Dynamic Contracts

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Issues

- Many markets work through **intertemporal contracts**
- Labor markets, credit markets, intermediate input supplies, ...
- Contracts solve (or create) a number of problems:
    1. Insurance: firms insure workers against low productivity shocks.
    2. Incentives: work hard to keep your job.
    3. Information revelation: you can lie once, but not over and over again.

Optimal contracts

If there are no frictions, agents can write complete contracts.

Frictions prevent this:

1. Lack of **commitment**: borrowers can walk away with the loan.
2. Private **information**: firms don't observe how hard employees work.

We study optimal contracts for these frictions.

An analytical trick

- Dynamic contracts generally depend on the entire history of play.
  - "Three strikes and you are out"
- The set of possible histories grows exponentially with t.
- A trick, due to Abreu et al. (1990), makes this tractable.
- Use the **promised expected future utility** as a state variable.
- Then the current payoff can (often) be written as a function of today's play and promised value.

Money Lender Model

Money lender model

Thomas and Worrall (1990), Kocherlakota (1996)

The problem:

- A set of agents suffer income shocks.
- They borrow / lend from a "money lender".
- They cannot commit to repaying loans.
- How can a contract be written that provides some insurance?

Applications:

- Credit markets with default
- Sovereign debt

The contract may not be *explicitly* include state-contingent payoffs

Environment

- The world lasts forever.
- There is one non-storable good.
- A money lender can borrow / lend from "abroad" at interest rate β^{-1}.
- A set of agents receive random endowments y_t.
- They can only trade with the money lender.

Preferences

$$E \sum_{t=0}^{\infty} \beta^t \, u(c_t)$$

Note: β determines time preference and interest rate.

Endowments

- Each household receives iid draws y_t.
- y takes on S discrete values, $\bar{y}_s$.
- Probabilities are Π_s.

Complete markets

- Households could achieve full insurance by trading Arrow securities.
- Consumption would be constant at the (constant) mean endowment.

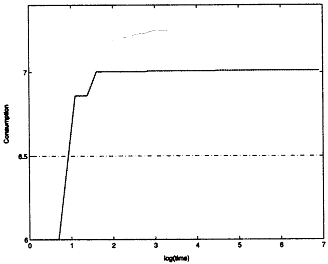
Incomplete markets

We consider 3 frictions:

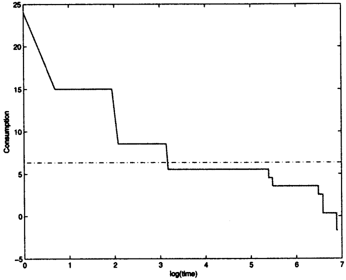
1. Households cannot commit not to walk away with a loan.
2. Households have private information about y_t.
3. Households have private information and a storage technology.

The optimal contracts in the 3 cases are dramatically different.

Sample consumption paths



(a) Lack of commitment



(b) Private information

Ljungqvist and Sargent (2004)

Sample consumption paths

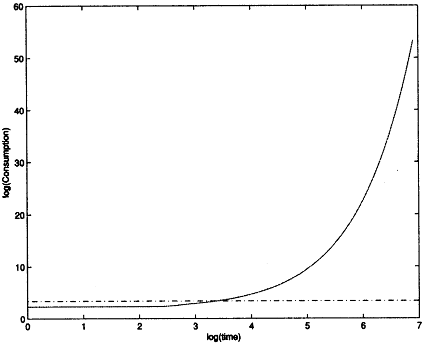


Figure 19.2.2: Typical consumption path in environment c.

(c) Private information and storage

Ljungqvist and Sargent (2004)

How to set up the problem

Assumptions:

1. the money lender offers the contract to the household
2. the household can accept or reject
3. the household accepts any contract that is better than autarky

How to set up the problem

- The optimal contract can be written as an **optimization problem**:
  - max profits
  - subject to: participation constraints.
- The state is the promised future value of the contract.
- To characterize, take first-order conditions.

One Sided Commitment

One sided commitment

Assumption:

- The money lender commits to a contract.
- Households can walk away from their debt.
- As punishment, they live in autarky afterwards.

The contract must be self-enforcing.

Applications:

- Loan contracts.
- Labor contracts.
- International agreements.

Contract

- We can study an economy with one person - there is no interaction.
- A contract specifies an allocation for each history: $h_t = \{y_0, ..., y_t\}$
- An allocation is simply household consumption:

$$c_t = f_t(h_t) \tag{1}$$

- The money lender collects y_t and pays c_t.

Contract

- Money lender's profit:

$$P = E\sum_{t=0}^{\infty}\beta^t (y_t - f_t(h_t)) \tag{2}$$

- Agent's value:

$$v = E\sum_{t=0}^{\infty}\beta^t\ u(f_t(h_t)) \tag{3}$$

- These are complicated!

Participation constraint

- With commitment, the lender would max P subject to the resource constraint.
  - What would the allocation look like?
- Lack of commitment adds a participation constraint:

$$\underbrace{E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} u(f_t(h_t))}_{\text{stay in contract}} \geq \underbrace{u(y_\tau) + \beta v_{AUT}}_{\text{walk away}} \tag{4}$$

- This must hold for every history h_t.

Autarky Value

- If the agent walks, he receives

$$v_{AUT} = E \sum_{t=0}^{\infty} \beta^t \ u(y_t) = \frac{E \ u(y_t)}{1-\beta} \tag{5}$$

Recursive formulation

- The contract is not recursive in the natural state variable y_t.
- History dependence seems to destroy a recursive formulation.
- We are looking for a state variable x_t so that we can write:

$$
\begin{aligned}
c_t &= g(x_t, y_t) \\
x_{t+1} &= l(x_t, y_t)
\end{aligned}
$$

Recursive formulation

- The correct state variable is the promised value of continuation in the contract:

$$v_t = E_{t-1} \sum_{j=0}^{\infty} \beta^j \, u(c_{t+j}) \tag{6}$$

- The household enters the period with promised utility v_t, then learns y_t.
- The contract adjusts c_t and v_{t+1} to fulfill the promise v_t.
- Proof: Abreu et al. (1990)

Recursive formulation

The state variable for the lender is v.

The obective is to design payoffs, c_s and w_s, for this period to max discounted profits

$$P(v) = \max_{c_s, w_s} \sum_{s=1}^{S} \Pi_s [(\bar{y}_s - c_s) + \beta P(w_s)] \tag{7}$$

w_s is the value of v' promised if state s is realized today.

Constraints

1. Promise keeping:

$$\sum_{s=1}^{S} \Pi_s \left[u(c_s) + \beta w_s \right] \geq v \tag{8}$$

2. Participation:

$$u(c_s) + \beta w_s \geq u(\bar{y}_s) + \beta v_{AUT}; \quad \forall s \tag{9}$$

3. Bounds:

$$c_s \in [c_{\min}, c_{\max}] \tag{10}$$

$$w_s \in [v_{AUT}, \bar{v}] \tag{11}$$

Cannot promise less than autarky or more than the max endowment each period.

Lagrangian / Bellman equation

$$P(v) = \max_{c_s, w_s} \sum_{s=1}^{S} \Pi_s \left[(\bar{y}_s - c_s) + \beta P(w_s) \right] \tag{12}$$

$$+\mu \left[\sum_{s=1}^{S} \Pi_s \left[u(c_s) + \beta w_s \right] - v \right] \tag{13}$$

$$+\sum_{s} \Pi_s \lambda_s \left[u(c_s) + \beta w_s - u(\bar{y}_s) - \beta v_{AUT} \right] \tag{14}$$

Notes:

1. W.l.o.g. I wrote the multipliers as $\Pi_s \lambda_s$.
2. Participation constraints may not always bind. Then $\lambda_s = 0$.

FOCs

$$c_s \quad : \quad \Pi_s = u'(c_s)\,\Pi_s\,[\lambda_s + \mu] \tag{15}$$
$$w_s \quad : \quad -\Pi_s P'(w_s) = \Pi_s\,[\lambda_s + \mu] \tag{16}$$

Assumption: P is differentiable. (Verify later)

Envelope:

$$P'(v) = -\mu \tag{17}$$

What do these say in words?

FOCs

Simplify:

$$u'(c_s) = -P'(w_s)^{-1} \tag{18}$$

This implicitly defines the consumption part of the contract: $c_s = g(w_s)$.

Properties:

- Later we see that $P(v)$ is concave ($P' < 0, P'' < 0$).
- Therefore: $u''(c_s)\, dc_s = \frac{P''(w_s)}{[P'(w_s)]^2} dw_s$ and $dc/dw > 0$.
- A form of consumption smoothing / insurance.
- If something makes the agent better off, the benefits are spread out over time.

Promised value

Sub Envelope in for μ:

$$P'(w_s) = P'(v) - \lambda_s \tag{19}$$

This describes how v evolves over time.

What happens depends on whether the participation constraint binds.

Case 1: Participation constraint does not bind

$\lambda_s = 0$

Therefore $P'(w_s) = P'(v)$ and

$w_s = v$ regardless of the realization y_s.

Consumption is a function of v, given by the FOC
$u'(c_s) = -P'(v)^{-1}$

- also constant over time

The household is fully insured against income shocks

- Intuition: this happens for low y.
- The lender may lose in such states: he pays out the promise.

Case 2: Participation constraint binds

$$\begin{aligned} \lambda_s &> 0 \\ P'(w_s) = P'(v) - \lambda_s &< P'(v) \end{aligned}$$

Therefore $w_s > v$: promised value rises.

Participation constraint:

$$u(c_s) + \beta w_s = u(\bar{y}_s) + \beta v_{AUT} \tag{20}$$

implies

$$c_s < \bar{y}_s \tag{21}$$

because $w_s \geq v \geq v_{AUT}$ (any contract must be better than autarky - otherwise the agent walks).

Intuition

Walking away from the contract is attractive in good states (high y_s).

The money lender must collect something in order to finance insurance in bad states: $c_s < \bar{y}_s$

The household gives up consumption in good times in exchange for future payoffs.

To make this incentive compatible, the lender has to raise future payoffs: $w_s > v$.

Amnesia

When the participation constraint binds, c and w are solved by

$$\begin{aligned} u(c_s) + \beta w_s &= u(\bar{y}_s) + \beta v_{AUT} \\ u'(c_s) &= -P'(w_s)^{-1} \end{aligned}$$

This solves for

$$\begin{aligned} c_s &= g_1(\bar{y}_s) \\ w_s &= l_1(\bar{y}_s) \end{aligned}$$

v does not matter!

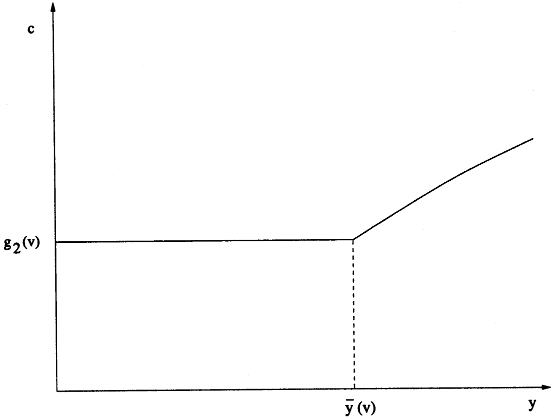
Intuition: The current draw y_s is so good that walking into autarky pays more than v.

The continuation contract must offer at least $u(\bar{y}_s) + \beta v_{AUT}$, regardless of what was promised in the past.

The optimal contract

- Intuition: For low y the participation constraint does not bind, for high y it does.
- The threshold value $\bar{y}(v)$ satisfies:
  1. Consumption obeys the no-participation equation $u'(c_s) = -P'(v)^{-1}$.
  2. The participation constraint binds with $w_s = v$: $u(c_s) + \beta v = u(\bar{y}[v]) + \beta v_{AUT}$
- $\bar{y}'(v) > 0$: Higher promised utility makes staying in the contract more attractive.

Consumption function

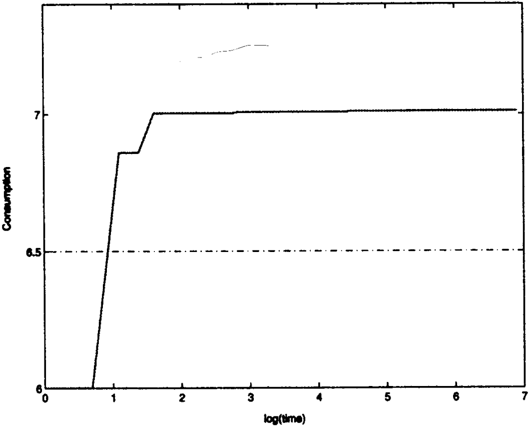


Ljungqvist and Sargent (2004)

Properties of the contract

1. For $y \leq \bar{y}(v)$: Pay constant $c = g_2(v)$ and keep c, v constant until the participation constraint binds.
2. For $y > \bar{y}(v)$: Incomplete insurance. $v' > v$.
3. v never decreases.
4. c never decreases.
5. As time goes by, the range of y's for which the household is fully insured increases.
6. Once a household hits the top $y = \bar{y}_S$: c and v remain constant forever.

Sample consumption path



Ljungqvist and Sargent (2004)

Intuition

- With two-sided commitment, the firm would offer a constant c.
  - It would collect profits from lucky agents and pay to the unlucky ones.
  - Because of risk aversion, the average c would be below the average y.
  - The firm earns profits.
- With lack of commitment:
  - Unlucky households are promised enough utility in the contract, so they stay. Full insurance.
  - Lucky households have to give up some consumption to pay for future payouts in bad states.
  - To compensate, the firm offers higher future payments every time a "profit" is collected.

Implications

Think about this in the context of a labor market.

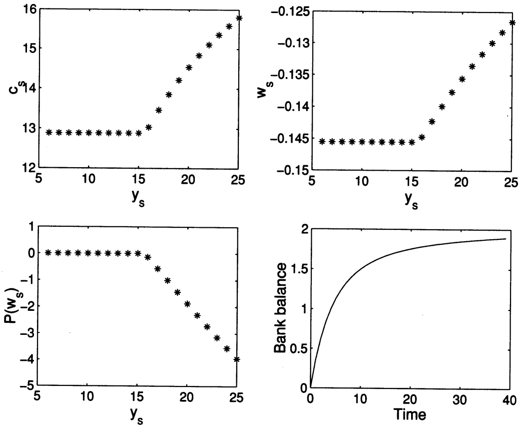
- "Young" households are poor (low v and c).
- Earnings rise with age.
- Earnings volatility declines with age (because the range of full insurance expands).
- Old workers are costly to employ. Firms would like to fire them.

This broadly lines up with labor market data.

Implications

- Inequality is first rising, then falling.
- Young households are all close to v_0 initially.
- Old households are perfectly insured in the limit.
- Middle aged households differ in their histories and thus payoffs.

Numerical example



Outcomes as function of y_s.

Ljungqvist and Sargent (2004)

Reading

- Ljungqvist and Sargent (2004), ch. 19.
- Abreu et al. (1990) - the paper that introduced the idea of using promised values as the state variable.

References I

Abreu, D., D. Pearce, and E. Stacchetti (1990): "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica: Journal of the Econometric Society*, 1041–1063.

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