Macroeconomics Qualifying Examination

August 2009
Department of Economics
UNC Chapel Hill

Instructions:

- This examination consists of four questions. Answer any three questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- Do not consult any books, notes, calculators, or cell phones.
- The questions are stated as clearly as possible. If after careful consideration you believe a question is ambiguously stated, and think it is absolutely necessary to modify the question in any way, then please state clearly all the changes you are making, before proceeding to answer the question. Unnecessary simplifications of questions will be penalized.
- Write legibly!
- Good luck!

Question 1

Consider an overlapping generations endowment economy with a single non-storable good. Time is indexed by $t = 1, 2, 3...$ The representative consumer of the generation born in period t lives in periods t and $t+1$, has preferences represented by $c_t^y + \alpha \ln c_{t+1}^o$, and endowment stream $(e_t^y, e_t^o) = (1, 1)$. There is no fiat money.

a. Define an Arrow-Debreu equilibrium.

b. Calculate the unique equilibrium (do not prove it is unique).

c. Define a Pareto-efficient allocation for this economy.

d. Suppose that $\alpha = 2$. Show that the equilibrium is not Pareto-efficient.

Question 2

Consider a neoclassical growth model with uncertainty. Time is discrete: $t = 0, 1, 2, ...$

Demographics: There is a unit mass of identical households.

Preferences: $\mathbb{E} \sum_{t=0}^{\infty} \beta^t \ln c_t$.

Endowments: The household is endowed with one unit of labor time in each period and with initial capital stock k_0.

Technology: Output per capita is given by $\theta_t k_t^\alpha$ where k is the capital-labor ratio. The resource constraint is given by $k_{t+1} = \theta_t k_t^\alpha - c_t$. θ_t is an i.i.d. random variable that takes on the values θ^1 with probability π and θ^2 with probability $1 - \pi$.

a. State the social planner's problem in recursive form.

b. Solve for the value function and the policy function. Start from the guess $V(k, \theta^i) = A + B \ln k + C \ln \theta^i$.

c. Define a Competitive Equilibrium, assuming: (i) θ_t is an aggregate shock; (ii) trading takes place in a sequence of markets; (iii) households trade a complete set of Arrow securities (an Arrow security pays one unit of the good if history $\theta^t = (\theta_1, ..., \theta_t)$ occurs).

Question 3:

The economy is populated by a representative household and a large number of firms, indexed by i, whose number is normalized to 1. The production function for the representative firm is given by

$$y_{it} = Ak_{it}^{\alpha}(K_t l_{it})^{1-\alpha}$$

where $0 < \alpha < 1$, $A > 0$, and y_{it}, k_{it} and l_{it} are output, capital and labor input of firm i at date t. The aggregate, economy-wide capital stock is given by K_t, that is, the sum of all capital stocks in the economy. When making their choices, firms act competitively, that is, take as given the aggregate capital stock K_t and the rental price of capital r_t and the wage rate w_t. The presence of K_t in the production function of firm i indicates a production externality.

The representative consumer has a unit time endowment to work and faces the standard problem of choosing a stream of consumption and asset holdings $(c_t, a_t)_{t=[0,\infty)}$ to maximize

$$\int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt$$

subject to

$$c_t + \dot{a}_t = i_t a_t + w_t$$

where i_t is the real interest rate and w_t is the wage rate. Let $a_0 = K_0$ be the initial endowment of capital of the representative consumer. Assume that the population is not growing over time, so that the aggregate resource constraint is given by

$$c_t + \dot{K}_t + \delta K_t = Y_t$$

where Y_t is the aggregate output of all firms and $\delta > 0$ is the depreciation rate of capital.

a. Define a competitive equilibrium for this economy

b. Along a balanced growth path c_t, Y_t and K_t are growing at a constant rate γ_c. Derive the formula for γ^{CE} as a function of the model parameters and interpret your results.

c. Compare the growth rate γ^{CE} to the growth rate γ^{SP} that a social planner would choose (along a BGP) who chooses consumption c_t and the aggregate capital stock K_t to maximize utility of the representative agent, subject to the aggregate resource constraint. Interpret your results.

Question 4:

Consider a standard growth model with heterogeneous households.

Demographics: There are $j = 1, ..., J$ types of households. The mass of type j households is μ_j.

Preferences: The representative **household** of type j maximizes $\sum_{t=0}^{\infty} \beta^t u_j(c_{jt})$. Note that households differ in their utility functions. Assume that u_j is increasing and strictly concave and obeys Inada conditions.

Technology: The resource constraint is given by $F(K_t, L_t) + (1 - \delta) L_t = C_t + K_{t+1}$ where C_t is aggregate consumption, K_t is aggregate capital, and L_t is aggregate labor input. F is constant returns to scale and obeys Inada conditions.

Endowments: Each household is endowed with one unit of labor in each period. At $t = 0$ household j is endowed with k_{j0} units of capital and with $b_{j0} = 0$ units of one period bonds.

Market arrangements are standard. Competitive firms that maximize period profits. Bonds are issued by households. Households can borrow and lend, so that b_{jt} may be negative, but k_{jt} must be non-negative.

Notation: Denote the rental prices for capital and labor by (r_t, w_t). The gross interest rate on the bond is R_t.

(a) State the household's **Bellman equation** and define a solution to the household problem in functional and in sequence form. Assume an interior solution ($k' > 0$).

(b) Define a **competitive equilibrium** in sequence language. Make sure the number of equations matches the number of unknowns.

(c) Define the economy's **steady state**. Is it possible to find the steady state interest rate without further restrictions on the utility function? Hint: What is the household's Euler equation in steady state?

The following questions require **verbal explanations** that should be supported by reference to equilibrium conditions. You need not prove your answers.

(d) Are there steady states that feature persistent inequality? Or do all households converge to the same level of per capita consumption, even if their endowments k_{j0} differ? Hint: Write down the household's present value budget constraint in steady state. It gives you steady state c_j as a function of prices and endowments (k_{j0}, b_{j0}).

(e) How does the steady state allocation change when a unit of capital is taken from household j and given to household j'? This is a comparison of steady states, not of equilibrium paths.

(f) Now imagine households differ in their β's, but not in their u functions. For simplicity, assume that $u(c) = c^{1-\sigma}/(1 - \sigma)$. What would the asset distribution look like in the limit as $t \to \infty$? Here you will need some intuition.