Macroeconomics Qualifying Examination

January 2010

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of three questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Good luck!

Question 1

1A. Consider the following one good planning problem.

- Time is discrete and goes on forever.
- Demographics: There is a single, infinitely lived household.
- Preferences: $\sum_{t=0}^{\infty} \beta^t \{\ln(C_t + \chi G_t) + v(h_t)\}$, $0 < \sigma < 1$, $\phi > 0$. C is consumption and h is hours worked.
- Endowments: At the beginning of time, the household is endowed with K_0 units of the good. In each period, he is endowed with 1 unit of time that can be split into work h and leisure $1 - h$.
- Technology: Output is produced according to $Y_t = (A_t h_t)^{\alpha} K_t^{1-\alpha}$. The resource constraint is

  $$K_{t+1} = Y_t - C_t - G_t + (1 - \delta) K_t \tag{1}$$

  $A_t = (1 + x)^t$. K is capital. G is government spending. $0 < \alpha < 1$.

Throughout, assume that the constraint $h \leq 1$ never binds.

Questions: For part 1A assume that $G = 0$ and that $v(h_t) = -\frac{\phi}{1-\sigma} h_t^{1-\sigma}$. But it will help you save time in part 1B if you carry G and the generic v around as long as possible.

1. State the planner's dynamic program. Derive Euler equations for consumption and hours.
2. Derive conditions that characterize the balanced growth path.
3. Does the utility function imply constant hours worked in spite of growing A?

1B. For part 1B assume that $G_t > 0$, but simplify by assuming that A is constant over time and that preferences for leisure are logarithmic: $v(h_t) = \phi \ln(1 - h_t)$.

Questions:

1. Derive how increasing G from 0 to $\tilde{G}$ affects the steady state values of: h, the marginal product of capital, the capital-labor ratio, C/Y, and the investment-output ratio.
2. How do your answers differ for the cases $\chi = 0$ and $\chi = 1$? Provide intuition.

Question 2

Consider the following AK model in continuous time.

- Demographics: There is a single, representative household who lives forever.
- Endowments: The household is endowed with K_0 units of the good at the beginning of time and with one unit of work time at each instant.
- Preferences:
$$\int_0^\infty e^{-(\rho-n)t} u(c_t) dt \tag{2}$$
- Technology: A single good is produced according to $Y_t = \tilde{A}_t K_t^\alpha L_t^{1-\alpha}$ with the resource constraint
$$\dot{K}_t = Y_t - c_t - \delta K_t \tag{3}$$
where $\tilde{A}_t = A(K_t/L_t)^{1-\alpha}$.

Household problem: the household maximizes (2) subject to the budget constraint

$$\dot{v}_t = (r_t - n)v_t + (1-\tau_w)w_t - (1+\tau_c)c_t \tag{4}$$

where the τs denote tax rates that could be negative. v is the household's capital holding. w is the wage and r is the interest rate.

Questions:

1. Define a solution to the household problem (a list of objects and equations they satisfy).
2. Define a solution to the firm's problem. The firm takes $\tilde{A}_t$ as given.
3. Solve for the balanced growth rates of c, y, and $k = K/L$.
4. Explain which (constant) taxes can affect balanced growth and which cannot.
5. Find a combination of tax rates (some negative) that maintain a balanced budget and achieve the socially optimal allocation.

Question 3

Consider the two (human and physical) capital version of the endogenous growth model.

Notation: c is consumption, k is physical capital, h is human capital, x is investment. $0 < \alpha < 1$. $0 < \delta < 1$. τ_{kt} and τ_{nt} denote tax rates on capital and labor incomes, respectively. g_t denotes government purchases.

Model description:

- Time is discrete and lasts forever.
- Demographics: There is a representative, infinitely lived household.
- Preferences:
$$\sum_{t=0}^{\infty} \beta^t \ln(c_t) \tag{5}$$
- Endowments: The household is endowed with k_0 units of capital and with h_0 units of human capital at the beginning of time.
- Technologies:
  - goods are produced according to
$$c_t + x_{ht} + x_{kt} + g_t = A k_t^{\alpha} h_t^{1-\alpha} \tag{6}$$
  - both types of capital are produced from goods according to
$$k_{t+1} \leq (1-\delta)\, k_t + x_{kt} \tag{7}$$
$$h_{t+1} \leq (1-\delta)\, h_t + x_{ht} \tag{8}$$
- Market arrangements:
  - there are Arrow-Debreu markets for goods; the price of the date t good is p_t.
  - human and physical capital are not traded;
  - the household rents human and physical capital to the firm at rental prices r_t and w_t.

The household solves: max (5) subject to the budget constraint

$$\sum_{t=0}^{\infty} p_t \left[c_t + x_{kt} + x_{ht}\right] \leq \sum_{t=0}^{\infty} p_t \left[(1-\tau_{kt})\, r_t k_t + (1-\tau_{nt})\, w_t h_t\right] \tag{9}$$

and the capital accumulation constraints (7) and (8).

Firms maximize period profits. The government uses tax revenues to finance government purchases g_t. Assume that all equilibrium quantities are strictly interior.

Questions:

1. Derive the household's first-order conditions.
2. Solve for the equilibrium ratio k_t/h_t as a function of parameters and tax rates.
3. If tax rates are fixed across time, show how the growth rate of c_t depends on these tax rates.
4. Show that for arbitrary sequences of taxes, in interior equilibria, only the initial values of h and k enter the consumer's budget constraint. Specifically, show that

$$\sum_{t=0}^{\infty} p_t c_t = (1 - \tau_{k0})\, r_0 k_0 + (1 - \delta)\, k_0 + (1 - \tau_{h0})\, w_0 h_0 + (1 - \delta)\, h_0$$

5. Formulate a Ramsey, optimal taxation, problem for this economy for a fixed sequence of government expenditures. Hint: To find the implementability constraint, substitute the Euler equation into the budget constraint to eliminate the p_t.
6. In the Ramsey problem, show that at every t, $\tau_{kt} = \tau_{nt}$.
7. Moreover, if the solution to the Ramsey problem converges to a balanced growth path, then both τ_k and τ_n converge to zero.

End of exam.

Answer: Question 1

1.

$$L = \beta\left(\ln c_t - \frac{\phi v}{1+v} h_t^{\frac{1+v}{v}} \right) + \lambda_t \left[(A_t h_t)^{\alpha} K_t^{1-\alpha} - c_t - K_{t+1} + (1-\delta) K_t \right]$$

FOCs

$$\begin{aligned}
\frac{\beta}{c_t} &= \lambda_t \\
\beta \phi h_t^{\frac{1}{v}} &= \alpha \lambda_t (A_t h_t)^{\alpha-1} A_t K_t^{1-\alpha} \\
\lambda_t &= \lambda_{t+1} \left[\beta(1-\delta) + (1-\alpha)(A_t h_t)^{\alpha} K_t^{-\alpha} \right]
\end{aligned}$$

This implies equations

$$\frac{c_t}{c_{t+1}} = \beta \left[(1-\delta) + (1-\alpha) \frac{Y_t}{K_t} \right]$$

$$h_t = \left[\frac{\alpha Y_t}{\phi(c_t + \chi G_t)} \right]^{\frac{v}{1+v}}$$

2. Balanced growth. For this section there are 3 key equations that solve for Y/K, C/Y, and h (given G/Y):

$$\begin{aligned}
(1+x)^{-1} &= \beta \left[(1-\alpha) \frac{Y}{K} + 1 - \delta \right] \rightarrow Y/K \\
(1+x)K/Y &= 1 + (1-\delta)K/Y - C/Y - G/Y \rightarrow C/Y \\
v'(h)h &= \alpha Y/(C + \chi G) \rightarrow h
\end{aligned}$$

3. Hours are stationary, even when A grows.

Part 1B

1. Effect of G. Regardless of χ: no effect on Y/K and thus the marginal product of capital. No effect on $(C+G)/Y$, so C/Y drops by an amount independent of χ.

When $\chi = 1$: no change in the RHS of the FOC for v and thus no change in v. We just relabeled consumption.

When $\chi = 0$: the RHS of the FOC for h rises. $v'(h)h = h/(1-h)$. Therefore, h rises.

2. Intuition. In both cases, no intertemporal distortion - no change in K/Y. When $\chi = 1$: just labeling - no real effect. When $\chi = 0$: simply an income effect.

Answer: Question 3

1. The FOCs wrt k_{t+1} and h_{t+1} give

$$\frac{p_t}{p_{t+1}} = (1-\tau_{kt+1}) A\alpha \left(\frac{h_{t+1}}{k_{t+1}}\right)^{1-\alpha} + 1 - \delta \tag{10}$$

$$\frac{p_t}{p_{t+1}} = (1-\tau_{ht+1}) A (1-\alpha) \left(\frac{k_{t+1}}{h_{t+1}}\right)^{\alpha} + 1 - \delta \tag{11}$$

Prices evolve according to the Euler equation

$$\frac{p_t}{p_{t+1}} = \frac{u'(c_t)}{\beta u'(c_{t+1})} \tag{12}$$

Combining the above yield the two EE's.

2. Setting the RHS of the first two eqs equal to each other gives

$$\frac{k_{t+1}}{h_{t+1}} = \frac{(1-\tau_{kt+1})\alpha}{(1-\tau_{ht+1})(1-\alpha)}$$

3. The growth rate is given by

$$\begin{aligned}\frac{c_{t+1}}{c_t} &= \beta\left[(1-\tau_{kt+1}) A\alpha \left(\frac{h_{t+1}}{k_{t+1}}\right)^{1-\alpha} + 1 - \delta\right] = \\ &= \beta\left[A(\alpha(1-\tau_k))^{\alpha}((1-\alpha)(1-\tau_h))^{1-\alpha} + 1 - \delta\right].\end{aligned}$$

It depends negatively on both tax rates

4. The BC is

$$\sum_{t=0}^{\infty} p_t c_t = \sum_{t=0}^{\infty} p_t \left[(1-\tau_{kt}) r_t k_t + (1-\delta) k_t - k_{t+1} + (1-\tau_{nt}) w_t h_t + (1-\delta) h_t - h_{t+1}\right]$$

Using conditions [1] and [2] to simplify the RHS we get

$$\sum_{t=0}^{\infty} p_t c_t = (1-\tau_{k0}) r_0 k_0 + (1-\delta) k_0 + (1-\tau_{h0}) w_0 h_0 + (1-\delta) h_0$$

5. The implementability constraint (IC) is the above after substituting from (12) for p_t (normalize $p_0 = 1$) and for the marginal products

$$[IC] : \sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = (1-\tau_{k0}) F_k(0) k_0 + (1-\delta) k_0 + (1-\tau_{h0}) F_h(0) h_0 + (1-\delta) h_0$$

The optimal tax sequence $\tau := \{\tau_{kt}, \tau_{ht}\}$ for a given $\{g_t\}$ is such that the implied equilibrium quantities $\{c_t(\tau), k_t(\tau), h_t(\tau)\}$ can be found by solving

$$\begin{aligned}
[RP_1] \quad : \quad & \max \sum_{t=0}^{\infty} \beta^t u(c_t) \\
& s.t.\ IC \text{ and } MC.
\end{aligned}$$

This simplifies to

$$\begin{aligned}
[RP_2] \quad : \quad & \max \sum_{t=0}^{\infty} \beta^t V(\lambda, c_t) + W(c_0, h_0, k_0, \lambda) \\
& s.t.\ MC
\end{aligned}$$

where λ is the multiplier on IC in $[RP_1]$ and $V() = \sum_{t=1}^{\infty} \beta^t u(c_t) - \lambda \sum_{t=1}^{\infty} \beta^t u'(c_t) c_t$
Solving this problem gives

$$\begin{aligned}
[5] \quad : \quad & \frac{V'(t)}{V'(t+1)} = \beta [F_k(t+1) + 1 - \delta] \\
[6] \quad : \quad & \frac{V'(t)}{V'(t+1)} = \beta [F_h(t+1) + 1 - \delta]
\end{aligned}$$

which imply

$$\frac{k_{t+1}}{h_{t+1}} = \frac{\alpha}{(1-\alpha)}$$

Since $k(\tau)$ and $h(\tau)$ must also satisfy $[4]$, we obtain that the optimal tax sequence must satisfy $\tau_{kt} = \tau_{ht}$.

If the solution to Ramsey problem converges to a steady state, then the above [5] and [6] converge to

$$\begin{aligned}
[5] \quad : \quad & 1 = \beta [F_k(t+1) + 1 - \delta] \\
[6] \quad : \quad & 1 = \beta [F_h(t+1) + 1 - \delta]
\end{aligned}$$

Again, since $k(\tau)$ and $h(\tau)$ must also satisfy the two EE from part a, we have $\tau_k, \tau_h \to_{t\to\infty} 0$.

End of exam.