Midterm Exam. Econ720. Fall 2012

Professor Lutz Hendricks

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

---

1 Krueger and Ludwig (2006 JME)

Time t is discrete. There are $i = 1, ..., I$ countries which trade goods and capital services.

Demographics: In each period $N_{t,i} = N_{0,i}\gamma_{Ni}^{t}$ young persons are born in country i. γ_{Ni} is the population growth rate. Each person lives for 2 periods.

Preferences: $\ln(c_{t,i}^{y}) + \beta \ln(c_{t+1,i}^{o})$.

Endowments: Each young is endowed with one unit of work time. The initial old each endowed with x_i units of capital.

Technologies: Country i produces output according to

$$Y_{t,i} = K_{t,i}^{\alpha}(Z_i A_t N_{t,i})^{1-\alpha} \tag{1}$$

$Z_i > 0$ is a country-specific productivity parameter. $A_t = \gamma_A^t$ is a common productivity trend. All countries produce the same good. Output is used for consumption and investment. Capital depreciates at rate δ.

Markets: In each country, a representative firm rents labor from the local households (at wage rate $w_{t,i}$) and capital from households in all countries. Capital and consumption goods are perfectly mobile across borders.

Given the preferences and technology, we know that the young save $s_{t,i} = \frac{\beta}{1+\beta} w_{t,i}$.

Questions:

1. State the firm's problem for country i and derive its first-order conditions. Show that $k_t = K_{t,i}/(Z_i A_t N_{t,i})$ is the same for all countries.

2. Define a competitive equilibrium.

3. Derive the law of motion for k_t:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)k_t^{\alpha}\gamma_A^{-1}\tilde{\gamma}_N^{-1} \tag{2}$$

   where $\tilde{\gamma}_N = \tilde{N}_{t+1}/\tilde{N}_t$ is the efficiency weighted growth rate of the world population and where $\tilde{N}_t = \sum_i Z_i N_{t,i}$.

4. Derive an expression for properly *detrended* output per person on the balanced growth path. Explain how it depends on γ_N and provide intuition.

5. Derive the balanced growth values for the investment rate of country i, $ir_i = I_{t,i}/Y_{t,i} = (K_{t+1,i} - K_{t,i})/Y_{t,i}$, and for the growth rate of the asset to income ratio (sort of a saving rate), $sr_i = (S_{t,i} - S_{t-1,i})/Y_{t,i}$. Hint: you know $S_{t,i}/Y_{t,i}$ from the saving function. You get $K_{t,i}/Y_{t,i}$ simply from the production function. And you know that S and K grow at known rates.

6. Note that the current account position of country i is given by $ca_i = sr_i - ir_i$. What kinds of countries run current account deficits? Explain the intuition.

2 Answer: Krueger and Ludwig

1. Firm: $\max K_{t,i}^{\alpha}(Z_i A_t N_{t,i})^{1-\alpha} - q_t K_{t,i} - w_{t,i} N_{t,i}$. FOCs:

$$q_t = \alpha k_t^{\alpha-1} \tag{3}$$
$$w_{t,i} = (1-\alpha) Z_i A_t k_t^{\alpha} \tag{4}$$

   where $k_t = K_{t,i}/(Z_i A_t N_{t,i})$ is equalized across countries because of capital flows that equalize q_t.

2. Equilibrium: $\{s_{t,i}, c_{t,i}^y, c_{t,i}^o, K_{t,i}, k_t\}$ and $\{q_t, w_{t,i}, r_t\}$ that solve:

   (a) household: saving function and 2 budget constraints;

   (b) firm: 2 first-order conditions

   (c) capital market clearing

$$K_{t+1} = \sum_i N_{t,i} s_{t,i} = \sum_i K_{t+1,i} \tag{5}$$

   (d) goods market clearing:

$$K_{t+1} + C_t = (1-\delta) K_t + \sum_i Y_{t,i} \tag{6}$$

   where $C_t = \sum_i N_{t,i} c_{t,i}^y + N_{t-1,i} c_{t,i}^o$.

   (e) identity: $r_{t+1} = q_{t+1} - \delta$

3. Start from law of motion for K_t and define the saving rate as $\bar{s} = (1-\alpha)\beta/(1+\beta)$:

$$K_{t+1} = \bar{s} \quad A_t k_t^{\alpha} \sum_i N_{t,i} Z_i \tag{7}$$
$$= \bar{s} A_t k_t^{\alpha} \tilde{N}_t \tag{8}$$

   Definition:

$$\begin{aligned} K_{t+1} &= \sum_i K_{t+1,i} = \sum_i k_{t+1} A_{t+1} Z_i N_{t+1,i} \\ &= k_{t+1} A_t \gamma_A \tilde{N}_t \tilde{\gamma}_N \end{aligned}$$

   Equate the two expressions and rearrange. Done.

4. Balanced growth: k is constant at

$$k^{1-\alpha} = \sigma = \bar{s} \gamma_A^{-1} \tilde{\gamma}_N^{-1} \tag{9}$$

   Note that detrended output per young person (or worker) is given by

$$\frac{Y_{t,i}}{N_{t,i} Z_i A_t} = k_t^{\alpha} = \sigma^{\alpha/(1-\alpha)} \tag{10}$$

where the last equality applies in steady state. Detrended per capita output is given by

$$y_{t,i} = \frac{Y_{t,i}}{(N_{t,i} + N_{t-1,i}) Z_i A_t} = \frac{\sigma^{\alpha/(1-\alpha)}}{1 + \gamma_{N,i}^{-1}} \tag{11}$$

For a small country k is not affected by population growth (it depends on the world growth rate of population). Then higher own population growth raises y. The reason is mechanical: the young do the producing. With high $\gamma_{N,i}$ there are a lot of young people in the economy.

A secondary effect is that higher population growth increases world population growth. That reduces the capital-labor ratio (the old save and the young work) and therefore output per worker.

5. Investment rate:

$$\frac{I_{t,i}}{Y_{t,i}} = \frac{K_{t+1,i} - K_{t,i}}{Y_{t,i}} \tag{12}$$

$$= \frac{(\gamma_A \gamma_{N,i} - 1) K_{t,i}}{Y_{t,i}} \tag{13}$$

$$= (\gamma_A \gamma_{N,i} - 1) k / k^{\alpha} \tag{14}$$

Sub in $k^{1-\alpha} = \sigma$ and we have

$$\frac{I_{t,i}}{Y_{t,i}} = \frac{\gamma_A \gamma_{N,i} - 1}{\gamma_A \tilde{\gamma}_N} \bar{s} \tag{15}$$

$$= \bar{s}(1 - \gamma_A^{-1} \gamma_{N,i}^{-1}) \frac{\gamma_{N,i}}{\tilde{\gamma}_N} \tag{16}$$

The "saving rate," defined as change in assets owned by country i divided by output is given by

$$\frac{S_{t,i} - S_{t,i-1}}{Y_{t,i}} = \bar{s}(1 - \gamma_A^{-1} \gamma_{N,i}^{-1}) \tag{17}$$

6. The current account

$$ca_i = sr_i (1 - \gamma_{N,i} / \tilde{\gamma}_N) \tag{18}$$

is negative for countries with faster than average population growth. The old do the saving, but the young need the capital at work.

End of file.