuition:

- suppose r is high; then $g(C)$ is high
- innovation is not profitable (strong discounting of profits)
- $g(Y)$ will be slow

Output Growth

Define average quality: $Q(t) = \int_0^1 q(v,t)dv$.

Final output with $x(v,t) = L$:

$$Y_t = (1-\beta)^{-1} L_t^{\beta} \int_0^1 q(v,t) L^{1-\beta} dv \tag{37}$$

$$= (1-\beta)^{-1} L Q(t) \tag{38}$$

Output growth: $g(Y) = g(Q)$.

Quality Growth

- Consider an interval Δt - small.
- Fraction $z^* \Delta t$ varieties experience 1 innovation.
- The rest experiences no innovation.
- For small Δt the probability of multiple innovation is negligible.
- Therefore:

$$
\begin{aligned}
Q(t+\Delta t) &= \int_0^1 [(z^* \Delta t)\lambda q(v,t) + (1 - z^* \Delta t) q(v,t)] dv \qquad (39) \\
&= (z^* \Delta t)\lambda Q(t) + (1 - z^* \Delta t) Q(t) \qquad (40)
\end{aligned}
$$

- Growth rate:

$$
g(Q(t)) = (\lambda - 1) z^* \qquad (41)
$$

Balanced Growth

$$
\begin{aligned}
g(Q) &= (\lambda - 1)z^* && (42)\\
&= g(C) && (43)\\
&= \frac{\lambda\eta\beta L - z^* - \rho}{\theta} && (44)
\end{aligned}
$$

Solve for z^* and

$$
g(C) = \frac{\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}} \tag{45}
$$

Properties of Balanced Growth

- No transitional dynamics.
  - it turns out only Q matters, not the entire distribution of $q(v)$
- Symmetry: all varieties share the same rate of innovation z^* - this is what makes the model tractable.
- The static allocation is not optimal
  - monopoly pricing distorts $x(v,t)$
- The growth rate may be above or below the Pareto optimal one (see Acemoglu 2009, ch. 14.1).

Applications

Optimal patent design:

- Hall (2007), Jones and Williams (2000), Jones and Williams (1998)

Effects of taxes on growth:

- Peretto (2007)

Trade and growth:

- Acemoglu et al. (2013)

Reading

- Acemoglu (2009), ch. 14.
- Aghion et al. (2014)
- Aghion and Howitt (2009): a text on R&D driven growth models.

Only best quality is used in equilibrium

- Let's focus on one good and suppress the (v,t) arguments for notational clarity.
- In the production function (6) all qualities s of the same good are perfect substitutes.
- The Firm minimizes the cost of $X(v,t) = \int q(s)^{(1/1-\beta)} x(s) ds$.
- The cost is $\int p(s) x(s) ds$.
- The firm uses the goods with the highest ratio of "quality" to price: $q(s)^{1/(1-\beta)} / p(s)$.
- The monopolist charges markup ψ: $p(s_{Mon}) = \psi q_{Mon}$.
- Competitors charge at least marginal cost $p(s) = q(s)$.
- The innovation is drastic if the monopolist has the highest quality/price ratio:

$$\lambda^{1/(1-\beta)} / (\lambda \psi) > 1 \tag{46}$$

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