Final Exam. Econ720. Fall 2013

Professor Lutz Hendricks

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- Clearly number your answers.
- The total time is 2 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Shimer's MP Model

[50 points] Workers: A unit mass of workers lives forever. Lifetime utility is given by $\mathbb{E}\int_0^\infty e^{-rt}w_t$, where w_t denotes the wage. When unemployed, $w_t = z$.

Firms: An infinite mass of firms can post vacancies at a flow cost of c.

Matching: The number of new matches formed is $m(u,v)$, where v is the number of vacancies and u is the number of unemployed workers. m has *constant* returns to scale.

When a worker meets a vacancy, they produce output $p(t)$. Wages are determined by Nash bargaining. The worker receives fraction β of the joint surplus.

Breakups: Matches break up with Poisson probability $s(t)$.

Shocks: The state of the economy is a pair (p,s). Shocks arrive with Poisson probability λ. When a shock arrives, a new pair (p,s) is drawn from a distribution that depends on the current (p,s) state. Between shocks, (p,s) is constant.

Equilibrium: We are looking for an equilibrium where the state variables are (p,s). It is not obvious that these are the right state variables, but we won't worry about this problem. Let's assume that the wage only depends on (p,s) as well.

Questions:

1. [3 points] Show that the job finding rate f and the vacancy filling rate q depends only on "labor market tightness" $\theta = v/u$.

2. [3 points] Find a differential equation of the form $\dot{u} =$ inflows - outflows.

3. [9 points] The value of being unemployed is given by: $rU_{p,s} = z + f\left(E_{p,s} - U_{p,s}\right) + \lambda\left(\mathbb{E}_{p,s}U_{p',s'} - U_{p,s}\right)$, where $E_{p,s}$ is the value of being employed. $\mathbb{E}_{p,s}U_{p',s'}$ is the expected value of U, if today's state is (p, s) and and aggregate shock occurs. Explain this equation.

4. [12 points] State and explain analogous equations for the value of being employed $E_{p,s}$ and the value of a filled vacancy $J_{p,s}$. Note that an aggregate shock changes the value of being employed, even if the worker stays matched.

5. [8 points] Let $V_{p,s}$ denote the joint surplus of a match that is divided between workers and firms: $V_{p,s} = E_{p,s} - U_{p,s} + J_{p,s}$. After substituting out a few terms, the equation becomes

$$rV_{p,s} = p - z - f\left(E_{p,s} - U_{p,s}\right) - sV_{p,s} + \lambda\left(\mathbb{E}_{p,s}V_{p',s'} - V_{p,s}\right) \tag{1}$$

   Explain these equations in words.

6. [4 points] State and explain the free entry condition.

7. [3 points] What does Nash bargaining with weight β imply for the relationship between $V_{p,s}$, $J_{p,s}$, and $E_{p,s} - U_{p,s}$?

8. [8 points] Define an equilibrium.

Final notes: Having done all this work, it is easy to derive a closed form solution for $\theta_{p,s}$. Substitute the Nash bargaining weights into the equation for $V_{p,s}$ to obtain

$$rV_{p,s} = p - z - f(\theta)\,\beta V_{p,s} - sV_{p,s} + \lambda\left(\mathbb{E}_{p,s}V_{p',s'} - V_{p,s}\right) \tag{2}$$

Combine this with free entry to obtain

$$\frac{r + s + \lambda}{q(\theta)} + \beta\theta = (1 - \beta)\,\frac{p - z}{c} + \lambda\mathbb{E}_{p,s}\frac{1}{q(\theta)} \tag{3}$$

Now one ask question such as: does this model imply reasonable fluctuations in vacancies, unemployment, or labor market tightness with respect to productivity? The whole point of Shimer's paper is: it does not.

2 Lucas Fruit Trees With Crashes

[50 points] Demographics: There is a single, representative household who lives forever.

Preferences: $U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $u(c) = c^{1-\sigma}/(1-\sigma)$.

Endowments: The agent is endowed at $t = 0$ with 1 tree. In each period, the tree yields stochastic consumption d_t, which cannot be stored. d_t evolves as follows:

- If $d_t = d_{t-1}$, then $d_{t+1} = d_t$ forever after.
- If $d_t \neq d_{t-1}$, then $d_{t+1} = \gamma d_t$ with probability π and $d_{t+1} = d_t$ with probability $1 - \pi$. $\gamma > 1$.

In words: d grows at rate $\gamma - 1$ until some random event occurs (with probability $1 - \pi$), at which point growth stops forever.

Markets: There are competitive markets for consumption (numeraire) and trees (price p_t). Assume that p_t is *cum dividend*, meaning that d_t accrues to the household who buys the tree in t and holds it into $t + 1$.

Questions:

1. [8 points] State the household's dynamic program.
2. [10 points] Derive the Euler equation.
3. [10 points] Define a recursive competitive equilibrium. Key: what is the state vector?
4. [14 points] Characterize the stochastic process of p_t. Is p_t a Markov process? Hint: there are 2 phases: before and after dividends have stopped growing. Assume that p/d is constant during the phase with growth.
5. [8 points] What happens to the stock market when the economy stops growing? Does it crash? Under what condition?

End of exam.