Macroeconomics Qualifying Examination

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UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Two Countries

Demographics: There are two countries, A and B. In each lives an infinitely lived representative household.

Preferences: $\sum_{t=0}^{\infty} \beta_j^t u(c_{j,t})$ where $j \in \{A, B\}$, with $\beta_A > \beta_B$.

Endowments: At $t = 0$, each agent is endowed with capital k_0. In each period, the agent is endowed with one unit of work time.

Technologies: Each country produces output according to

$$y_{j,t} = f(k_{j,t}, l_{j,t}) = k_{j,t}^{\alpha} l_{j,t}^{1-\alpha} \tag{1}$$

where $k_{j,t}$ is the capital employed in j (though it may be owned elsewhere). If the economy is closed, the resource constraint is

$$k_{j,t+1} = f(k_{j,t}, l_{j,t}) + (1-\delta) k_{j,t} - c_{j,t} \tag{2}$$

If the economy is open, goods can be traded costlessly and capital can be rented internationally. Labor cannot cross borders.

Markets: goods (numeraire), capital rental (q), labor rental (w), one period bonds in zero net supply (rate of return R).

Questions:

1. [6 points] Assume closed economies. Solve for the steady state capital stocks in A and B. You do not need to derive standard equations, such as the Euler equation.
2. Now consider the equilibrium as $t \to \infty$ with free trade in goods and capital mobility. Labor is still immobile.
   (a) [4 points] What happens to c_A and c_B as $t \to \infty$? Explain.
   (b) [5 points] Find the steady state capital stocks in both countries. Explain what you find.
   (c) [9 points] Find steady state debt in B. Explain in words what you find. Hint: Write out the budget constraint and apply what you know about c_B.
3. [6 points] What does this model imply for the role of preference heterogeneity for wealth inequality? Which assumptions are responsible for the model's implications? How could you modify the model to obtain more reasonable implications?

1.1 Answer: Two Countries

1. We know the Euler equation is $u'(c) = \beta R' u'(c')$, so that in steady state $R = 1/\beta = f'(k^*) + 1 - \delta$. Therefore
$$k_j^* = \left(\frac{\alpha}{1/\beta_j - (1-\delta)}\right)^{1/(1-\alpha)} \tag{3}$$
More patience is associated with more capital.

2. Free trade.

   (a) The key: The Euler equation is supposed to hold for both countries with the same R. This cannot be. Hence, we must have $c_B \to 0$ and c_A constant.

   (b) Hence, the interest rate is the same as the autarky rate in A. Hence, the steady state capital stock (in each country) is still given by (3) with β_A. The more patient agents saves; the less patient one borrows. Eventually, A owns all the capital and consumes everything. Then its saving decision decides the capital stock.

   (c) In B, the budget constraint with $c_B = 0$ is
$$a'_B = Ra_B + w \tag{4}$$
where a denotes the assets owned by B. Set assets constant over time and we have $a_B = -w/(R-1)$. B borrows up to the point where all their future income is pledged for repaying debt. The wage is given by the marginal product of labor
$$w = (1-\alpha)(k^*)^\alpha \tag{5}$$

3. Key assumptions: agents live forever and their preferences never change. Then arbitrarily small differences in discount rates produce degenerate wealth distributions where the most patient agent holds all wealth. A more sensible model would have finite lives (or the Krusell-Smith approximation of finite lives).

2 RCE with heterogeneity

Demographics: There is a unit mass of households who live forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_{j,t})$.

Endowments: Household j is endowed with $k_{j,0}$ units of capital at $t = 0$ and with $e_{j,t}$ efficiency units of labor in period t. $e_{j,t}$ is drawn i.i.d. from density D_e.

Technology:

- $K_{t+1} = F(K_t, z_t L_t) + (1 - \delta) K_t - C_t$. F has constant returns to scale.
- $K_t = \int_0^1 k_{j,t} dj$. $L_t = \int_0^1 e_{j,t} dj$. $C_t = \int_0^1 c_{j,t} dj$.
- z_t is an aggregate productivity shocks with law of motion (conditional density) $D_z(z_{t+1}; z_t)$.

Markets: There are competitive markets for goods (numeraire), labor (w), and capital rental (q).

Firms solve a standard static profit maximization problem. They rent capital and labor to maximize period profits.

Questions:

1. [6 points] What is the aggregate state of the economy? Denote it by S.
2. [11 points] Define a *recursive* competitive equilibrium. You do not need to write out the household or firm problem (they are both standard). Just state the objects and equilibrium conditions. Be precise.
3. [8 points] Explain why this kind of model is hard to compute and how Krusell and Smith have proposed to solve this problem.

2.1 Answer: RCE with heterogeneity

1. $S = (z, \Gamma)$ where Γ is the joint distribution of households over $(k_{j,t}, e_{j,t})$.
2. RCE objects:
   (a) household: value function $V(s_j, S)$, saving function $k'_j = \kappa(s_j, S)$, and consumption function $h(s, S)$, where the individual state is $s_j = (k_j, e_j)$.
   (b) firm: price functions $w(S), q(S)$.
   (c) law of motion for the aggregate state: Since the law of motion of z is exogenous, we just need $\Gamma' = M(z, \Gamma)$

RCE equations:

(a) market clearing:

  i. goods: $C(S)+K(S')=F(K(S),L(S))+(1-\delta)K(S)$ where $C(S)=\int_s h(s,S)\Gamma(s)\,ds$, etc.

  ii. labor: $L(S)=\int_s e(s)\Gamma(s)\,ds$, where $e(s)$ means the e endowment that is part of s.

  iii. capital rental: analogous.

(b) consistency:

$$\Gamma'(s')=M(z,\Gamma)(s')=\int_s \Gamma(s)\,\mathbb{I}[\kappa(s,S)=k'(s')]\,D_e(e(s'))\,ds \tag{6}$$

In words: the fraction of persons with $s'=(e',k')$ is given by the sum of the probabilities that a person of each s transitions into that state. That probability is $D_e(e')$ for states where persons choose k' and 0 otherwise.

3. The problem: Γ is an infinite dimensional object (meaning: it cannot be exactly represented using a finite number of pieces of information, such as parameters).

   KS propose to solve this problem by replacing Γ with moments of Γ, such as the mean of k_j or the covariance of (k_j,e_j). The model then replaces the true law of motion with that of a finite vector of moments; call those α_j. That is, everywhere a Γ shows up in the model, it is replaced by $\hat{\Gamma}=[\alpha_1,...,\alpha_J]$, which is a vector of scalar moments. Then M is replaced by $\hat{M}$ such that $\alpha_j'=\hat{M}_j(\alpha_1,...,\alpha_J)$.

3 Optimal Policy in the New Keynesian Model

Setup We consider a standard New Keynesian model with sticky prices.

- Demographics: There is a representative infinitely lived household.
- Endowments: The household has an initial bond position B_0. The household supplies labor elastically each period.
- Preferences: The lifetime utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + \xi \ln \left(\frac{M_t}{P_t} \right) \right)$$

  where $\beta \in (0,1)$, $\chi \geq 0$, $\theta > 0$, $\xi > 0$. C_t denotes consumption, N_t denotes labor, and $\frac{M_t}{P_t}$ denotes the real money balance.

- Technology: Monopolistic intermediate goods producers use only labor and have the constant returns to scale production function

$$Y_{j,t} = A_t N_{j,t}$$

  where A_t is a stochastic productivity term and common to all intermediate goods producers.

  There is a perfectly competitive final goods sector with a CES technology that aggregates intermediate inputs to a final good,

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

  where j are the input varieties and $\epsilon > 1$ is the elasticity of substitution. Each period, only a fraction $1 - \phi$ of intermediate goods producers can update their price (Calvo pricing).

  We assume that the government uses taxes and subsidies (not modeled here) to correct distortions from imperfect competition associated with the <u>time-invariant</u> markup $\frac{\epsilon}{\epsilon-1}$.

A log-linearized version of the above model is as follows, where " $\tilde{}$ " denote deviations form the steady state.

$$\tilde{Y}_t = \mathbb{E}_t \tilde{Y}_{t+1} - \tilde{r}_t \tag{7}$$

$$\tilde{\pi}_t = \frac{(1-\phi\beta)(1-\phi)}{\phi}(1+\eta)(\tilde{Y}_t - \tilde{Y}_t^e) + \beta \mathbb{E}_t \tilde{\pi}_{t+1} \tag{8}$$

$$\tilde{Y}_t^e = \tilde{A}_t \tag{9}$$

$$\tilde{A}_t = \rho_a \tilde{A}_{t-1} + \varepsilon_{a,t} \tag{10}$$

where $\rho_a \in (0,1)$, r_t is the real interest rate, and π_t is inflation. Superscript e denotes output from the efficient allocation. Superscript f will denote the flexible-price equilibrium, which coincides with the efficient output level given the assumptions so far.

Questions

1. [3 points] Provide a description and intuition for equations (7) and (8).

2. [5 points] Since productivity is an AR(1) process, the efficient output will also be AR(1)

$$\tilde{Y}^e_t = \rho \tilde{Y}^e_{t-1} + \varepsilon_t$$

   where $\rho \in (0,1)$ and ε_t has mean zero.

   Furthermore, the real interest rate that is consistent with the efficient allocation can be written as the following process (you don't need to derive this)

$$\tilde{r}^e_t = \rho \tilde{r}^e_{t-1} + (\rho - 1)\varepsilon_t \tag{11}$$

   Let's define the output gap as the difference between actual output and the output that would be achieved in the efficient allocation, $\tilde{X}_t \equiv (\tilde{Y}_t - \tilde{Y}^e_t)$.

   In what follows, we focus on the problem of a central bank that sets policy each period (discretion, no commitment) and has the following objective function

$$\min_{\tilde{\pi}_t, \tilde{X}_t} \frac{1}{2}\left(\tilde{\pi}_t^2 + \omega \tilde{X}_t^2\right)$$

   subject to

$$\tilde{\pi}_t = \gamma \tilde{X}_t + \beta \mathbb{E}_t \tilde{\pi}_{t+1}$$

   where $\gamma = \frac{(1-\phi\beta)(1-\phi)}{\phi}(1+\chi)$, $\omega > 0$. We assume that the central bank takes $\mathbb{E}_t \tilde{\pi}_{t+1}$ as given when setting policy in period t.

   (a) Can the central bank achieve $\tilde{X}_t = \tilde{\pi}_t = 0 \; \forall t$?

   (b) Does your answer in 2a imply $\tilde{i}_t = \tilde{r}_t = \tilde{r}^e_t$? Show your derivation.

3. [11 points] Assume that the economy is initially in the steady state with zero inflation and no price dispersion. The central bank still sets policy each period as described above (discretion) and it takes $\mathbb{E}_t \tilde{\pi}_{t+1}$ as given. Now there is a negative transitory productivity shock in period 0 ($\varepsilon_0 < 0$).

   (a) Can the central bank achieve $\tilde{X}_t = \tilde{\pi}_t = 0$ in every period?

   (b) Describe how $\tilde{Y}^e_t, \tilde{Y}_t$, and $\tilde{X}_t$ evolve (state the sign on impact and how they evolve over time).

   (c) How would intermediate producers want to change prices after the shock? Can the central bank eliminate short-term distortions? Explain and provide an intuition.

4. [7 points] We now assume that the desired markups are a stochastic process around the steady state value $\frac{\epsilon}{\epsilon-1}$. We define the firms' desired (i.e. if prices were flexible) log markup μ_t^f as

$$\mu_t^f \equiv \ln \frac{\epsilon_t}{\epsilon_t - 1}$$

The steady state value is (as before)

$$\mu = \ln \frac{\epsilon}{\epsilon - 1}$$

We assume that the government corrects long-run distortions from μ (as discussed), but not from fluctuations in μ_t^f. It can be shown (you don't have to derive this) that Equation (8) then becomes

$$\tilde{\pi}_t = \gamma \tilde{X}_t + \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \tilde{u}_t$$

where $\tilde{u}_t = \lambda(\mu_t^f - \mu)$ can be called 'cost-push-shocks'. We assume it has the following AR(1) process

$$\tilde{u}_t = \rho_u \tilde{u}_{t-1} + \varepsilon_{u,t}$$

where $\rho_u \in (0,1)$ and $\mathbb{E}_t \varepsilon_{u,t} = 0$.

Write down the central bank's problem and the optimality condition under discretion. You can again assume that the central bank treats $\mathbb{E}_t \tilde{\pi}_{t+1}$ as given.

5. [12 points] The economy is initially in the steady state with zero inflation and no price dispersion. Then there is a positive shock $\varepsilon_{u,t}$. The central bank sets policy each period under discretion and treats $\mathbb{E}_t \tilde{\pi}_{t+1}$ as given.

   (a) Can the central bank achieve $\tilde{X}_t = \tilde{\pi}_t = 0$ in every period? Explain.

   (b) Assume that the policy function for inflation is

   $$\pi_t = \Phi \tilde{u}_t$$

   where $\Phi > 0$ is a constant (you can take this as given, you don't have to solve for Φ). Find the policy function for the output gap.

   (c) Describe how $\tilde{Y}_t^e, \tilde{Y}_t$, and $\tilde{X}_t$ evolve (state the sign on impact and how they evolve over time).

   (d) How do intermediate producers want to change prices after the shock? Can the central bank eliminate short-term distortions? Explain and provide an intuition.

6. [7 points] Briefly compare the effects of the two types of shocks (negative productivity shock and positive cost-push shock), under discretionary central bank policy as described above. Are the responses qualitatively different? Why, why not? Provide a brief explanation.

3.1 Solutions for Optimal Policy in the New Keynesian Model (shorter)

1.
   - Equation (7) can be interpreted as demand, it describes a relationship between consumption (output) growth and the interest rate and is given by the linearized Euler equation.
   - Equation (8) can be interpreted as the New Keynesian Phillips Curve, it describes the relationship between the nominal variable inflation and the real variable output gap, where γ is the slope of the Phillips curve. γ is decreasing in the price stickiness, i.e. the Phillips curve gets flatter. When prices are fully flexible ($\phi \to 0$), then the Phillips curve is vertical and we are in the flexible-price equilibrium where real output does not depend on nominal variables like inflation.

2. (a) Yes. $\tilde{X}_t = \tilde{\pi}_t = 0 \; \forall t$ minimizes the loss function and satisfies the constraint.

   (b) The Fisher relation is

   $$\tilde{r}_t = \tilde{i}_t - \mathbb{E}_t \tilde{\pi}_{t+1}.$$

   The Euler equation in terms of the output gap is

   $$\tilde{X}_t = \mathbb{E}_t \tilde{X}_{t+1} - (\tilde{r}_t - \tilde{r}_t^e) \tag{12}$$

   The central bank chooses zero output gap in each period, as in the flexible price equilibrium. The equation for aggregate demand (Euler equation) then implies $\tilde{r}_t = \tilde{r}_t^e$. Since inflation is zero, the nominal interest rate equals the real interest rate, and this equals the flexible-price real rate of interest (and thus the interest rate consistent with the efficient allocation) $\tilde{i}_t = \tilde{r}_t = \tilde{r}_t^f = \tilde{r}_t^e$.

3. (a) Yes, the central bank can achieve $\tilde{X}_t = \tilde{\pi}_t = 0$ in every period.

   (b) $\tilde{Y}_t^e$ will jump down on impact and then converge back to the steady state. $\tilde{Y}_t$ will follow the same path as $\tilde{Y}_t^e$, hence the output gap is zero.

   (c) The central bank achieves a zero output gap by mimicking the flexible price equilibrium (and thus the efficient allocation). By choosing inflation equal to zero in every period, no intermediate good producer has an incentive to update the price, which implies that there is no dispersion in prices and output is equal to the flexible price output and to the efficient allocation.

4. The central bank solves

   $$\min_{\tilde{\pi}_t, \tilde{X}_t} \frac{1}{2} \left(\tilde{\pi}_t^2 + \omega \tilde{X}_t^2 \right)$$

   subject to

   $$\tilde{\pi}_t = \gamma \tilde{X}_t + \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \tilde{u}_t$$

The Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\left(\tilde{\pi}_t^2 + \omega \tilde{X}_t^2\right) \\ + \lambda\left(\tilde{\pi}_t - \gamma \tilde{X} - \beta \mathbb{E}_t \tilde{\pi}_{t+1} - \tilde{u}_t\right)$$

The additive exogenous cost shock does not change the first order conditions. The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \tilde{\pi}_t} = 0 \Leftrightarrow \tilde{\pi}_t = \lambda$$
$$\frac{\partial \mathcal{L}}{\partial \tilde{X}_t} = 0 \Leftrightarrow \omega \tilde{X}_t = -\lambda\gamma$$

The optimality conditions yield

$$\tilde{X}_t = -\frac{\gamma}{\omega}\tilde{\pi}_t$$
$$\tilde{\pi}_t = \gamma \tilde{X}_t + \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \tilde{u}_t$$

5. (a) $\tilde{X}_t = \tilde{\pi}_t = 0$ is inconsistent with the Phillips Curve. Hence, the central bank cannot achieve zero inflation and zero output gap simultaneously.

   (b) The optimality condition implies

   $$\tilde{X}_t = -\frac{\gamma}{\omega}\tilde{\pi}_t \\ = -\frac{\gamma}{\omega}\Phi \tilde{u}_t$$

   Inflation is an increasing linear function of the cost shock, thus the optimality condition implies that the output gap is a decreasing linear function of the cost shock.

   (c) The central bank will choose some inflation and some negative output gap in response to a positive cost shock. Inflation and output gap are proportional to the cost shock and over time (as the cost shock dies out), both will converge to their steady state value of zero. The efficient output is unaffected by the markup shock, but the actual output will be lower and thus the gap is negative.

   (d) The markup shock implies that firms want to set higher prices, which would create price dispersion because only some firms can update their price. To prevent that, the central bank would have to stabilize inflation at zero, but then the Phillips curve implies a negative output gap. Given the optimality condition, the central bank will only partially stabilize inflation and thus does not fully eliminate the distortion from price dispersion.

6. The central bank can achieve zero inflation and output gap (and thus the efficient allocation) when there are productivity shocks, but not when there are markup shocks. The reason why

it cannot achieve the efficient allocation when there are markup shocks is that they also create a real distortion that implies a deviation between the efficient allocation and the flexible price equilibrium. A productivity shock could trigger that firms want to update their price and this could lead to price dispersion, but the central bank can address that as described above.

End of exam.