Midterm Exam. Econ720. Fall 2015

Professor Lutz Hendricks. UNC.

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Capital Depreciates in 2 Periods

Consider a standard growth model in discrete time where capital fully depreciates in 2 periods.

Demographics: There is a representative agent of mass 1 who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$.

Endowments: k_0 at $t = 0$.

Technology: $f(k_t) = c_t + i_t$ with $k_{t+1} = i_t + i_{t-1}$.

Questions:

1. Write down the planner's Bellman equation.
2. Define a solution to the planner's problems.

2 Dynamics of Warm Glow Bequests

Demographics: In each period, a unit mass of persons is born. Each person lives for 2 periods.

Preferences: Households value consumption only when old. They also value the bequests they leave to their children. Lifetime utility of a person born in $t-1$ is given by $\ln c_{i,t} + \beta \ln b_{i,t+1}$. In this model, the young really don't do anything.

Endowments: The initial old at $t = 1$ are endowed with $b_{i,1}$ units of the good. Denote the cdf of $b_{i,1}$ by Λ_1. Each old is endowed with 1 unit of work time.

Technology: $F(K_t, L_t) = C_t + K_{t+1}$ with constant returns to scale. The capital in this economy consists of the bequests given by the old.

Markets: There are competitive markets for goods (numeraire), capital rental (price q), labor rental (w).

The solution to the firm's problem is the same as always: $q_t = F_1(K_t, L_t) = f'(k_t)$ and $w_t = f(k_t) - q_t k_t$ with $k = K/L$.

The solution to the household problem is simple. The budget constraint is given by: $c_{it} + b_{i,t+1} = w_t + q_t b_{i,t}$. Note the timing: In t, the old give $b_{i,t+1}$ to their kids. Firms use this in production in $t+1$. The kids receive $b_{i,t+1}$ plus interest.

Because of log utility, the household bequeathes a constant fraction of income, regardless of prices:

$$b_{i,t+1} = \frac{\beta}{1+\beta} \left[w_t + q_t b_{i,t} \right] \tag{1}$$

Questions:

1. State the market clearing conditions.
2. Define a recursive competitive equilibrium.
3. Derive a law of motion of the aggregate capital stock k_t.

End of exam.

3 Answer: Capital Depreciates in 2 Periods

Based on Krusell's text, example 4.6.

1. Define two auxiliary state variables: $z_t = i_{t-1}$ and $x_t = i_{t-2}$. Then $k_t = z_t + x_t$. The laws of motion are: $x_{t+1} = z_t$ and $z_{t+1} = f(k_t) - c_t$.

   Bellman: $V(x,z) = \max u(c) + \beta V(z, f(z+x) - c)$.

2. FOC: $u'(c) = \beta V_2(.')$. $V_1 = \beta V_2(.') f'(x+z)$. $V_2 = \beta V_1(.') + \beta V_2(.') f'(x+z)$.

   Euler: $u'(c) = \beta u'(c') f'(k') + \beta^2 u'(c'') f'(k'')$.

   Solution: Sequences $\{c_t, k_{t+1}\}$that solve the Euler equation and the resource constraint. With boundary conditions k_0 given and $\lim \beta^t u'(c_t) k_t = 0$.

4 Answer: Dynamics of warm glow bequests[1]

1. Market clearing:

   (a) goods: resource constraint, but we need to define $C_t = \int_0^1 c_{it} di$ and $K_t = \int_0^1 b_{i,t} di$.

   (b) labor: $L_t = 1$.

   (c) capital rental: see definition of K.

2. RCE: The state variable is the distribution of wealth, $b_{i,t}$. Let's denote its cdf by $\Lambda(b)$.

   Equilibrium objects:

   (a) Household value and policy functions: $V(b), c(b), b'(b)$ (abusing notation here).

   (b) Price functions: $w(\Lambda), q(\Lambda)$.

   (c) The law of motion for Λ (which, in this degenerate mode, does not affect the household solution).

   Equilibrium conditions:

   (a) $V(b) = \max \ln c + \beta \ln b$ subject to $c + b' = w(\Lambda) + q(\Lambda) b$. Policy functions are stated in the question.

   (b) the firm's first order conditions.

   (c) market clearing conditions.

   (d) consistency: $\Lambda'(x) = \int_0^1 I(b'(b) \leq x) d\Lambda(b)$. This could be written in different ways. In words: The fraction of households with $b' \leq x$ tomorrow is given by the fraction of households today who leave less than x to their kids.

[1] Based on Acemoglu, Introduction to Modern Economic Growth, ch. 9.6.

3. Law of motion: From

$$
\begin{aligned}
k_{t+1} &= \int b_{i,t+1} di \qquad (2) \\
&= \frac{\beta}{1+\beta} \int \left[w_t + q_t b_{it} \right] di \qquad (3)
\end{aligned}
$$

we have

$$
k_{t+1} = \frac{\beta}{1+\beta} f\left(k_t\right) \qquad (4)
$$

as if there were a single agent with a fixed saving rate (a Solow model).

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