Contracts and Incentives

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Contracts and Incentives

Contracts can provide insurance, but that messes up incentives

Example: Unemployment Insurance

- An unemployer worker searches for a job.
- The job finding rate depends on search effort a.
- Income is low during unemployment.
- The worker is risk averse and likes smooth consumption.
- Full insurance implies: the worker has no incentive to search for a job.

The task: Design an unemployment insurance scheme that trades off **consumption smoothing** and **incentives** to search hard.

Environment

Preferences:

$$E \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - a_t \right] \quad (1)$$

$$c_t, a_t \geq 0 \quad (2)$$

The worker starts unemployed with income 0.

Consumption equals income. No storage.

All jobs pay w.

The job finding rate is $p\left(a\right)$ with $p\left(0\right) = 0$, $p' > 0$, $p'' < 0$.

Autarky

When employed:

- $c = w$. $a = 0$.
- $V^e = \frac{u(w)}{1-\beta}$.

Autarky

When unemployed

$$V^u = \max_a u(0) - a + \beta \left[p(a) V^e + (1 - p[a]) V^u \right] \quad (3)$$

FOC:

$$\beta p'(a) \ \left[V^e - V^u \right] \leq 1$$

with equality if $a > 0$.

Solution is time invariant a and V^u.

Optimal search effort equates

- marginal cost of effort (1)
- marginal wage gain from searching

Full information and control

The insurance agency can observe and control search effort.

This eliminates the incentive problem and yields full insurance.

Contract design problem

- Assume the worker is promised utility V.
- The cost of delivering V is $C(V)$.
- The unemployment agency designs a contract to minimize $C(V)$.
- In each period: assign a search effort a and consumption c.
- If search fails: update V.

The agency's problem

$$C\left(V\right)=\min_{c,a,V^{u}} c+\beta\left[1-p\left(a\right)\right]C\left(V^{u}\right) \tag{4}$$

subject to promise keeping

$$u\left(c\right)-a+\beta\left[p\left(a\right)V^{e}+\left(1-p\left[a\right]\right)V^{u}\right]\geq V \tag{5}$$

where $V^{e}=w/\left(1-\beta\right)$.

FOCs

θ is the Lagrange multiplier on the promise keeping constraint.

$$c : 1 = u'(c)\,\theta$$

$$V^u : \beta p(a)\, C'(V^u) = \theta \beta p(a)$$

$$\implies C'(V^u) = \theta$$

$$a : \beta p'(a)\, C(V^u) = \theta \{1 - \beta p'(a)\,[V^e - V^u]\}$$

Envelope:

$$C'(V) = \theta$$

Characterization

$C'(V) = \theta = C'(V^u)$.

Assumption: C is strictly convex (verify later).

Then

$$V^u = V \tag{6}$$

Constant θ implies constant c and a.

Intuition: Without incentive issues, the problem of the unemployed is stationary.

Contract when effort is not observable

If the agency makes transfers to the household $\left(V^u > V^{aut}\right)$, incentives for search are reduced.

If a is not contractable, the worker chooses a below "optimal"

Example: $V = V^e \Longrightarrow a = 0$.

A contract must provide a penalty for not finding a job quickly.

In the data: Unemployment benefits typically declines with unemployment duration.

Optimal contract with asymmetric information

The agency cannot observe a.

It still controls c through unemployment benefits.

The agency's problem:

$$C(V) = \min_{c,a,V^u} c + \beta [1 - p(a)] C(V^u) \tag{7}$$

subject to promise keeping

$$u(c) - a + \beta [p(a) V^e + (1 - p[a]) V^u] = V \tag{8}$$

and incentive compatibility.

Incentive compatibility

The assigned a must be consistent with the household's first-order condition from

$$\max u(c) - a + \beta \left[p(a) V^e + (1 - p[a]) V^u \right] \tag{9}$$

The FOC is the same as under autarky:

$$\beta p'(a) \; \left[V^e - V^u \right] \leq 1 \tag{10}$$

Optimal contract

$$
\begin{aligned}
C(V) &= \min_{c,a,V^u} c + \beta\left[1-p(a)\right] C\left(V^u\right) \\
&\quad +\theta\left[V - u(c) + a - \beta p(a) V^e - \beta\left[1-p(a)\right] V^u\right] \\
&\quad +\eta\left[1 - \beta p'(a)\left\{V^e - V^u\right\}\right]
\end{aligned}
$$

FOCs

$$
\begin{aligned}
c &: \theta u'(c) = 1 \\
a &: C\left(V^u\right) = \theta\left[\frac{1}{\beta p'(a)} - \left(V^e - V^u\right)\right] - \eta \frac{p''(a)}{p'(a)}\left(V^e - V^u\right) \qquad (11) \\
V^u &: C'\left(V^u\right) = \theta - \eta \frac{p'(a)}{1-p(a)}
\end{aligned}
$$

Optimal contract

Envelope:

$$C'(V) = \theta$$

Notes:

- With $\eta = 0$ the FOCs with full control emerge.
- $[.] = 0$ in (11) because of incentive compatibility.

Characterization

Assume: $C(V^u) > 0$ so that promise keeping is binding $(\theta > 0)$.
Then

$$C'(V) = \theta = C'(V^u) + \eta \frac{p'(a)}{1 - p(a)} > C'(V^u)$$

Assumption (tricky): C is convex. Then

$$V^u < V$$

For a household who remains unemployed: $V' = V^u$ and V is falling over time.

Then θ rises over time $(C'' < 0)$ and c falls over time.

Characterization

From the household's FOC

$$\beta p'(a)\ [V^e - V^u] \leq 1$$

It follows that a rises over time.

Intuition: Agency interprets long unemployment as evidence of low search effort.

Reading

- Ljungqvist & Sargent, "Recursive methods," 2nd ed. ch. 21.