Partial Equilibrium R&D Models

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Issues

- We study models where **intentional innovation** drives productivity growth.
- We start by describing the demand block (common to essentially all models).
- Later we embed it into a GE model.

Background

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.
- How important are the 2 cases? – An open question.

How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
  - blueprints can be used simultaneously in the production of several goods.

How to model innovation

There are N consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- Therefore downward sloping demand curves

Approach 1: **Quality ladders**

- Each good can be made by many firms.
- Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: **Increasing variety**

- Each firm can invest to create a new variety $(N \to N+1)$
- Then it becomes the monopolist for that variety

The Demand Block

Modeling the Demand Side

- The trick in all R&D models:
  a demand side that generates a **constant price elasticity**
- This makes the monopoly price essentially exogenous
  $p_M = MC/(1 - 1/\varepsilon_D)$

Dixit Stiglitz Model

- The world is static.
- There are N consumption goods c_i with prices p_i.
- There is one "other" consumption good y with price 1.
  - It purpose is to absorb income effects.
- Household income is m.

Preferences

- Preferences: $u(C, y)$
- C is a CES composite consumption good:

$$C = \left(\sum_{i=1}^{N} c_i^{\theta} \right)^{1/\theta} \tag{1}$$

- $\theta = (\varepsilon - 1)/\varepsilon > 0$.
- Elasticity of substitution $\varepsilon > 1$.
- The trick: constant substitution elasticity implies constant price elasticity.

Love for variety

A key implication: simply having more varieties increases welfare.

Assume you have $\bar{C}$ units of "stuff" that can be made (1-for-1) into any variety:

$\sum_{i=1}^{N} c_i = \bar{C}$.

Consider the symmetric case: $c_i = \bar{C}/N$.

Then

$$\begin{aligned} C &= \left(\sum_{i=1}^{N} [\bar{C}/N]^{\theta} \right)^{1/\theta} \\ &= \left(N \; \left[\bar{C}/N \right]^{\theta} \right)^{1/\theta} && (2) \\ &= N^{(1-\theta)/\theta} \; \bar{C} && (3) \end{aligned}$$

Spreading $\bar{C}$ over more varieties (N) increases utility.

Demand functions

The household's demand functions are iso-elastic.

The household solves:

$$\max u(C, y)$$

subject to

$$\sum_{i=1}^{N} p_i c_i + y = m \tag{4}$$

Given m, this is just a CES cost minimization problem.

Demand functions

$$\max u\left(\left[\sum_{i=1}^{N} c_i^{\theta}\right]^{1/\theta}, m - \sum p_i c_i\right)$$

FOC

$$\begin{aligned}
\frac{\partial u}{\partial y} &= \frac{\partial u}{\partial c_i}\frac{1}{p_i} \\
&= \frac{\partial u}{\partial C}\frac{1}{\theta}\left[\sum_{i=1}^{N} c_i^{\theta}\right]^{1/\theta-1} \theta\ \frac{c_i^{\theta-1}}{p_i}
\end{aligned}$$

Demand functions

A useful feature:

$$[c_i/c_j]^{-1/\varepsilon} = p_i/p_j \tag{5}$$

Equal for all goods:

$$c_i^{-1/\varepsilon}/p_i \tag{6}$$

Demand function:

$$c_i = Xp_i^{-\varepsilon} \tag{7}$$

for some endogenous constant X (which we need to find).

Price elasticity is constant at ε.

Demand functions

Claim:

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \tag{8}$$

where C is the composite consumption good

$$C = \left[\sum_{i=1}^{N} c_i^{\theta}\right]^{1/\theta} \tag{9}$$

and P is the "ideal price index" for the household (the cost minimizing cost of C:

$$P = \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{10}$$

Note: This is just a CES cost function.

Finding X

Now we have a simple two good problem:

$$\max u(C, y) \tag{11}$$

subject to

$$PC + y = m \tag{12}$$

FOC:

$$u_y / u_C = 1/P \tag{13}$$

Example: $u(C, y) = \alpha \ln(C) + (1-\alpha)\ln(y)$.

- $1/P = \frac{1-\alpha}{\alpha}\frac{C}{y}$
- with budget constraint: $y = (1-\alpha)m$ and $PC = \alpha m$.

Ideal price index

Another way of thinking about the household problem:

1. For given C, find the cost minimizing c_i. Define the price index as

$$PC = \sum p_i c_i \tag{14}$$

   1.1 max $u(C, y)$ subject to $PC + y = m$.

The cost minimizing price index is

$$P = \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{15}$$

Ideal price index I

Proof:

$$\min \sum_i p_i c_i + \lambda \left[\left(\sum_j c_j^\theta \right)^{1/\theta} - C \right] \tag{16}$$

FOC:

$$p_i = \lambda \left(\sum_j c_j^\theta \right)^{(1/\theta)-1} c_i^{\theta-1} \tag{17}$$

$$= \lambda C^{1-\theta} c_i^{\theta-1} \tag{18}$$

Solve for λ:

$$c_i = (\lambda / p_i)^{1/(1-\theta)} C \tag{19}$$

Ideal price index II

$$\left(\sum c_i^{\theta}\right)^{1/\theta} = C\lambda^{1/(1-\theta)}\left(\sum p_i^{\theta/(1-\theta)}\right)^{1/\theta} \tag{20}$$

$$\lambda = \left(\sum p_i^{\theta/(1-\theta)}\right)^{(1-\theta)/\theta} \tag{21}$$

Substitute and simplify.

The demand functions $c_i/C = (p_i/P)^{-\varepsilon}$ emerge.

QED

Digression: An Alternative Derivation

By definition:

$$PC = \sum p_i c_i \tag{22}$$

We need to express C and $\sum p_i c_i$ as functions of prices to solve for P.

First-order conditions determine relative demands:

$$c_i / c_1 = p_i^{-\varepsilon} / p_1^{-\varepsilon} \tag{23}$$

Sub into expression for

$$\begin{aligned} \sum p_i c_i &= c_1 \sum p_i \left(c_i / c_1\right) \\ &= c_1 p_1^{\varepsilon} \sum p_i^{1-\varepsilon} \end{aligned}$$

Alternative Derivation

Sub the same into expression for

$$
\begin{aligned}
C &= c_1 \left(\sum (c_i/c_1)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\
&= c_1 \left(\sum (p_i/p_1)^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\
&= c_1 p_1^{\varepsilon} \left(\sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}
\end{aligned}
$$

Take the ratio:

$$
P = \frac{PC}{C} = \frac{c_1 p_1^{\varepsilon}}{c_1 p_1^{\varepsilon}} \frac{\sum p_i^{1-\varepsilon}}{\left(\sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}}
$$

Simplify to get the solution for P.

Alternative Derivation

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \tag{24}$$

Proof:

$$p_i c_i = p_i c_1 \; (p_i/p_1)^{-\varepsilon}$$

$$\begin{aligned} \sum p_i c_i &= PC = c_1 p_1^{\varepsilon} \; \sum p_i^{1-\varepsilon} \\ &= c_1 p_1^{\varepsilon} \; P^{1-\varepsilon} \end{aligned}$$

$$PC \; P^{\varepsilon-1} = c_1 p_1^{\varepsilon}$$

Rearrange. QED.

Household summary

- Assume a Dixit-Stiglitz composite consumption good in preferences.
- Then demand is isoelastic.
  - the elasticity is determined by the elasticity of substitution across varieties in C.
- The cost of the optimal bundle C is given by P.
- The household reduces to a 2 good problem with standard solution.

Firms

- Each firm has a monopoly over a variety i.
- The demand elasticity is ε.
- Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\varepsilon} \tag{25}$$

- Assumption: The firm is small enough to neglect its effect on C and P.

Equilibrium

- Assume symmetry.
- Price index:

$$
\begin{aligned}
P &= \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \\
&= N^{\frac{1}{1-\varepsilon}} \frac{\psi}{1-1/\varepsilon}
\end{aligned}
$$

- More goods of the same price $\rightarrow$ it costs less to achieve the same utility.

Equilibrium: Profits

$$\begin{aligned}\pi_i &= c_i\,(p_i - \psi) \\ &= C\;P^{\varepsilon}\;p_i^{-\varepsilon}\;(p_i - \psi) \\ &= C\;N^{\varepsilon/(1-\varepsilon)}\;\frac{\varepsilon}{\varepsilon - 1}\psi \qquad (26)\end{aligned}$$

More varieties can increase profits:

- Direct effect: P falls - more competitors erode profits.
- "Aggregate demand externality": C may rise (depends on preferences)
  - Higher N raises marginal utility for a given variety.
  - Innovators impose pecuniary externality on competitors.

Continuum of varieties

- Nothing changes when i is continuous.
- Replace all $\sum$ with $\int$.

Reading

- Acemoglu (2009), ch. 12.
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.

Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.