Midterm Exam. Econ720. Fall 2018

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- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- The total number of points is 100.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 OLG endowment economy

Demographics: In each period, a unit mass of households are born. Each lives for 2 periods.

Preferences: $\ln c_t^y + c_{t+1}^o$ – linear in old consumption.

Endowments: $e^y = 2$ when young and $e^o = 2$ when old.

Technology: None. Endowments can only be eaten: $e^y + e^o = c_t^y + c_t^o$.

Markets: We consider Arrow-Debreu and sequential trading.

Questions:

1. Consider the Arrow-Debreu equilibrium.
   (a) [15 points] Solve the household problem; obtain closed form solutions for young and old consumption.
   (b) [4 points] What is the marginal propensity to consume when young out of young income? What is the intuition?
   (c) [8 points] Define an Arrow-Debreu equilibrium.
   (d) [12 points] Show that there is a unique Arrow-Debreu equilibrium with $\pi_{t+1} = p_{t+1}/p_t = 2$ (for all t).
2. Consider the sequential trading equilibrium. There are now markets for goods and bonds (in zero net supply) in each period.
   (a) [15 points] Define the equilibrium.
   (b) [5 points] Show that the unique equilibrium looks like the Arrow-Debreu equilibrium.
3. Add a storage technology that transforms 1 unit of the good into $\gamma > 0$ units in the next period.
   (a) [15 points] Define a sequential markets equilibrium.
   (b) [5 points] Under what conditions will the storage technology be used?

2 Answers: OLG model

1. Arrow-Debreu
   (a) Household
      i. $\max u\left(c_t^y\right) + \beta c_{t+1}^o$ subject to $p_t e^y + p_{t+1} e^o = p_t c_t^y + p_{t+1} c_{t+1}^o$. Euler: $u'\left(c_t^y\right) = p_t/p_{t+1}$. This is a special case of the usual Euler equation where $u'\left(c_{t+1}^0\right) = 1$.

ii. Solution: c_t^y, c_{t+1}^o that solve Euler and budget constraint.

iii. With log utility: $c_t^y = p_{t+1}/(p_t\beta)$ and $c_{t+1}^o = e^y p_t/p_{t+1} + e^o - 1$.

(b) The marginal propensity to consume is 0. Intuition: Linear utility at any age fixes marginal utility at all ages. There are no income effects.

(c) Arrow-Debreu CE: $\{c_t^y, c_t^o, p_t\}$ that solve

i. household: see above (2)

ii. goods market clearing: $c_t^y + c_t^o = e^y + e^o$.

(d) Solution for Arrow-Debreu: Let $\pi_{t+1} = p_{t+1}/p_t$. Goods market: $2+2 = \pi + 2/\pi + 2 - 1$. The quadratic formula gives $\pi = 1$ or $\pi = 2$.

i. Steady state utilities are: with $\pi = 1$: $\ln(1) + 3 = 3$; with $\pi = 2$: $\ln(2) + 2 < 3$. As expected, the $\pi = 1$ equilibrium dominates; because it gets the golden rule.

ii. But: $\pi = 1$ is not an equilibrium. The initial old would violate their budget constraint.

2. Sequential trading equilibrium:

(a) Equilibrium:

i. Household: $1/c_t^y = R_{t+1}$. $c_{t+1}^o = e^y R_{t+1} + e^o - 1$.

ii. Goods market: unchanged.

iii. But now we also have bond market clearing: $c_t^y = 2$.

(b) Clearly the same equilibrium allocation as Arrow-Debreu where $R_{t+1} = 1/\pi_{t+1} = 1/2$.

3. Storage technology:

(a) Equilibrium:

i. Household now maximizes utility subject to $e^y = c_t^y + k_{t+1}$ and $c_{t+1}^o = e^o + \gamma k_{t+1}$ and $k_{t+1} \geq 0$. The Euler equation is $u'(c_t^y) \geq \gamma$ with equality if $k_{t+1} > 0$.

ii. Goods market clearing: $e^y + e^o + k_{t+1} = \gamma k_t + c_t^y + c_t^o$.

(b) If $k_{t+1} > 0$: $c_t^y = 1/\gamma$, $c_{t+1}^o = e^o + \gamma e^y - 1/\gamma$. $k_{t+1} = e^y - 1/\gamma$. Hence, the storage technology is used when $1/\gamma < e^y = 2$ or $R = \gamma > 1/2$. As expected.

4. Planner (not asked in the exam):

(a) The planner solves

$$\max \sum_t \omega^t \left(\ln c_t^y + e^y + e^o - c_{t+1}^o\right) \tag{1}$$

with first-order conditions $c_t^y = \omega < 1$ and $c_t^o = e^y + e^o - \omega$.

(b) Clearly, the $\pi = 2$ equilibrium is not efficient, but we knew that. It implies that the interest rate $r = 1/\pi - 1$ is below the population growth rate (0).

3 RCE with human capital

Demographics: There are J types of households, indexed by j. Each type has unit mass and lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$

Endowments: Type j is endowed with $k_{j,0}$ units of capital and $h_{j,0}$ units of human capital in $t = 0$. In each t, a household has one unit of time that can be used for studying $(l_{j,t})$ or working $(1 - l_{j,t})$.

Technology:

- Output of goods: $F(K_t, H_t) + (1 - \delta) K_t = C_t + K_{t+1}$ where K_t is aggregate capital and $H_t = \sum_j h_{j,t}(1 - l_{j,t})$ is aggregate labor input in human capital units.
- Human capital: $h_{j,t+1} = G(h_{j,t}, l_{j,t})$.

Questions:

1. [8 points] Write down the Bellman equation of agent j.
2. [4 points] What is the aggregate state of the economy?
3. [9 points] Write down the consistency condition for the aggregate state that intuitively says: expected S' is consistent with the actions of the agents.

4 Answers

1. Bellman
$$V(k, h; S) = \max u(c) + \beta V(k', h'; S') \tag{2}$$
subject to $h' = G(h, l)$ and $wh(1 - l) + Rk = c + k'$ and aggregate law of motion $S' = \sigma(S)$.
2. Aggregate state: $S = (\kappa, \eta)$ where $\kappa_j = k_j$ and $\eta_j = h_j$ in equilibrium.
3. Consistency: Given $\kappa' = \phi(\kappa, \eta)$ we have $\phi_j(\kappa, \eta) = k'(\kappa_j, \eta_j, \kappa, \eta)$. Given $\eta' = \varphi(\kappa, \eta)$ we have $\varphi_j(\kappa, \eta) = h'(\kappa_j, \eta_j, \kappa, \eta)$.

End of exam.