Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **three** questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**. Number your answers.
- Good luck!

1 Subsistence Consumption

Consider a two-period OLG model with production. Time is discrete, $t = 1, 2, ...$

Demographics: $N = 1$ young agents are born at each date. Each agent lives for 2 periods.

Preferences:

$$\ln \left(c_t^y - \theta a_t\right) + \beta \ln \left(c_{t+1}^o\right)$$

where $\theta > 0$ and $\beta > 0$ are parameters. $a_t = \bar{c}_{t-1}^y$ is the average consumption of the young at $t-1$. The idea is that agents want to "keep up" with the consumption levels of their parent's generation.

Endowments: Young agents are endowed with 1 unit of work time and no capital. The initial old are endowed with S_0 units of capital.

Technology: Output is produced according to $Y_t = AK_t^\alpha L_t^{1-\alpha}$ where K_t and L_t are aggregate capital and labor inputs, respectively. Capital fully depreciates, so that $Y_t = C_t + K_{t+1}$ where C_t is aggregate consumption.

Questions:

1. State the household's problem and solve for the saving function.
2. Define a competitive equilibrium.
3. Show that the equilibrium laws of motion are given by

$$\begin{aligned} k_{t+1} &= \frac{\beta}{1+\beta}\left[(1-\alpha) A k_t^\alpha - \theta a_t\right] \\ a_{t+1} &= \frac{1}{1+\beta}(1-\alpha) A k_t^\alpha + \frac{\beta}{1+\beta}\theta a_t \end{aligned}$$

   where k_t is the capital labor ratio.

4. Simple algebra implies that the unique steady state is given by

$$\begin{aligned} a &= \frac{k}{\beta(1-\theta)} \\ k &= \left(\frac{\beta(1-\alpha)(1-\theta) A}{1+\beta(1-\theta)}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

   (you don't have to show this). How does k respond to θ? Give a clear intuition.

2 Barro-Gordon Model

In the basic Barro-Gordon model workers commit to a one-period nominal wage contract. Assuming aggregate demand is given by

$$y_t - \overline{y} = -(w_t - p_t) - z_t$$

where y_t is the log of output, $\overline{y}$ is the natural, flexible-price equilibrium level of output, $(w_t - p_t)$ is the log of the real wage, and z_t is an $i.i.d.(0, \sigma^2)$ supply shock. We will further assume that workers' commitment to nominal wage contracts is such that

$$E_{t-1} y_t = \overline{y}$$

That is, expected output is at its natural level. The expectations operator is with respect to time $(t-1)$ information.

1. From the above commitment rule find an expression for w_t in terms of the (expected) price level.

2. Suppose the monetary authority sets inflation $\pi_t = p_t - p_{t-1}$ so as to minimize the loss function given by
$$L = (y_t - \widetilde{y})^2 + b\pi_t^2$$
where $\widetilde{y}$ is the output target level of the monetary authority and $\widetilde{y} - \overline{y} = k > 0$. First, re-express the loss function in terms of inflation and its expectations, and shocks.

3. Is there an (expected) inflation bias?

4. If households were to expect an inflation rate of zero, would it be optimal for the monetary authority to choose an inflation rate of zero? (You should demonstrate this. In doing so you are allowed to set the shocks to zero).

5. Now adding back the shocks, explain why something like a constitutional amendment that forces the authority to commit to zero inflation would or would not be optimal.

3 Basic New Keynesian Model

Recently the New Keynesian model has been used as the workhorse to study monetary policy effects on the economy. In this model competition is imperfect and there is inertia in price setting (a la Calvo or quadratic adjustment cost) which cause monetary policy to have real short term effects. Households are infinitely lived and seek to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right)$$

where

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

is an index of consumption. $C_t(i)$ is the quantity of good i consumed by households in period t. The household budget set is described by

$$\begin{aligned} \int_0^1 P_t(i) C_t(i) di + Q_t B_t &\leq B_{t-1} + W_t N_t + T_t \\ \lim_{T\to\infty} E_t\left(B_t\right) &= 0 \quad \text{all } t \end{aligned}$$

1. Solve the household's consumption sub-problem and show that demand for variety i is given by

   $$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

   where the price index is given by

   $$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

2. Plug above into budget constraint and show that the optimal conditions for consumption and labor supply are the same as in the traditional Real Business Cycle model.

3. Firms are assumed to set prices a la Calvo (1983) with $(1-\theta)$ being the per period probability that a firm resets its price, independent of the time elapsed since the last price adjustment. Intuitively explain the optimization problem of a firm that gets to set its price in period t:

   $$Max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi(Y_{t+k|t}) \right) \right]$$

   subject to

   $$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

   where $\Psi(Y_{t+k|t})$ is the cost function.

4. Find the optimal price-setting condition.
5. Demonstrate price setting in the frictionless case, stating clearly the value of the mark-up.

4 Answers

4.1 Answer: Subsistence consumption[1]

1. Household problem:
$$\max \ \ln\left(c_t^y - \theta a_t\right) + \beta \ln\left(c_{t+1}^o\right)$$
   subject to
$$\begin{aligned} c_t^y &= w_t - S_t \\ c_{t+1}^o &= R_{t+1} S_t \\ c_t^y &\geq \theta a_t \end{aligned}$$
   The saving function is given by
$$S_t = \frac{\beta}{1+\beta}\left(w_t - \theta a_t\right)$$

2. Competitive equilibrium: An allocation $\{c_t^y, c_t^o, S_t, a_t, k_t, K_t, L_t\}$ and a price system $\{w_t, R_t\}$ [9 objects] that satisfy

   (a) saving function and 2 budget constraints [3]

   (b) firm first-order conditions [2]:
$$\begin{aligned} w_t &= (1-\alpha) k_t^{\alpha} && (1) \\ R_t &= \alpha k_t^{\alpha - 1} && (2) \end{aligned}$$

   (c) market clearing: goods (given), labor: $L_t = 1$, capital: $K_{t+1} = N S_t$. [3]

   (d) definition of a_t and $k_t = K_t$ [2].

3. Laws of motion: From the saving function
$$S_t = k_{t+1} = \frac{\beta}{1+\beta}\left[(1-\alpha) A k_t^{\alpha} - \theta a_t\right].$$
   Further
$$\begin{aligned} a_{t+1} = c_t^y &= w_t - S_t \\ &= (1-\alpha) A k_t^{\alpha} - \frac{\beta}{1+\beta}\left[(1-\alpha) A k_t^{\alpha} - \theta a_t\right] \\ &= \frac{1}{1+\beta}(1-\alpha) A k_t^{\alpha} + \frac{\beta}{1+\beta}\theta a_t \end{aligned}$$

[1]Based on a question due to Joydeep Bhattacharya.

4. Divide the steady state equation by $(1-\theta)$ and it is apparent that $\partial k/\partial\theta < 0$. The intuition:
   (a) mechanical: a higher θ raises the marginal utility when young relative to old. This reduces saving.
   (b) economic: with high θ it is more important to "keep up" with the parents' consumption, which reduces saving.

4.2 Barro-Gordon Model

1. Taking expectations

$$\begin{aligned} E_{t-1}y_t - \overline{y} &= -E_{t-1}w_t + E_{t-1}p_t \\ w_t &= E_{t-1}p_t \end{aligned}$$

wage is set one-period in advance.

2.

$$\begin{aligned} L &= \left(y_t - \overline{y} + \overline{y} - \widetilde{y}\right)^2 + b\pi_t^2 \\ &= \left[(p_t - w_t) - z_t - k\right]^2 + b\pi_t^2 \\ &= \left(p_t - E_{t-1}p_t - z_t - k\right)^2 + b\pi_t^2 \\ &= \left(p_t - p_{t-1} + p_{t-1} - E_{t-1}p_t - z_t - k\right)^2 + b\pi_t^2 \\ &= \left(\pi_t - \pi_t^e - z_t - k\right)^2 + b\pi_t^2 \end{aligned}$$

3. Differentiate with respect to π_t

$$\begin{aligned} \frac{\partial L}{\partial \pi_t} &= 2\left(\pi_t - \pi_t^e - z_t - k\right) + 2b\pi_t = 0 \\ \pi_t &= \frac{\pi_t^e + z_t + k}{1+b} \end{aligned}$$

taking expectations

$$\begin{aligned} \pi_t^e &= E_{t-1}\pi_t = E_{t-1}\left(\frac{\pi_t^e + z_t + k}{1+b}\right) \\ \pi_t^e &= \frac{k}{b} \text{ positive inflation bias} \end{aligned}$$

4. Suppose $\pi_t^e = 0$ should monetary authority $\pi_t = 0$? According to F.O.C.

$$\frac{\partial L}{\partial \pi_t} = 2\left(\pi_t - \pi_t^e - z_t - k\right) + 2b\pi_t = 0$$

setting $\pi_t^e = \pi_t = 0$ and wlog $z_t = 0$

$$\frac{\partial L}{\partial \pi_t} = 2k > 0$$

therefore, it is suboptimal to set $\pi_t = 0$

5. Authorities would not want to set law to commit to zero inflation because it would limit policy's ability to stabilize the economy in the face of output shocks z.

5 New-Keynesian Model

1. To solve the subproblem assume an expenditure level Z

$$Z_t = \int_0^1 P_t(i) C_t(i) di$$

then solve Lagrangian

$$L = \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} + \lambda \left[Z_t - \int_0^1 P_t(i) C_t(i) di \right]$$

F.O.C. gives

$$C_t^{\frac{1}{\varepsilon}} C_t(i)^{\frac{-1}{\varepsilon}} = \lambda P_t(i)$$

which also holds for good j

$$C_t^{\frac{1}{\varepsilon}} C_t(j)^{\frac{-1}{\varepsilon}} = \lambda P_t(j)$$

combining

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)} \right)^{-\varepsilon}$$

substitute into expenditure equation

$$\begin{aligned} Z_t &= C_t(j) P_t(j)^{\varepsilon} \int_0^1 P_t(i)^{1-\varepsilon} di \\ &= C_t(j) P_t(j)^{\varepsilon} P_t^{1-\varepsilon} \end{aligned}$$

solve for $C_t(j)$

$$C_t(j) = \frac{Z_t}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon}$$

also holds for good i. Substitute into definition for C_t

$$\begin{aligned} C_t &= \left[\int_0^1 \left(\frac{Z_t}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \frac{Z_t}{P_t} \end{aligned}$$

$$P_t C_t = Z_t = \int_0^1 P_t(i) C_t(i) di$$

therefore,

$$C_t(j) = \frac{Z_t}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t$$

which also holds for good i.

2. Plug into budget constraint to get

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

same budget set as traditional RBC model. Similarly, the FOC's become

$$\frac{-U_{nt}}{U_{ct}} = \frac{W_t}{P_t}$$

and

$$Q_t = \beta E_t\left[\frac{U_{C_{t+1}}}{U_{C_t}}\frac{P_t}{P_{t+1}}\right]$$

3. Optimizing firms set prices the maximize profits not knowing when they will get the chance to readjust again. This is captured by the Bernouli factor θ. Given that households own firms, profits are treat like assets and are thus discounted using the asset pricing (Lucas equation) discount factor from the household's problem.

4.
$$\sum_{k=0}^{\infty}\theta^k E_t\left[Q_{t,t+k}\left((1-\varepsilon)\frac{P_t^{*-\varepsilon}}{P_{t+k}^{-\varepsilon}}C_{t+k} - \Psi^{/}\left(\frac{P_t^{*-\varepsilon}}{P_{t+k}^{-\varepsilon}}C_{t+k}\right)\left(-\varepsilon\frac{P_t^{*-\varepsilon-1}}{P_{t+k}^{-\varepsilon}}C_{t+k}\right)\right)\right] = 0$$

specify in terms of Y

$$\sum_{k=0}^{\infty}\theta^k E_t\left[Q_{t,t+k}\left((1-\varepsilon)Y_{t+k|t} + \frac{\varepsilon}{P_t^*}\Psi^{/}\left(Y_{t+k|t}\right)Y_{t+k|t}\right)\right] = 0$$

$$\sum_{k=0}^{\infty}\theta^k E_t\left[Q_{t,t+k}Y_{t+k|t}\left(P_t^* - \frac{\varepsilon}{\varepsilon-1}\Psi^{/}\left(Y_{t+k|t}\right)\right)\right] = 0$$

5. No price rigidity means $k = 0$.

$$P_t^* = \frac{\varepsilon}{\varepsilon-1}\Psi^{/}\left(Y_{t|t}\right)$$

Price is a constant mark-up over marginal cost.

End of exam.