Midterm Exam. Econ720. Fall 2011

Professor Lutz Hendricks

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Three period OLG model

Consider the following overlapping generations model.

Demographics: Agents live for three periods. At each date, a unit measure of households is born. Cohort t is born at $t-1$ and middle-aged at t.

Preferences: Generation t derives utility from consumption when middle aged and when old according to

$$U(c_{1t}, c_{2t+1}) = \ln(c_{1t} - \gamma a_t) + \ln(c_{2t+1})$$

where a represents an "aspiration" level which is inherited from the parents:

$$a_t = \gamma c_{1t-1}$$

and $0 < \gamma < 1$. The idea is that growing up rich raises the standards for a "good life."

Endowments: When middle aged, the household is endowed with one unit of time which he supplies inelastically to firms.

Technology: A single good is produced according to a constant returns to scale production function. Inputs are physical and human capital. Output is used for investment, education spending e, and consumption:

$$K_t^\alpha h_t^{1-\alpha} = K_{t+1} + e_{t+1} + c_{1t} + c_{2t}$$

When **young**, the household can produce human capital according to $h_t = e_t^\beta$ with $0 < \beta < 1$.

Market arrangements are standard and competitive. Households buy goods from firms when middle aged and old (at price 1). Households work for the firm. Each unit of human capital earns wage rate w_t. Households rent capital to firms at rental rate r_t.

The household of cohort t solves the following problem:

$$\max_{c_{1t},c_{2t+1},e_t,h_t,s_t} U\left(c_{1t},c_{2t+1}\right) \tag{1}$$

subject to the budget constraints

$$c_{1t}+s_t=w_t h_t-(1+r_t)e_t \tag{2}$$

and

$$c_{2t+1}=(1+r_{t+1})s_t \tag{3}$$

where $h_t=e_t^{\beta}$. We will solve this by backward induction, starting with the middle aged household, then proceeding to the young household's choice of e_t.

Questions:

1. [26 points] Solve the problem of a middle aged household. You should obtain a saving function in closed form. Also derive the indirect utility function

$$V(e_t,a_t)=2\ln(1/2)+\ln(1+r_{t+1})+2\ln\left(w_t h_t-(1+r_t)e_t-\gamma a_t\right)$$

2. [13 points] Solve for the optimal level of human capital investment by a young agent. Use the indirect utility function.

3. [10 points] Characterize optimal firm behavior.

4. [17 points] Define a competitive equilibrium. Make sure you explicitly state the market clearing conditions and that you have the same number of variables and equations.

5. [21 points] Consider the special case $\beta=0$, so that human capital is an exogenous constant ($h_t=h$) and $e_t=0$. Show that the steady state level of $k=K/h$ is characterized by

$$w(1-\gamma)=(2-\gamma)k \tag{4}$$

6. [13 points] What happens to the steady state level of k when γ rises? What is the intuition for this result?

2 Answer: Three period OLG model

1. The household is endowed with e_t and h_t. He solves

$$\max \ln (w_t h_t - (1+r_t)e_t - s_t - \gamma a_t) + \ln ((1+r_{t+1})s_t)$$

The FOC is

$$c_{2t+1}/(c_{1t} - \gamma a_t) = (1+r_{t+1})$$

Using the budget constraints we find the saving function

$$\frac{w_t h_t - (1+r_t)e_t - s_t - \gamma a_t}{(1+r_{t+1})s_t} = \frac{1}{(1+r_{t+1})}$$

$\Rightarrow$

$$s_t = (w_t h_t - (1+r_t)e_t - \gamma a_t)/2$$

This makes sense: Without "aspirations" the household would save a constant fraction of income due to log utility.

Indirect utility function: Substitute saving function into the objective function and collect terms to obtain the result to be shown.

2. When young: the household maximizes the indirect utility function. In this case, this is equivalent to maximizing the present value of earnings (why?).

$$\max w_t e_t^{\beta} - (1+r_t)e_t$$

Solution:

$$e_t = (\beta w_t/(1+r_t))^{1/(1-\beta)}$$

3. The firm's problem is standard. Define $k = K/h$. Then $w_t = (1-\alpha)\,k_t^{\alpha}$ and $1 + r_t = \alpha\ k_t^{\alpha-1}$. The same r as in the household problem because capital depreciates fully.

4. A competitive equilibrium is a sequence of quantities $(c_{1t}, c_{2t}, s_t, e_t, h_t, K_t, a_t)$ and prices (w_t, r_t) that satisfy:

- Household: saving function and 2 budget constraints; optimality of e;
- Firm: two FOCs
- Law of motion for a.
- Market clearing

Capital market clearing requires

$$s_t = K_{t+1} + e_{t+1}$$

The e term appears because education requires capital. Goods market clearing is the same as the resource constraint. Labor market clearing is implicit in the notation.

5. The saving function does not change because the adult household takes (h, e) as given. Now they are simply fixed. Capital market clearing is now

$$s = (wh - \gamma a)/2 = K = kh$$

We can substitute out a using $a = c_1$. The adult budget constraint implies $c_1 = wh - s$. Therefore,

$$s = (wh - \gamma(wh - kh))/2 = kh$$

Rearranging leads to the equation we are supposed to prove.

6. If children inherit aspirations from their parents, steady state capital is lower (they save less). Intuition: Ceteris paribus, a higher γ reduces saving; people require more consumption to maintain the same marginal utility.

End of file.