Review Problems: Innovation and Growth

Econ520. Fall 2022. Prof. Lutz Hendricks. November 2, 2022

Jones, Macroeconomics, problems 6.1-6.8.

1 Basics

- 1. What is meant by the non-rivalry of ideas?
 - (a) Give examples of rival and non-rival goods.
 - (b) If Roche holds a patent on a drug, does that make it a rival good?
- 2. Explain how non-rivalry lead to increasing returns to scale and scale effects.
- 3. What is meant by scale effects? Explain why they arise.
- 4. Define "balanced growth path."

1.1 Answers: Basics

- 1. See slides
- 2. See slides
- 3. See slides.
- 4. An equilibrium path along which all variables grow at constant rates.

2 Romer Model

- 1. Why is there sustained growth in the Romer model, but not in the Solow model?
- 2. Derive the balanced growth rate of ideas in the Romer model.
- 3. Suppose there is a one-time increase in the productivity of research. Describe the effect on the level and the growth rate of technology (A).
- 4. The government uses patent protection and R&D subsidies to foster growth. Could such policies overshoot their targets and actually reduce output and consumption, even in the long-run?

2.1 Answer: Romer model

1. Romer: constant returns to A in the production of A. Solow: diminishing returns to K in the production of K.

Of course, the Romer model with diminishing returns grows, if there is population growth. This is due to the non-rivalry of ideas.

So there are four cases: rival / non-rival \times constant returns / diminishing returns. All of them have sustained growth (with population growth), except for the Solow case (diminishing returns / rival).

- 2. We did this in class.
- **3.** Increase in \bar{z} : faster growth $(g = \bar{z}\ell\bar{N})$. No change in y at impact.

4. The short answer is: of course. Suppose we set the fraction of labor working in R&D to 1. Then output is zero.

3 Modified Romer Model

Consider the following modified Romer model:

• Production functions:

$$Y_t = A_t^{\alpha} L_{yt} \tag{1}$$

$$\dot{A}_t = BA_t L_{at} - dA_t \tag{2}$$

• Resource constraint:

$$L = L_{yt} + L_{at} \tag{3}$$

• Allocation of labor:

$$L_{at} = \ell \bar{N} \tag{4}$$

$$L_{yt} = (1-\ell)N \tag{5}$$

There are two changes relative to the original Romer model: the exponent α on labor in the production function for goods and the depreciation term dA_t in the production function for ideas (which is the same as the depreciation term in the Solow model). Assume that $0 < \alpha < 1$ and 0 < d < 1.

Questions:

- 1. Derive the growth rates of Y_t and of A_t as functions of exogenous parameters.
- 2. Plot the time paths of $log(Y_t)$ and $log(A_t)$ for an economy that experiences a permanent increase in depreciation (d rises) at date t_0 . Explain what you plot.
- 3. Explain why the non-rivalry of ideas leads to increasing returns to scale. What does non-rivalry mean?

3.1 Answers: Modified Romer Model

- 1. $g(Y) = \alpha g(A)$ and $g(A) = B\ell \overline{N} d$.
- 2. Straight lines with kinks at t_0 . The $\log(Y)$ line is flatter than $\log(A)$ line. Explanation: higher d reduces g(A). But no jump in A at t_0 ; we are not changing the stock of ideas, just the growth rate.
- 3. We think constant returns to all rival factors the replication argument. But non-rival ideas do not need to be replicated. Example: build 2 factories to double output. No need to double the number of blueprints. Nonrivalry means: an idea can be used at the same time by multiple users.

4 Romer Model with Diminishing Returns

Consider the Romer model with $\dot{A}_t = BA_t^{\phi} s_R L_t$, $L_t = e^{nt}$ and $\rho < 1$.

- 1. Derive the balanced growth rate.
- 2. Intuitively, why does the balanced growth rate rise with n?
- 3. [Harder] What is the effect of a permanent increase in s_R on the time path of A_t ? Set n = 0. Hint: use the law of motion for A_t to plot $g(A_t)$ against A_t .

4.1 Answers: Romer Model with Diminishing Returns

- 1. The same as in the slides, but with $\lambda = 1$: $g(A) = n/(1-\phi)$.
- 2. We did this in class: Without population growth, diminishing returns imply that growth peters out. Each time the population increases, there is an upward push to innovation (scale effect). We had a graph in the slides.

3. Use $g(A) = Bs_R L A^{\phi-1}$. This declines in A and has a steady state where g(A) = 0. Start in that steady state. Raising s_R pushes the g(A) curve up. The growth rate is now positive, but declines over time (moving along the g(A) curve towards the new steady state with higher A).