Modern Macro

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What Econ720 is about

Macro is built around a small number of workhorse models:

1. Overlapping generations
2. Ramsey in continuous and discrete time
   - aka standard growth model, Cass-Koopmans model, neoclassical growth model
3. Stochastic Ramsey model
4. Search and matching models

We study basic versions of the **models** and the **tools** needed to analyze them.

What is not covered

1. How the models are applied to study macro questions
   1.1 this is a **theory** course
   1.2 but we will make some exceptions
2. Computational issues
   2.1 see Econ821
3. Empirical issues.

Modern Macro

(Special Advertisement Section)

Or:

Why Most of Your Undergraduate Macro Courses Were Useless

Some of you will find the next few slides obvious...

Modern macro

Let's start by talking about how macroeconomists approach questions.

The main point is:

Macro is micro.

An Old-Fashioned Macro Model

- Consumption function: $C = C_0 + cY$.
- Investment function: $I = I_0 - bi$.
- Identity: $Y = C + I + G$.
- IS curve:

$$(1-c)Y = C_0 + I_0 + G - bi$$

- Money demand: $L = L_0 + kY - di$.
- Money supply: M/P.
- LM curve:

$$M/P = L_0 + kY - di$$

IS-LM Implications

1. Government spending always raises output and employment.
   - Constraints are missing (the supply side).
2. There is a fiscal multiplier.
   - It is a function of the parameters c, k, b, d.
   - Which parameters are stable?
3. Expectations do not matter (or do they affect I_0?).
4. Consuming more / saving less raises output.
   - The model lacks dynamics.

This cannot be right!

What is missing?

1. **Capital**: Saving less does not raise (future) output.
   - A good model must be **dynamic**.
2. **Budget constraints**: Taxing people reduces income.
   - A good model must be internally consistent.
3. **Expectations**: the parameters are not stable.
   - A good model should have stable parameters.
   - Stable parameters are "deeper" than marginal propensities to consume.
4. **Choices**: Taxing people may lead them to work harder.
   - A good model must capture how individuals respond to changing prices / expectations.
5. **Welfare**: Is raising Y good or bad?

Modern Macro

- Modern macro builds models bottom-up (**micro-foundations**).
- A model is an **artificial economy**.
- It is described by the list of **agents**, their **demographics**, their **preferences**, and the **technologies** they have access to.
- Individual behavior is the result of an **optimization problem**.
- Agents have **rational expectations**.
  - They understand how the economy works.
  - Their expectations are the best possible forecasts.
- Agents interact in markets.
  - Prices and quantities are determined by market clearing.

Digression

Are people really this rational?

Competitive Equilibrium

What this course is really about:

How do you translate the description of an economy into a set of equations that characterize the **competitive equilibrium**.

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Definition

A competitive equilibrium is an **allocation** (a list of quantities) and a **price system** (a list of prices) such that
- the quantities **solve all agents' problems**, given the prices;
- all **markets clear**.

How to Set Up a Competitive Equilibrium

1. Describe the economy
2. Solve each agent's problem
3. State the market clearing conditions
4. Define an equilibrium

Step 1: Describe the Economy

1. List the agents (households, firms).
2. For each agent define:
   - **Demographics**: e.g., population grows at rate n.
   - **Preferences**: e.g., households maximize utility $u(c)$.
   - **Endowments**: e.g., each household has one unit of time each period.
   - **Technologies**: e.g., output is produced using $f(k)$.
3. Define the **markets** in which agents interact.
   - E.g., households work for firms; households purchase goods from firms.

Step 2: Solve Each Agent's Problem

- Write down the maximization problem each agent solves.
  - E.g.: The household chooses c and s to maximize utility, subject to a budget constraint.
- Derive a set of equations that determine the agent's choice variables.
  - E.g.: A consumption function, saving function.

Step 3: Market Clearing

- For each market, calculate supply and demand by each agent.
- Aggregate supply $= \sum$ individual supplies.
- Aggregate supply $=$ aggregate demand.

Step 4: Define the Equilibrium

From steps 2-3:

Collect all endogenous objects

- e.g., consumption, output, wage rate, ...

Collect all equations

- first order conditions or policy functions
- market clearing conditions

You should have N equations that could (in principle) be solved for N endogenous objects

- prices
- quantities (the allocation)

What do we gain from this approach?

Consistency:

- Aggregate relationships by construction satisfy individual constraints.
- Example: the aggregate consumption function cannot violate any person's budget constraint.

Transparency:

- The assumptions about the fundamentals are clearly stated.

What do we gain from this approach?

Non-arbitrary behavior:

- In old macro, results depend on the assumed behavior.
- In modern macro, behavior is derived.

Expectations:

- Expectations are endogenous.
- They are automatically consistent with the way the economy behaves.

What do we gain from this approach?

Welfare:

- It is possible to figure out how a policy change affects the welfare (utility) of each agent.

Testing:

- Models can be tested against micro data.

Micro and macro become the same thing.

Static example

Static Example

- We study a very simple one period economy.
- There are many identical households.
- They receive **endowments** which they eat in each period.
- Nothing interesting happens in this economy - it merely illustrates the method.

Step 1: Describe the Economy

- **Demographics**:
  - There are N identical households.
  - They live for one period.
  - For now, there are no other agents (firms, government, ...).
- **Preferences**:
  - Households value consumption of two goods according to a utility function $u(c_1, c_2)$

Step 1: Describe the Economy

- **Technology**:
  - The technology is trivial: each agent receives **endowments** of the two goods (e_1, e_2).
  - There is no production. Endowments cannot be stored.
- **Markets**:
  - Agents trade goods in a market, where everyone behaves as a price-taker.
  - There are no financial assets.
  - The prices of the two goods are p_1 and p_2.
    What are prices denoted in?

Step 2: Household problem

There is only one agent: the household.

Households maximize $u(c_1, c_2)$ subject to a budget constraint.

State variables the household takes as given:

- market prices for the two goods, p_1 and p_2.
- endowments e_1 and e_2.

The **choice variables** are c_1 and c_2.

- We can normalize the price of one good to one (numeraire): $p_1 = 1$.
- Call the relative price $p = p_2/p_1$.

Household problem

Budget constraint: Value of endowments = value of consumption.

The household solves the **problem**:

$$
\begin{aligned}
& \max u(c_1, c_2) \\
s.t.\ c_1 + p\ c_2 &= e_1 + p\ e_2
\end{aligned}
$$

Solving the household problem

- A solution to the household problem is a pair (c_1, c_2).
- To find the optimal choices set up a **Lagrangean**:

$$\Gamma = u(c_1, c_2) + \lambda \ [e_1 + p\ e_2 - c_1 - p\ c_2]$$

- What happens if we write the budget constraint the other way around?

$$\Gamma = u(c_1, c_2) + \lambda \ [c_1 + p\ c_2 - e_1 - p\ e_2]$$

- It would actually be easier to substitute the constraint into the objective function and solve the unconstrained problem

$$\max \ u(e_1 + p\ e_2 - p\ c_2, c_2)$$

but the Lagrangean is instructive.

Household first-order conditions

- The **first order conditions** are

$$\partial \Gamma / \partial c_i = u_i(c_1, c_2) - \lambda p_i = 0 \tag{1}$$

- The multiplier λ has a useful interpretation: It is the marginal utility of relaxing the constraint a bit, i.e. the marginal utility of wealth.
- The solution to the household problem is then a vector (c_1, c_2, λ) that solves
  - 2 FOCs
  - the budget constraint.

Some tips

- Always explicitly state what variables constitute a solution and which equations do they have to satisfy.
- You should have a FOC for each choice variable and all the constraints.
- Make sure you have the same number of variables and equations. Later on, this will make it easier to assemble the equations needed for the competitive equilibrium.

Simplify the optimality conditions

- It is useful to substitute out the Lagrange multiplier λ.
- The ratio of the FOCs implies

$$u_2/u_1 = p \tag{2}$$

- This is the familiar tangency condition: marginal rate of substitution equals relative price. [Graph]
- Now the solution is a pair (c_1, c_2) that satisfies (2) and the budget constraint.
- Note: I can keep the Lagrange multiplier or drop it. If I keep it, I also need to keep another equation (e.g., the FOC for c_1).

Log utility example

- Assume log utility:

$$u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$$

- Then the problem can be solved in closed form:

$$\frac{u_2}{u_1} = \beta \frac{c_1}{c_2} = p$$

- Substitute this back into the budget constraint:

$$\begin{aligned} c_1 + \beta\ c_1 &= W = e_1 + p\ e_2 \\ c_1 &= \frac{W}{1+\beta} \\ c_2 &= \frac{\beta W}{1+\beta} \end{aligned}$$

Log utility example

- Tip: This is a peculiar (and often very useful) feature of log utility: the expenditure shares are independent of p. The reason is exactly the same as that of constant expenditure shares resulting from a Cobb-Douglas production function: unit elasticity of substitution.
- Tip: Recall that taking a monotone transformation of u doesn't change the optimal policy functions. In particular, we can replace u by

  $$u(c_1, c_2) = c_1 \ c_2^{\beta}$$

  Convince yourself that this yields exactly the same consumption functions.

Step 3: Market Clearing

There are two markets (for goods 1 and 2).

- Why isn't there just 1 market where good 1 is traded for good 2?

Each agent

- supplies the endowments e_i and
- demands consumption c_i in those markets.

Goods are traded for **units of account**.

I don't use the word **money** because there is no such thing in this economy.

Market Clearing

The market clearing condition is

"aggregate supply = aggregate demand."

Aggregate supply is simply the sum of individual supplies:

$$S_i = \sum_{h=1}^{N} e_i = N\ e_i \tag{3}$$

Aggregate demand:

$$D_i(p, e_1, e_2) = \sum_{h=1}^{N} c_i = N\ c_i(p, e_1, e_2) \tag{4}$$

Market clearing:

$$c_i = e_i \tag{5}$$

Everybody eats their own endowments.

Definition of Equilibrium

A **competitive equilibrium** is an allocation (c_1, c_2) and a price p that satisfy:

- 2 household optimality conditions (FOC and budget constraint).
- 2 goods markets clearing conditions.

Now we count equations and variables.

- We have $2N+1$ objects: $2N$ consumption levels and one price.
- We have $2N$ household optimality conditions and 2 market clearing conditions.

Why do we have one equation too many?

Arrow-Debreu versus Sequential Trading

Two Period Example

Demographics:

- N identical households live for 2 periods, $t = 1, 2$.

Commodities:

- there is one good in each period

Preferences: $u(c_1, c_2)$

Endowments: e_t

Markets

Now we have a choice between 2 equivalent arrangements

- Arrow-Debreu: all trades take place at $t = 1$
- Sequential trading: markets open in each period

Arrow-Debreu Trading

The arrangement:

- All trades take place at $t = 1$
- Agents can buy and sell goods for delivery at any date t
- Prices are p_t

Surprise: If we write out this model, it **looks exactly like the static 2 good model** (see above).

Equivalence of Dates and Goods

Fact

A model with T goods is equivalent to a model with T periods.

This is only true under "**complete markets**"

- roughly: there are markets that allow agents to trade goods across all periods and states of the world
- we will talk about details later

Sequential Trading

An alternative trading arrangement.

Markets open at each date.

Only the date t good can be purchased in the period t market.

Now we have **one numeraire for each trading period**: $p_t = 1$.

We need assets to transfer resources between periods.

Markets

At each date we have

1. a market for goods $(p_t = 1)$;
2. a market for 1 period discount bonds (price q_t)

A discount bond pays 1 unit of $t+1$ consumption.

Household problem

Now we have one budget constraint per period:

$$e_t + b_{t-1} = c_t + b_t q_t \tag{6}$$

With $b_0 = 0$.

Household solves:

$$\max_{b_1} u(e_1 - b_1 q_1, e_2 + b_1) \tag{7}$$

Household solution

FOC:

$$u_1 q_1 = u_2 \tag{8}$$

q_1 is the relative price of period 2 consumption.

Give up 1 unit of c_1 and get $1/q_1$ units of c_2.

Solution: c_1, c_2, b_1 that solve FOC and 2 budget constraints.

Market Clearing

- Goods: $e_t = c_t$
- Bonds: $b_t = 0$

Equivalence

Fact

When markets are complete, Arrow-Debreu and sequential trading equilibria are identical.

Summary

IS-LM is dead. Long-live general equilibrium

- The method outlined here is central to all of (macro) economics.
- Being able to translate a description of an economy into the definition of a competitive equilibrium is an important skill.

Reading

Krusell (2014), ch. 2 describes the ingredients of modern macro models.

Ch. 5 talks about Arrow-Debreu versus sequential trading.

References I

Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.