Exam 1. Econ520. Spring 2017

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UNC

Instructions:

- Answer all questions.
- Clearly number your answers. Write legibly.
- Do *not* write your answers on the question sheets.
- *Explain* your answers – do not just state them.
- *Show* your derivations – do not just state the final result.
- Do not refer to any notes or books. You may use a calculator.
- The total time is 75 minutes.
- The total number of points is 100.

1 Short Questions (35 points)

1. [9 points] Explain why long-run growth is not sustainable with capital accumulation only. Explain why it is sustainable through knowledge accumulation? What is the fundamental difference?

2. [11 points] Describe the "reversal of fortunes" evidence. How does this evidence support the case that institutions are important sources of cross-country income variation?

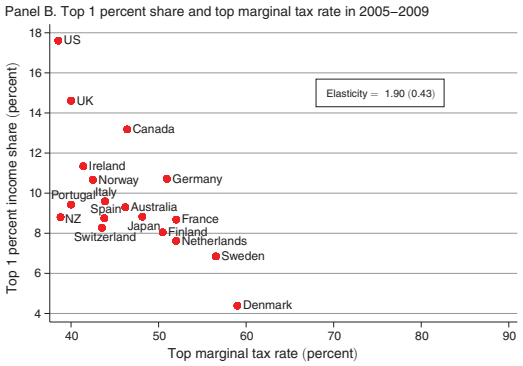
3. [15 points] In cross-country data, we see that countries with higher top marginal tax rates experience less income inequality.

   (a) Panel (a) of Figure 1 shows cross-country evidence at one point in time. It would be tempting to conclude that taxes cause inequality. What could go wrong with that argument? Be specific.

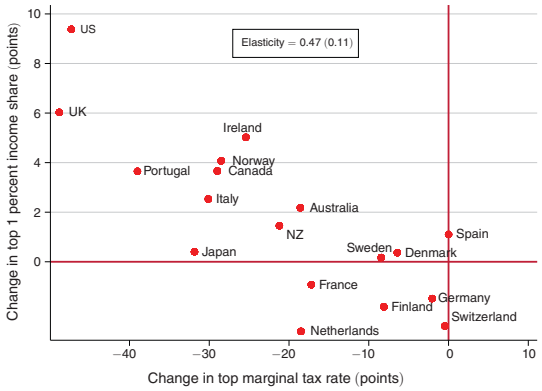
   (b) Panel (b) shows changes over time. How does this help address some of the issues that affect the interpretation of panel (a)? What issues remain?

Figure 1: Top marginal tax rates and income inequality

(a) Cross-country evidence



(b) Changes over time



2 Solow Model with Land (28 points)

Consider a Solow model with a fixed factor (land) denoted M. The production function is

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{\beta} M^{1-\alpha-\beta} \tag{1}$$

and the law of motion for $k = K/L$ is therefore

$$\dot{k}_t = s A_t^{\beta} k_t^{\alpha} m_t^{1-\alpha-\beta} - c_t - (n+\delta) k_t \tag{2}$$

where $m = M/L$ and n is population growth. Assume that A grows at a constant rate.

Questions:

1. [7 points] Show that the term $A_t^{\beta} k_t^{\alpha-1} m_t^{1-\alpha-\beta}$ must be constant on the balanced growth path. Hint: How can $g(k)$ be constant over time?

2. [7 points] Show that the balanced growth rate of capital is given by $g(k) = \frac{\beta g(A) - (1-\alpha-\beta)n}{1-\alpha}$. In the special case where $\alpha + \beta = 1$, we are back in the standard Solow model where $g(k) = g(A)$. Hint: What is the growth rate of $A_t^{\beta} k_t^{\alpha-1} m_t^{1-\alpha-\beta}$?

3. [7 points] The balanced growth rate of k may be negative. How is this possible, given that productivity grows at a positive rate?

4. [7 points] Holding $\beta/1-\alpha$ fixed at 1, how does faster population growth affect the balanced growth rate of capital? What is the economic intuition for this result?

3 Romer Model with Obsolescence (37 points)

Consider a Romer model where ideas become obsolete over time. The rate at which this happens depends on the rate of innovation. Specifically:

$$\dot{A}_t = B\left(s_A L_t\right)^{\lambda} A_t^{\phi} - \delta\left(g\left(A_t\right)\right) A_t \tag{3}$$

where $\lambda > 0$, $\phi < 1$, $B > 0$, and $0 < s_A < 1$ are parameters. The new part is the rate of obsolescence $\delta\left(g\left(A\right)\right)$, which is an increasing function of the growth rate of ideas.

Questions:

1. [8 points] Derive the balanced growth rate of ideas. If you get stuck here, assume for the remainder of the question that $g_{ss}\left(A\right) = \frac{\lambda n}{1-\phi}$.

2. [7 points] From here on assume that $\lambda = 1$ and $\phi = 0$. Derive the balanced growth **level** of A/L. Hint: You know $g\left(A\right)$ from the previous answer.

3. [13 points] For simplicity assume that $\delta\left(g\left(A\right)\right) = \bar{\delta} \times g\left(A\right)$ for some constant $\bar{\delta} > 0$. Graph how $g\left(A\right)$ depends on A/L. Explain your graph. What does it say about the stability of the balanced growth path?

4. [9 points] Based on this graph, can you explain the odd result that $\bar{\delta}$ affects the balanced growth level of A/L, but not the balanced growth rate?

End of exam.

4 Answers

4.1 Answer: Short questions

1. The difference is rivalry. Both types of capital are accumulated subject to diminishing returns. Without some other force to counteract diminishing returns, the marginal product of capital goes to zero as capital is accumulated. Eventually, the MPK is too small to support growth. Nonrival knowledge capital introduces increasing returns. Recall the example of a drug manufacturer. The increasing returns offset the diminishing returns that set in as knowledge is accumulated.

2. Reversal of fortunes: Among colonies, we see a negative relationship between incomes in 1500 and incomes and institutions today. The argument: In low income colonies, population density was low. The only profitable strategy was to settle. Thus, institutions were put in place that protect the rule of law and individual rights. In high income colonies, slavery was the most profitable strategy. Hence, dictatorial institutions were put in place.

3. Income inequality and top tax rates:

   (a) The general point: correlation is not causation. What could go wrong: Countries that do not like high inequality could adopt many policies aimed at reducing inequality (schooling, transfers, etc). Think Sweden and Finland.

   (b) Using differences over time addresses the point made in (a). It is still possible that countries experience time-varying "shocks" that affect inequality. It is less clear why countries with large shocks of this kind would choose large reductions in tax rates. A more plausible concern is: Perhaps countries that reduced tax rates a lot also phased out other policies that reduce inequality.

4.2 Answer: Solow Model with Land

All of this exactly follows the steps of the standard Solow model. The intuition is the same as for the model with fixed resource inputs. In fact, this is a special case of the fixed resource model where the resource input is constant over time.

1. $g(k) = A_t^{\beta} k_t^{\alpha-1} m_t^{1-\alpha-\beta} - c_t/k_t - n - \delta$. Balanced growth requires constant $g(k)$. This can only happen if the term $A_t^{\beta} k_t^{\alpha-1} m_t^{1-\alpha-\beta}$ is constant over time.

2. Take the growth rate of $A_t^{\beta} k_t^{\alpha-1} m_t^{1-\alpha-\beta}$ and set it to zero. Solve for $g(k)$.

3. The answer is the same as in the model with non-renewable resources. Population growth implies that m is falling over time. This operates like negative productivity growth.

4. If the share of land increases, $g(k)$ falls. This happens because the fixed input is now "more important" in production. A given reduction in m reduces output by more. It requires a larger increase in A to offset it.

4.3 Answer: Romer model with obsolescence

1. The argument is the same as in the standard Romer model.

2. Start from
$$g(A) = Bs_A L/A - \delta(g(A)) \tag{4}$$
On the BGP, $g(A) = g_{ss} = \lambda n/(1-\phi)$. Then we have
$$(A/L)_{ss} = \frac{Bs_A}{g_{ss} + \delta(g_{ss})} \tag{5}$$

3. Now we have
$$g(A) = Bs_A L/A - \bar{\delta} g(A) \tag{6}$$
or
$$g(A) = \frac{Bs_A L/A}{1+\bar{\delta}} \tag{7}$$
We get the usual graph, except that a higher $\bar{\delta}$ shifts the $g(A)$ curve down.

4. Remember how the balanced growth rate comes about in the model without obsolescence. Without population growth, diminishing returns lead growth to peter out (moving along the $g(A)$ curve). Population growth shifts the curve up at a rate determined by n. This offsets the diminishing returns (scale effects) and growth is sustained at a rate that also depends on n.

   The same happens here. A higher $\bar{\delta}$ simply slows growth by a fixed amount for any given A/L. To offset this, the economy must lower the level of A/L (less diminishing returns to A). The growth rate is still determined by the rate at which the $g(A)$ curve shifts out.

End of answers.