Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of three questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Good luck!

1 Wealth externality [40 points]

We study a standard growth model in continuous time where households get utility from holding wealth.

Demographics: There is a single representative household who lives forever.

Preferences: Households value consumption (c) and wealth (W) according to $\int e^{-\rho t} u(c_t, W_t/\bar{W}_t) dt$ with $u(c, W/\bar{W}) = \frac{c^{1-\gamma}}{1-\gamma}[W/\bar{W}]^{-\lambda}$. Wealth consists of capital and bonds: $W_t = K_t + B_t$. $\bar{W}_t$ denotes average wealth, which the household takes as given. $\gamma > 1$ and $\lambda > 0$ ensure that wealth is a good, not a bad.

Technologies: A single good is produced from capital according to $Y_t = AK_t$. The resource constraint is $\dot{K}_t = Y_t - c_t - G_t$. G denotes useless government spending.

Endowments: At date $t = 0$ the household is endowed with K_0 and bonds B_0. Bonds are issued by the government and pay interest $r = A$.

Government: the government taxes all income at rate τ. The flow budget constraint is $\dot{B}_t = G_t + (1-\tau) r B_t - \tau Y_t$. Government spending is a constant fraction of output: $G_t = gY_t$.

Markets: Output is produced by competitive firms.

Questions

1. Household:
   (a) Write down the Hamiltonian. State the states and controls. The household receives capital and bond income in the amount of $(1-\tau)rW$. There is no labor income.
   (b) Derive the first-order conditions. It is best not to eliminate the costate.
   (c) Define a solution to the household problem (objects and equations).
2. Define a competitive equilibrium (objects and equations).
3. Assume that the government does not issue bonds and sets G_t to balance the budget in each period.
   (a) Derive the balanced growth rate of output.
   (b) Compare the balanced growth rates for $\lambda = 0$ versus $\lambda > 0$.
      i. Explain the intuition.
   (c) How does the income tax affect output growth? Compare the cases with $\lambda > 0$ and $\lambda = 0$.
      i. Explain the intuition.

2 AK Model with Money [40 points]

We study an Ak model with cash and credit goods.

CONSUMER. The representative consumer solves

$$\begin{aligned}
& \max \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}\right) \\
& \text{s.t.} \\
p_t c_{1t} &\leq m_{t-1} \\
p_t c_{2t} + p_t x_t + m_t + B_t &\leq p_t r_t k_t + R_t B_{t-1} + m_{t-1} - p_t c_{1t} + T_t \\
k_{t+1} &\leq \left(1-\delta\right) k_t + x_t \\
& k_0, m_{-1} \text{ are given}
\end{aligned}$$

where

c_{1t} is the amount of cash good consumed in period t

c_{2t} is the amount of credit good consumed in period t

x_t is investment

m_t is money holdings carried from t to $t+1$

B_t is the number of bonds purchased in period t

r_t is the rental rate on capital

R_t is the nominal interest rate on bonds purchased in $t-1$

T_t is the nominal lump sum transfer

Assume

$$u\left(c_{1t}, c_{2t}\right) = \left(\alpha c_{1t}^{\rho} + \left(1-\alpha\right) c_{2t}^{\rho}\right)^{\frac{1-\sigma}{\rho}} / \left(1-\sigma\right)$$

FIRMS. Firms solve the static profit maximization problem

$$\max p_t A k_t - p_t r_t k_t \tag{1}$$

GOVERNMENT. The government expands the money supply at a constant rate

$$M_t^s = \mu M_{t-1}^s$$

and hands it back to the consumers: $T_t = M_{t+1}^s - M_t^s$.

Money market in equilibrium must satisfy $m_t = M_t^s$ for all t.

The resource constraint is: $c_{1t} + c_{2t} + x_t = A k_t$

Questions

1. Derive the relationships between the balanced growth rates of c_{1t}, c_{2t}, k_t, y_t.

2. Write down the household problem and derive the first-order conditions.

3. Derive the balanced growth rate of y. Hint: The growth rate of du/dc_2 equals $-\sigma g(c_2)$ when the ratio c_2/c_1 is constant over time.

4. Explain how μ affects the balanced growth rate. What is the intuition?

5. Let π denote the inflation rate for this economy, $\pi_t = p_t/p_{t-1}$. What is the effect of changes in μ on the long run value of π in this economy?

3 Asset Pricing [40 points]

Consider a household with an exogenous stream of income, y_t. It has preferences of the form

$$U = E_t \sum_{j=0}^{\infty} \beta^j \log(c_{t+j}),$$

where c_t represents consumption, β is a subjective discount factor, and $u(c_t) = \log(c_t)$ is the period utility function. The household must make two decisions, how much to save and consume in each period, and the form in which it holds its wealth. Suppose there are two assets, one that is safe (denoted B_t for 'bonds') and one that is risky (denoted S_t for 'stocks'). The safe asset has a gross rate of return from period t to $t+1$, R^B_{t+1}, that is known with certainty at date t. The gross rate of return for the risky asset, R^S_{t+1}, is uncertain at t.

Questions

1. What is the household's flow budget constraint? It is important to get this right to avoid problems down the road.

2. Derive and interpret the first-order conditions for optimal allocations in the two assets.

3. Under what conditions does the average return on the risky asset exceed that on the safe asset? What is the economic intuition for this result?

4. In US data, aggregate consumption growth is virtually uncorrelated with stock returns. In the context of this model, what does this imply for the average spread between stock and bond returns, $E(R^S_{t+1} - R^B_{t+1})$?

4 Answers

4.1 Answer: Wealth externality[1]

1. Household:

   (a) The household budget constraint is $\dot{W} = (1-\tau)AW_t - c_t$. Hamiltonian:
$$H = u(c, W/\bar{W}) + \mu\left((1-\tau)AW - c\right) \tag{2}$$

   (b) First-order conditions:
$$u_c = \mu \tag{3}$$
$$\dot{\mu} = \rho\mu - \mu(1-\tau)A - u_W \tag{4}$$

   (c) Solution: Functions of time c_t, W_t, μ_t that satisfy the 2 FOCs, the budget constraint, and a TVC.

2. Equilibrium: objects $\mu_t, c_t, K_t, B_t, W_t, \bar{W}_t, G_t, r_t$ that satisfy

   - 3 household conditions
   - government: budget constraint and $G = gY$.
   - market clearing: goods (resource constraint) and bonds ($W = B + K$).
   - $\bar{W} = W$.
   - No arbitrage: $r = A$.

3. Balanced growth with simplified government:

   (a) balanced growth rate: Note that $u_c/u_w = \frac{1-\gamma}{-\lambda}\frac{W}{c}$. Substitute into the first-order condition to obtain
$$g(\mu) = \rho - (1-\tau)A + \frac{\lambda}{1-\gamma}\frac{c}{K} \tag{5}$$
$$g(K) = A(1-\tau) - \frac{c}{K} \tag{6}$$
$$g(\mu) = g(u_c) = -\gamma g(c) \tag{7}$$

   After some rearranging:
$$-\gamma g(c) = \rho - (1-\tau)A + \frac{\lambda}{1-\gamma}[(1-\tau)A - g(c)] \tag{8}$$

   Write this as
$$g(c)\left\{\gamma + \frac{\lambda}{\gamma-1}\right\} = (1-\tau)A\left\{1 + \frac{\lambda}{\gamma-1}\right\} - \rho \tag{9}$$

[1] Based on Liutang and Mengying (2008).

(b) From the Euler equation: If the household cares about wealth, the growth rate of u_c is more negative. Consumption grows faster. Intuition: the utility of wealth acts like an additional return to saving. Households save more for a given interest rate. In an AK model this means the economy grows faster.

(c) If $\lambda = 0$ we are in a standard AK model. The capital tax reduces growth. Intuition: lower interest rate implies lower consumption growth by the Euler equation.
If $\lambda > 0$:

$$\frac{dg(c)}{d(1-\tau)} = A\frac{1+\lambda/(\gamma-1)}{\gamma+\lambda/(\gamma-1)} \tag{10}$$

As λ rises, so does the effect of taxes on growth. Intuition: Lower taxes raise the saving rate through the usual channel. In addition, the lower c/K raises the marginal value of wealth, which raises saving further.

4.2 Answer: AK Model with Money

1. Balanced growth rates: From the constraints we find that $\gamma = g(c_1) = g(c_2) = g(k)$.

2. First-order conditions: Denote the multipliers on the cia constraints with λ_t and the budget constraints by θ_t

FOC w.r.t c_{1t}, c_{2t}, m_t, k_{t+1}, B_{t+1} are

$$\begin{aligned}
\beta^t \left(\alpha c_{1t}^{\rho} + \alpha c_{2t}^{\rho}\right)^{\frac{1-\sigma}{\rho}-1} \alpha c_{1t}^{\rho-1} &= (\lambda_t + \theta_t)\, p_t && (11)\\
\beta^t \left(\alpha c_{1t}^{\rho} + \alpha c_{2t}^{\rho}\right)^{\frac{1-\sigma}{\rho}-1} (1-\alpha)\, c_{2t}^{\rho-1} &= \lambda_t p_t && (12)\\
\lambda_{t+1} + \theta_{t+1} &= \theta_t && (13)\\
\theta_t p_t &= \theta_{t+1} p_{t+1} \left(r_{t+1} + 1 - \delta\right) && (14)\\
\theta_t &= \theta_{t+1} R_{t+1}
\end{aligned}$$

(11)/(12) implies $\frac{\alpha}{1-\alpha}\left(\frac{c_{1t}}{c_{2t}}\right)^{\rho-1} = \frac{\lambda_t+\theta_t}{\lambda_t}$.

Using (11),

$$\frac{\beta\left(\alpha c_{1t+1}^{\rho} + (1-\alpha)\, c_{2t+1}^{\rho}\right)^{\frac{1-\sigma}{\rho}-1} c_{1t+1}^{\rho-1}}{\left(\alpha c_{1t}^{\rho} + (1-\alpha)\, c_{2t}^{\rho}\right)^{\frac{1-\sigma}{\rho}-1} c_{1t}^{\rho-1}} = \frac{(\lambda_{t+1}+\theta_{t+1})\, p_{t+1}}{(\lambda_t+\theta_t)\, p_t}$$

using (13)

$$= \frac{\theta_t p_{t+1}}{\theta_{t-1} p_t}$$

using CIA

$$= \frac{\theta_t m_{t+1} c_{1t}}{\theta_{t-1} c_{1t+1} m_t}$$

using (4)

$$= \frac{p_{t-1} m_{t+1} c_{1t}}{p_t (A+1-\delta) c_{1t+1} m_t} = \frac{m_{t-1} c_{1t}^2 m_{t+1}}{c_{1t-1} m_t^2 (A+1-\delta) c_{1t+1}}$$

3. Balanced growth rate: One of the Euler equations implies:

$$u_2 = \beta(A+1-\delta) u_2(.') \tag{15}$$

From the hint, it follows that we have the standard non-monetary balanced growth condition

$$\begin{aligned} \beta\gamma^{1-\sigma-\rho}\gamma^{\rho-1} &= \frac{\mu}{\mu(A+1-\delta)} \\ \gamma &= (\beta[A+1-\delta])^{-\frac{1}{\sigma}} \end{aligned}$$

4. Money and growth: Thus, μ does not affect the long run growth rate of the economy. Intuition: Same as in the CIA model with exogenous growth. Inflation does not distort the consumption / saving decision. It only distorts the decision to consume the cash or the credit good.

5. Inflation:

$$\frac{p_t}{p_{t-1}} = \frac{m_t c_{1t-1}}{c_{1t} m_{t-1}} = \frac{\mu}{\gamma}$$

Thus, μ positively affects inflation in the long run, one-for-one. This follows directly from the fact that inflation does not affect the growth rate of real variables.

4.3 Answer Sketch: Asset Pricing[2]

(a) The budget constraint is $c_t + B_{t+1} + S_{t+1} = y_t + B_t R_{B,t} + S_t R_{S,t}$.

(b) $V(S,B) = \max u(c) + \beta E V(S', B')$. First-order conditions:

$$\begin{aligned} u'(c) &= \beta E V_B(.') \\ &= \beta E V_S(.') \\ V_B &= u'(c) R_B \\ V_S &= u'(c) R_S \end{aligned}$$

As a result, familiar looking asset pricing equations emerge:

$$u'(c) = \beta E \{u'(c') R_i'\}; \; i = B, S$$

[2] Based on a question due to Tim Cogley.

Or, expressed in terms of the equity premium ($EP = R'_S - R'_B$) and the marginal rate of substitution ($MRS = \beta\, u'(c')/u'(c)$):

$$E\,\{MRS\,EP\} = 0$$

(c) To determine when the equity premium is positive, rewrite as

$$Cov\,(MRS, EP) + E\,\{MRS\}\; E\,\{EP\} = 0$$

With log utility, $MRS = \beta\, c/c'$. Hence, the equity premium is positive, iff stock returns and consumption growth are inversely correlated. Intuition: Assets are more valuable, if they pay out in bad times.

(d) Small covariance implies low equity premium.

End of exam.