Problem Set 6: Innovation and Growth

Econ720. Fall 2017. Prof. Lutz Hendricks

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1 Stochastic patent duration

[Due to Matt Doyle] Consider a version of the "Expanding Variety of Goods" model in which innovators' monopoly power diminishes over time. Otherwise the model is standard.

Demographics: There is a single representative household.

Endowments: The household is endowed with L units of labor, which can only be used for work.

Preferences:

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} \cdot e^{-\rho t} dt. \tag{1}$$

Technology:

- Final goods are produced from labor and intermediate inputs according to

$$Y = AL^{1-\alpha} \cdot \sum_{j=1}^{N} (X_j)^\alpha, \tag{2}$$

where $0 < \alpha < 1$, Y is output, L is labor input, X_j is the input of the $j'th$ type of the intermediate good, and N is the number of varieties.

- It takes one unit of final goods to produce one unit of intermediates.
- It costs η units of the final good to create a new type of intermediate good.

Market arrangements:

- The final goods sector is perfectly competitive.
- Intermediate goods producers hold monopolies.
- There is free entry for innovators.
- Households own all firms in the economy.

Patents: Upon innovation, the innovator receives a patent. If intermediate good j is currently monopolized, it becomes competitive in the next instant dT with probability $p \cdot dT$, where $p \geq 0$. Thus, if good j is invented at time t, the probability of it still being monopolized at some future date $v \geq t$ is $e^{-p \cdot (v-t)}$.

Notation: Denote by N^c, the number of intermediate goods produced competitively and by N the total number of intermediate goods.

Answer the following questions:

1. State the household problem and its solution.
2. Solve the problem of the final goods producer.
3. Solve the problem of the intermediate input producer.
4. State the free entry condition for innovation.
5. Define an equilibrium.
6. Derive the quantity of X_j produced when the j'th producer is a monopolist. D
7. Derive the quantity of X_j produced when the j'th intermediate good is produced competitively.
8. Using free entry and the definition of profits, show that:

   $$r = (L/\eta) \cdot A^{1/(1-\alpha)} \cdot \frac{1-\alpha}{\alpha} \cdot \alpha^{2/(1-\alpha)} - p \tag{3}$$

   Note that a higher p (shorter patents) reduces growth in this model. This is, of course, not a general result.
9. Solve for a balanced growth values of $\dot{c}/c, N^c/N$, and Y/N. Hint: Use the following approximation: $\dot{N}^c = p \cdot (N - N^c)$.