The Romer Model

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We study models where intentional innovation drives productivity growth.

Romer model:
- Innovations are produced like any other good using R&D labor as input.

Policy effects
- Policies, such as R&D subsidies, can change the rate at which innovations are produced.
- Surprisingly, it turns out that policies have no effect on long-run growth.
Learning Objectives

In this section you will learn:

1. how to analyze the Romer model
2. why R&D policies do not change the long-run growth rate of the economy
The Romer model

Solow block

▶ Production of goods works exactly like in the Solow Model
▶ Aggregate production function:

\[ Y_t = K_t^\alpha (A_tL_Yt)^{1-\alpha} \]  \hspace{1cm} (1)

▶ Capital accumulation as in the Solow model

\[ \dot{K}_t = s_KY_t - \delta K_t \]  \hspace{1cm} (2)

▶ Labor input grows at a constant rate

\[ g(L) = n \]  \hspace{1cm} (3)
Solow Block

What has changed?

Final goods production function has:

- constant returns to rival inputs: $K$ and $L_Y$.
- has **increasing returns** to all inputs (including $A$)

Labor is divided into production ($L_Y$) and R&D ($L_A$).
R&D Block

- Ideas are produced just like other goods.
- The input is labor \((L_{At})\)
  - not much changes if capital is an input, too.
- The output is a number of new ideas.
  - \(A_t\) is the number of ideas that have been invented up to \(t\).
  - \(\dot{A}_t\) is the number of ideas discovered today (or the rate at which they are discovered).
R&D Block

- The **ideas production function**: 
  \[
  \dot{A}_t = \bar{B}L_{At}^\lambda
  \]  
  (4)

- \( \lambda \) determines returns to scale.
- \( \bar{B} \) is a productivity parameter.
Ideas are inputs to innovation

- How easy it is to produce a new idea depends on how much has already been discovered.
  \[ \bar{B} = B A^\phi \]  

- If ideas help produce new ideas: \( \phi > 0 \): \( A \uparrow \rightarrow \bar{B} \uparrow \).
- If there is "fishing out": \( \phi < 0 \).
- Assume \( \phi \leq 1 \). (If \( \phi > 1 \) odd things happen...).
- The ideas production function is then
  \[ \dot{A} = B L_A^\lambda A^\phi \]  
  \[ g(A) = B L_A^\lambda A^{\phi-1} \]
Ideas production function

Even though ideas foster innovation ($\phi > 0$), more ideas imply slower $g(A)$. 

Even though ideas foster innovation ($\phi > 0$), more ideas imply slower $g(A)$.
Note how similar this is to the law of motion for capital in the Solow model.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \dot{K}_t ) =</th>
<th>Productivity</th>
<th>“Capital”</th>
<th>Labor</th>
<th>Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solow</td>
<td>( sA^{1-\alpha} )</td>
<td>( K_t^\alpha )</td>
<td>( L_t^{1-\alpha} )</td>
<td>( -\delta K_t )</td>
<td></td>
</tr>
<tr>
<td>Romer</td>
<td>( \dot{A}_t ) =</td>
<td>( B )</td>
<td>( A_t^\phi )</td>
<td>( L_{At}^\lambda )</td>
<td>( -0 )</td>
</tr>
</tbody>
</table>

It follows that there cannot be long-run growth in \( A/L \) when \( \lambda + \phi < 1 \) (details follow).

But we still can get long-run growth in \( Y/L \).
The Romer model

Behavior

So far we have described technologies. To describe behavior, we make a **Solow assumption:**

- A constant saving rate

\[ \frac{S}{Y} = \frac{I}{Y} = s_K \]

- A constant labor allocation:

\[ L_A = s_A L \quad (8) \]
\[ L_Y = (1 - s_A) L \quad (9) \]
Model summary

The Solow block:

\[ Y = K^\alpha \ (A \ L_Y)^{1-\alpha} \quad (10) \]
\[ \dot{K} = s_K \ Y - \delta \ K \quad (11) \]
\[ L_t = L_0 \ e^{nt} \quad (12) \]

Production of ideas:

\[ \dot{A} = B \ L_A^\lambda \ A^\phi \quad (13) \]

Constant behavior:

\[ L_Y = s_Y \ L; \quad L_A = s_A \ L \quad (14) \]

The growth rate of ideas:

\[ g(A) = B \ (s_A \ L)^\lambda \ A^{\phi-1} \quad (15) \]
Model summary

- This looks complicated, but isn’t.
- We have tricked the model such that $Y$ and $K$ don’t matter for how $A$ evolves.
  \[ \dot{A} = B L_A^\lambda A^\phi \]  

- This would change, if we let $\dot{A}$ depend on $K$
  - but that would not affect the results
  - only the algebra would be more complicated (see Romer 2011)
Does the Model Make Sense?

- The production functions are arbitrary.
  - But what matters are certain qualitative features, not the exact functional form.
  - We will get back to this.

- There is only one input. Only one good.
  - All of this can be relaxed without changing anything too important.

- Where are the households, consumption, population growth ...
  - We can add those - it does not make any difference.

- The labor allocation is fixed.
  - This is important.
  - The literature does not make this assumption. It can talk about patents, policy, ...

- Ideas are produced like goods.
Balanced growth path

**Definition**

A BGP is a path along which all variables grow at **constant rates**.

Why might this be interesting?
Balanced growth path

At what rates do the endogenous objects grow on the BGP?

Result 1: \( g(k) = g(y) \)

Proof:
Balanced growth path

Result 2: \( g(y) = g(A) \)

Proof:

Result

All long-run growth is due to R&D.
Growth rate of ideas

\[ g(A) = \frac{\lambda n}{1 - \phi} \]  

(17)

**Proof:**

Ideas production:

\[ g(A) = B \frac{L_A^\lambda}{A^{1-\phi}} \]  

(18)

BGP: \( g(A) \) is constant \( \implies \) \( L_A^\lambda A^{\phi-1} \) is constant

Take growth rates of that

\[ g(g(A)) = \lambda g(L_A) - (1 - \phi) g(A) = 0 \]  

(19)

With constant time allocation, \( s_A: g(L_A) = n \).

Solve for \( g(A) \). Done.
Summary: Balanced growth

Balanced growth in the Romer model is characterized by:

\[ g(y) = g(k) = g(A) \tag{20} \]
\[ g(A) = \frac{\lambda n}{1 - \phi} \tag{21} \]

All growth is due to innovation.
Why is this true?
Why is all growth due to innovation?

Solow model:

Romer model:
Balanced growth: Intuition

\[ g(A) = \frac{\lambda \ n}{1 - \phi} \]  (22)

Growth is simply a multiple of population growth.
Behavior does not matter: \(s_K\) and \(s_A\) do not appear in (22).
Intuition

▶ Consider the case $\phi = 0$.
▶ Ideas production is then

$$\dot{A} = B L_A^\lambda$$

(23)

▶ If the population is constant, $L_A$ is constant.
▶ In each period, the economy produces a constant number of ideas.
▶ The growth rate of ideas, $g(A) = B L_A/A$, falls to zero over time.
▶ A fixed number of people cannot produce a growing stream of ideas.

Population growth is necessary for sustained innovation (at a constant rate).
How growth is sustained

\[ g(A) = B A^{\phi - 1} L_A^\lambda \]
Special Case: Phi = 1

With $\phi = 1$, idea production becomes

$$g(A) = B \, L_A^\lambda$$

This is the case studied by Romer (1990).
The model has exploding growth, unless the population is constant.
This is clearly contradicted by post-war data: $L_A$ rose dramatically, while $g(y)$ was at best constant.
Reality check

1. The model says: constant population - no growth.
   ▶ But we are still producing new ideas all the time.
   ▶ How can we reconcile this?

2. What if the population shrinks over time?
   ▶ Is the long-run growth rate negative?
Reading

▶ Jones (2013b), ch. 5.

Optional:

▶ Romer (2011), ch. 3.1-3.4
▶ Jones (2013a), ch. 6
Advanced Reading

- Jones (2005) talks in some detail about the economics of ideas.


