Midterm Exam. Econ720. Fall 2016

Professor Lutz Hendricks. UNC.

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 CIA with cash and credit goods

85 points total.

Demographics: There is a representative household of unit mass who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t U(c_{1,t}, c_{2,t})$ with the usual assumptions on U.

Endowments: y_t units of goods; M_0 units of money at $t = 0$; B_0 units of nominal one period bonds at $t = 0$

Government: Prints new money and hands it out as lump sum transfer: $M_{t+1} - M_t = T_t$

Technology:

- The endowment can be made into two types of consumption goods: $y_t = c_{1,t} + c_{2,t}$. This implies that the price of both goods is the same (p_t).
- CIA: $p_t c_{1,t} \leq M_t$.

Markets: competive markets for goods (price p_t for both), bonds (price 1), money (price 1).

Questions:

1. Write down the household problem. Derive the first-order conditions and define a solution. Show that

$$u_2 R = u_2 + \lambda = u_1 \tag{1}$$
$$u_2 (1 + \pi') = R' \beta u_2 (') \tag{2}$$

   where λ is the Lagrange multiplier on the CIA constraint and $R - 1$ is the nominal interest rate.

   Hint: the budget constraint is

$$M_{t+1} + B_{t+1} + p_t c_{1,t} + p_t c_{2,t} = p_t y_t + M_t + T_t + R_t B_t \tag{3}$$

2. Interpret the first-order conditions (1) and (2).
3. Define a competitive equlibrium.
4. Suppose that the government fixes the interest rate at a constant $R > 1$ in every period. Characterize the equilibrium consumption allocation.
5. Show hat the Friedman Rule is optimal (zero nominal interest rate). Hint: this is very short.

2 Carrol et. al (2000)

15 points total.

An infinitely lived household in continuous time solves $\max \int_0^\infty e^{-\theta t} U\left(c_t, h_t\right)$ subject to $\dot{k}_t = r_t k_t + w_t - c_t$ and $\dot{h}_t = \rho\left(c_t - h_t\right)$. The control is c_t. The interpretation is habit formation where past consumption affects current marginal utility.

Questions:

1. Write down the current value Hamiltonian.
2. Derive the necessary first-order conditions.

End of exam.

3 Answer: CIA with cash and credit goods[1]

1. Budget constraint in real terms:

$$\left(m'+b'\right)\left(1+\pi'\right)+c_1+c_2=y+m+\tau+Rb \tag{4}$$

where $m=M/P$, $b=B/p$, $\tau=T/p$.

Bellman:

$$\begin{aligned} V\left(m,b\right) &= \max U\left(c_1, y+m+\tau+Rb-c_1-\left(m'+b'\right)\left(1+\pi'\right)\right) && (5)\\ &+\beta V\left(m',b'\right)+\lambda\left(m-c_1\right) && (6)\end{aligned}$$

FOCs:

$$\begin{aligned} u_1 &= u_2+\lambda && (7)\\ u_2\left(1+\pi'\right) &= \beta V_b\left('\right)=\beta V_m\left('\right) && (8)\\ V_m &= u_2+\lambda && (9)\\ V_b &= u_2 R && (10)\end{aligned}$$

Simplify:

$$\begin{aligned} u_2 R &= u_2+\lambda=u_1 && (11)\\ u_2\left(1+\pi'\right) &= R'\beta u_2\left('\right) && (12)\end{aligned}$$

Solution: $\{c_1,c_2,m,b,\lambda\}$ that satisfy 3 FOCs, budget constraint, and CIA or $\lambda=0$.

2. Interpretation:

   (a) $u_1=u_2+\lambda$: give up a unit of c_1. Then you can eat one unit of c_2 and you relax the CIA constraint.

   (b) $u_2R=u_1$: In order to buy good 2, you can bring a bond into the period. This earns nominal interest R. In order to buy good 1, you need cash with nominal interest 1. So R is effectively the relative price of the 2 goods.

   (c) $u_2=R'/\left(1+\pi'\right)\beta u_2\left('\right)$: this is the standard Euler equation for the credit good, where $R/\left(1+\pi\right)$ is the real interest rate.

3. CE: $\{c_1,c_2,m,b,\lambda\}$ and $\{p,R\}$ that satisfy

   (a) household: 5

   (b) government budget constraint

   (c) goods market (RC)

[1] Based on the Minnesota Qualifying Exam 2008.

(d) money market: implicit

(e) bond market: $b = 0$

4. If $R > 1$: CIA constraint binds in every period. $u_2 R = u_1$ and the resource constraint fully pin down a unique allocation $\{c_{1,t}, c_{2,t}\}$.

5. Zero nominal interest rate ($R = 1$): We get the same allocation as in an economy without money. So the Friedman Rule is optimal.

   To see this: $u_1 = u_2$ means: no distortion to the static allocation. This determines consumption in each period (with the resource constraint).

4 Answer: Carrol et al (2000)

1. $H = U(c, h) + \lambda [rk + w - c] + \mu\rho [c - h]$

2. Optimality requires:

$$U_c = \lambda - \rho\mu \tag{13}$$

$$\dot{\lambda} = (\theta - r)\lambda \tag{14}$$

$$\dot{\mu} = (\theta + \rho)\mu - U_h \tag{15}$$

End of file.