

Models of Creative Destruction

Firm Dynamics

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Motivation

We extend the Schumpeterian model to have innovation by incumbents.

This produces a model of firm size dynamics.

Environment

Demographics, preferences, commodities: unchanged.

Resource constraint:

$$Y = C + X + Z \quad (1)$$

where

$$X(t) = \int_0^1 \psi x(v, t) dv \quad (2)$$

$$Z(t) = \int_0^1 [z(v, t) + \hat{z}(v, t)] q(v, t) dv \quad (3)$$

z and \hat{z} are innovation inputs by incumbents and their rivals.

Final goods technology

$$Y(t) = \frac{1}{1-\beta} L(t)^\beta \int_0^1 q(v,t)^\beta x(v,t|q)^{1-\beta} dv \quad (4)$$

- ▶ the only change: quality is taken to power β
- ▶ implies: sales vary with quality (so the model has firm size implications)

Intermediate goods technology

- ▶ constant marginal cost ψ

Innovation technology for incumbents

- ▶ let $q(v, s)$ be the quality at the time the incumbent invented it
- ▶ investing zq implies a flow probability of innovation of ϕz
- ▶ the quality step is λ

Innovation technology for entrants

- ▶ investing $\hat{z}q$ implies a flow probability of innovation of $\eta(\hat{z})$ (decreasing)
- ▶ the quality step is $\kappa > \lambda$ (leapfrogging)
- ▶ innovators take η as given (an externality)

Solving each agent's problem

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Household:

$$g(C) = \frac{r - \rho}{\theta} \quad (5)$$

Final goods producer:

$$x(v, t|q) = p^x(v, t|q)^{-1/\beta} q(v, t) L \quad (6)$$

$$w(t) = \beta Y(t) / L(t) \quad (7)$$

Intermediate goods producer

Assume drastic innovation

$$p^x(v, t|q) = \frac{\psi}{1 - \beta} = 1 \quad (8)$$

Innovation by entrants

Free entry:

$$\eta(\hat{z}(v, t|q)) V(v, t|\kappa q) = q(v, t) \quad (9)$$

This assumes an equilibrium with entry.

The aggregate flow probability that any competitor replaces the incumbent is $\hat{z}\eta(\hat{z})$.

Innovation by incumbents

Again assuming positive innovation.

Increase z until the marginal value equals marginal cost:

$$\phi z(v, t|q) [V(v, t|\lambda q) - V(v, t|q)] = q(v, t) z(v, t|q) \quad (10)$$

Value of the firm

Expected discounted value or profits

$$rV(v, t|q) = \pi(v, t|q) + \dot{V}(v, t|q) - z(v, t|q)q(v, t) \quad (11)$$

$$+ \phi z(v, t|q) [V(v, t|\lambda q) - V(v, t|q)] \quad (12)$$

$$- \hat{z}(v, t|q) \eta(\hat{z}(v, t|q)) V(v, t|q) \quad (13)$$

Note: Terms 3 and 4 cancel by the incumbent's FOC.

Profit

$$\pi(v, t|q) = [p^x(v, t|q) - \psi]x(v, t|q) \quad (14)$$

$$= \beta qL \quad (15)$$

because $p^x = 1$ and $x = qL$.

Equilibrium

Allocation

$\{C(t), X(t), Z(t), Y(t), L(t), z(v, t), \hat{z}(v, t), x(v, t), \pi(v, t), V(v, t)\}$

Prices $\{p^x(v, t), w(t), r(t)\}$

that satisfy:

- ▶ household: Euler (and TVC)
- ▶ final goods firm: 3
- ▶ intermediate goods firm: 1
- ▶ free entry of incumbents and entrants: 2
- ▶ market clearing: goods, labor (2)
- ▶ definitions of X, Z, π (3)
- ▶ definition of V (differential equation) (1)

Balanced Growth Path

Assert $\dot{V} = 0$, $z(q)$, and $\hat{z}(q)$ constant over time (verify later)

Law of motion for V implies: $V(q) = vq$.

- ▶ so that rV and $\pi = \beta Lq$ can grow at the same rate

Free entry for entrants:

- ▶ $\eta(\hat{z}(v, t|q)) V(v, t|\kappa q) = q(v, t)$
- ▶ implies \hat{z} is the same for all q
- ▶ but z for incumbents may vary with q

Innovation for incumbents

$\phi [V(v, t | \lambda q) - V(v, t | q)] = q(v, t)$ implies

$$V(q) = \frac{q}{\phi(\lambda - 1)} \quad (16)$$

Law of motion for V :

$$rV(q) = \beta Lq - \hat{z}\eta(\hat{z})V(q) \quad (17)$$

- ▶ the term reflecting incumbent innovation drops out (by its FOC)

Combine the two:

$$\eta(\hat{z}) = \frac{\phi(\lambda - 1)}{\kappa} \quad (18)$$

This solves for \hat{z} .

Equilibrium growth rate

Substitute back into free entry

$$r = \phi (\lambda - 1) \beta L - \hat{z} \eta (\hat{z}) \quad (19)$$

Together with the Euler equation, this solves for the growth rate.

Implications for firm dynamics

Since $x(v, t|q) = qL$, firm size (sales) are governed by the evolution of q

For a given firm: x

- ▶ increases by factor λ with probability $\phi z \Delta t$
- ▶ stays the same with probability $\hat{z} \eta(\hat{z}) \Delta t$
- ▶ drops to 0 with complementary probability

Applications

Garcia-Macia et al. (2016)

- ▶ how much of output growth is due to innovation by incumbents vs competitors?

Acemoglu et al. (2013)

- ▶ tax policy in a model with R&D and firm quality heterogeneity

Hottman et al. (2016)

- ▶ measures sources of firm heterogeneity

Reading

- ▶ Acemoglu (2009), ch. 14.3.
- ▶ Aghion et al. (2014), survey of Schumpeterian growth models

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