# Models of Creative Destruction
# Firm Dynamics

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# Motivation

We extend the Schumpeterian model to have innovation by incumbents.

This produces a model of firm size dynamics.

# Environment

Demographics, preferences, commodities: unchanged.

Resource constraint:

$$Y = C + X + Z \tag{1}$$

where

$$Z(t) = \underbrace{\int_0^1 \hat{z}(v,t)\, q(v,t)\, dv}_{\text{entrants}} + \underbrace{\int_0^1 z(v,t)\, q(v,t)\, dv}_{\text{incumbents}} \tag{2}$$

# Final goods technology

$$Y(t) = \frac{1}{1-\beta} L(t)^{\beta} \int_0^1 q(v,t)^{\beta} x(v,t|q)^{1-\beta} dv \tag{3}$$

The only change: quality is taken to power $\beta$

Implies: sales vary with quality (so the model has firm size implications)

# Intermediate goods technology

Constant marginal cost $\psi$

- previously $\psi q$

Therefore

$$X(t) = \int_0^1 \psi x(v,t)\,dv \tag{4}$$

# Innovation technology for incumbents

- let $q(v,s)$ be the quality at the time the incumbent invented it
- investing $zq$ implies a flow probability of innovation of $\phi z$
- the quality step is $\lambda$

# Innovation technology for entrants

Investing $\hat{z}q$ implies a flow probability of innovation of $\eta(\hat{z})\hat{z}$

- $\eta$ is **decreasing**
- marginal cost of innovation is rising in $\hat{z}$
- innovators take $\eta$ as given (an externality)

Why rising marginal costs?

- If incumbents and entrants have constant marginal cost, only one of them innovates in equilibrium.

The quality step is $\kappa$

# Summary of changes

| Agent | New | Old |
|---|---|---|
| Final goods | $\int_0^1 q(v,t)^{\beta} x(v,t\|q)^{1-\beta} dv$ | Was $q(v,t)^1$ |
| Intermediates | Marginal cost $\psi$ | Was $q\psi$ |
| Incumbents | Innovate | Don't innovate |
| Entrants | probability of innovation $\eta(\hat{z})\hat{z}$ | $\eta z$ |

# 3. Solving each agent's problem

# Solving each agents' problem

Household (unchanged):

$$g(C) = \frac{r - \rho}{\theta} \tag{5}$$

Final goods producer (barely changed):

$$x(v,t|q) = p^x(v,t|q)^{-1/\beta} q(v,t) L \tag{6}$$
$$w(t) = \beta Y(t) / L(t) \tag{7}$$

The only change: exponent on $q$ was $1/\beta$.

# Intermediate goods producer

Assume drastic innovation.

Then price follows the usual monopoly formula:

$$p^x(v,t|q) = \frac{\psi}{1-\beta} = 1 \tag{8}$$

with normalization $1-\beta = \psi$

# Innovation by entrants

Free entry:

Investing $q\hat{z}$ gives a flow of $\eta\hat{z}$ new patents "per period"

$$\underbrace{\eta(\hat{z})\hat{z}}_{\text{probability}} \underbrace{V(v,t|\kappa q)}_{\text{payoff}} = \underbrace{q(v,t)\hat{z}}_{\text{cost}} \tag{9}$$

or

$$\boxed{V(v,t|\kappa q) = \frac{q}{\eta(\hat{z})}} \tag{10}$$

Note the $\kappa q$.

This assumes an equilibrium with entry.

The flow probability that any competitor replaces the incumbent is $\hat{z}\eta(\hat{z})$.

# Innovation by incumbents

Again assuming positive innovation.

Increase $z$ until the marginal value equals marginal cost:

$$\underbrace{\phi z(v,t|q)}_{\text{probability}}\underbrace{[V(v,t|\lambda q)-V(v,t|q)]}_{\text{payoff}}=\underbrace{q(v,t)z(v,t|q)}_{\text{cost}} \tag{11}$$

We show later that $V$ is proportional to quality $q$. Then

$$\phi V(v,t|q)[\lambda-1]=q(v,t) \tag{12}$$

or

$$\boxed{V(v,t|q)=\frac{q}{\phi(\lambda-1)}} \tag{13}$$

# Value of the firm

Expected discounted value of profits

$$V(v,t|q) = \mathbb{E}\int_0^{\infty} e^{-rt}\pi(v,\tau|q)\,d\tau \tag{14}$$

where profits are constant over time

until the firm is hit by a shock:

- another firm replaces the incumbent
  flow probability $\hat{z}(v,t|q) \times \eta(\hat{z}(v,t|q))$
- incumbent successfully innovates
  flow probability $\phi z(v,t|q)$

This type of problem has a generic solution...

# Generic derivation

Take the generic discounted present value

$$V = \mathbb{E} \int_0^\infty e^{-rt} \pi(t)\, dt \tag{15}$$

where profits change stochastically according to a Poission process. With flow probability $\rho$, profits change so that the continuation value becomes $\hat{V}$.

We show that

$$rV = \pi + \dot{V} + \rho \left( \hat{V} - V \right) \tag{16}$$

# Generic derivation I

Evaluate the flow payoffs over a short period $\Delta t$:

$$V = \int_0^{\Delta t} e^{-(r+\rho)t} \pi_t dt \tag{17}$$

$$+ e^{-r\Delta t} \left[ e^{-\rho \Delta t} V_{\Delta t} + \left[ 1 - e^{-\rho \Delta t} \right] \hat{V} \right] \tag{18}$$

Note the discounting at $r + \rho$.

- Because the probability of still receiving profits is $e^{-\rho t}$

At the end of the interval, discounted by $e^{-r\Delta t}$, the payoffs are

- $V_{\Delta t}$: the value of continuing at the end of $\Delta t$; with probability $e^{-\rho \Delta t}$
- $\hat{V}$: the value of continuing with a shock; with complementarity probability.

# Generic derivation II

Assume that $\pi$ is constant over the interval $\Delta t$. Then the first integral is

$$\frac{1-e^{-(r+\rho)\Delta t}}{r+\rho}\pi \tag{19}$$

Add and subtract $V$ in the second term and it becomes

$$e^{-\rho\Delta t}(V_{\Delta t}-V)+\left[1-e^{-\rho\Delta t}\right]\hat{V}+e^{-\rho\Delta t}V \tag{20}$$

Substituting back into the definition of $V$ gives

$$V\left[1-e^{-(r+\rho)\Delta t}\right]=\frac{1-e^{-(r+\rho)\Delta t}}{r+\rho}\pi \tag{21}$$

$$+e^{-r\Delta t}\left[e^{-\rho\Delta t}[V_{\Delta t}-V]+\left[1-e^{-\rho\Delta t}\right]\hat{V}\right] \tag{22}$$

# Generic derivation III

Divide by $\left[1-e^{-(r+\rho)\Delta t}\right]$ and take $\Delta t \to 0$.

The first term becomes $\frac{\pi}{r+\rho}$.

Set $[V_{\Delta t} - V] = \dot{V}\Delta t$. Then the second term becomes

$$\frac{e^{-(r+\rho)\Delta t}}{1-e^{-(r+\rho)\Delta t}}\dot{V}\Delta t \tag{23}$$

Using L'Hopital's rule this becomes:

$$\frac{-(r+\rho)e^{-(r+\rho)\Delta t}\Delta t + e^{-(r+\rho)\Delta t}}{(r+\rho)e^{-(r+\rho)\Delta t}} = \frac{1}{r+\rho} \tag{24}$$

Similarly, using L'Hopital's rule the third term becomes

$$\frac{\rho}{r+\rho}\hat{V} \tag{25}$$

# Generic derivation IV

Putting it all together gives

$$(r+\rho)V = \pi + \dot{V} + \rho\hat{V} \tag{26}$$

or

$$rV = \pi + \dot{V} + \rho\left[\hat{V} - V\right] \tag{27}$$

# Value of the firm

Applying the generic formula:

$$rV(v,t|q) = \underbrace{\pi(v,t|q)}_{\text{flow profit}} + \underbrace{\dot{V}(v,t|q)}_{0} - \underbrace{z(v,t|q)\,q(v,t)}_{\text{R\&D cost}} \tag{28}$$

$$+ \underbrace{\phi z(v,t|q)}_{\text{prob success}} \underbrace{[V(v,t|\lambda q) - V(v,t|q)]}_{\text{payoff}} \tag{29}$$

$$- \underbrace{\hat{z}(v,t|q)\,\eta(\hat{z}(v,t|q))}_{\text{prob lost patent}} \underbrace{V(v,t|q)}_{\text{loss}} \tag{30}$$

Note: Terms 3 and 4 cancel by the incumbent's FOC.

Therefore

$$rV = \pi + \underbrace{\dot{V}}_{=0} - \hat{z}\eta(\hat{z}) \times V \tag{31}$$

# Value of the firm

Profit (unchanged):

$$\pi(v,t|q) = [p^x(v,t|q) - \psi]\, x(v,t|q) \tag{32}$$
$$= \beta q L \tag{33}$$

because $p^x = 1$ and $x = qL$. Therefore

$$rV = \beta q L - \hat{z}\eta(\hat{z})\, V \tag{34}$$

or

$$\boxed{V = \frac{\beta q L}{r + \hat{z}\eta(\hat{z})}} \tag{35}$$

The usual story: losing the patent just increases the effective interest rate.

# 4. Equilibrium

Allocation

$\{C(t), X(t), Z(t), Y(t), L(t), z(v,t), \hat{z}(v,t), x(v,t), \pi(v,t), V(v,t)\}$

Prices $\{p^x(v,t), w(t), r(t)\}$

that satisfy:

- household: Euler (and TVC)
- final goods firm: 3
- intermediate goods firm: 1
- free entry of incumbents and entrants: 2
- market clearing: goods, labor (2)
- definitions of $X, Z, \pi$ (3)
- definition of $V$ (differential equation) (1)

# Balanced Growth Path

Euler equation

$$g(C) = \frac{r - \rho}{\theta} \tag{36}$$

We now have 3 expressions for the value of the firm:

1. Free entry by incumbents (13)
2. Free entry by entrants (10)
3. The present value of profits (35)

$$V(q) = \underbrace{\frac{\beta q L}{r + \hat{z}\eta(\hat{z})}}_{\text{incumbents}} = \underbrace{\frac{q/\kappa}{\eta(\hat{z})}}_{\text{entrants}} = \underbrace{\frac{q}{\phi(\lambda - 1)}}_{\text{present value}} \tag{37}$$

These jointly solve for $r, \hat{z}$.

The Euler equation (36) then gives the growth rate.

# Implications for firm dynamics

We now begin to have a model of firm dynamics.

- We have firm entry and exit (innovation by entrants)
- We have firm sales growth (stochastic) with firm age

Firm sales are given by$x(v,t|q) = qL$.

For a given firm: $x$

- increases by factor $\lambda$ with probability $\phi z \Delta t$
- stays the same with probability $\hat{z}\eta(\hat{z})\Delta t$
- drops to 0 with complementary probability

# Applications

Garcia-Macia et al. (2016)

- how much of output growth is due to innovation by incumbents vs competitors?

Acemoglu et al. (2013)

- tax policy in a model with R&D and firm quality heterogeneity

Hottman et al. (2016)

- measures sources of firm heterogeneity

# Reading

- Acemoglu (2009), ch. 14.3.
- Aghion et al. (2014), survey of Schumpeterian growth models

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